Short-term generation scheduling in a hydrothermal power system.

Xiong, Min

SHORT-TERM GENERATION SCHEDULING

IN

A HYDROTHERMAL POWER SYSTEM

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MIN XIONG, BSc

University of Durham

School of Engineering

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To my Parents

ABSTRACT

This thesis is concerned with the theoretical development of efficient optimisation techniques and computer program algorithms that are practically applicable to short-term generation scheduling in large scale mixed hydrothermal power systems. It contains the intensive results of the research project obtained on application of various mathematical programming methods to the short-term hydrothermal generation scheduling problem. All the work was carried out at the University of Durham.

The algorithms developed are used in operation planning process for determining the loading schedules of all the generating units over a daily and weekly period in a hydrothermal power system to meet the forecasted load demand while maintaining the minimum production and operating cost. The scheduling problem is modelled in great detail. Cascaded reservoirs in a multi-river hydro subsystem are modelled with its operating constraints. The entire hydrothermal generation scheduling is formulated as a large-scale, dynamic, mixed-integer programming problem. By applying the solution methodology of Lagrangian relaxation based on mathematical decomposition techniques, the hydrothermal scheduling problem can be decomposed into hydro and thermal subproblems. This makes it possible to exploit the special features of the subproblems and to solve the entire optimisation problem very efficiently.

For thermal subproblems, dynamic programming and merit-order schemes were proposed. To exploit the network structure of the hydro subproblems, the network flow concept was used. The simplex method on a graph, a minimal cost out-of-kilter network flow algorithm and a sparse dual revised simplex method were applied. To take into account the small nonlinearity in the hydro subproblems, the Frank-Wolfe method was used. The Lagrangian relaxation technique was also used to solve the purely thermal unit commitment problem

and generation scheduling in a purely hydroelectric power system. Above all, several efficient solution procedures were developed for the coordination of the master problem, these algorithms ensure the feasible and near-optimal solution of hydrothermal generation schedules.

The application of these algorithms to some hypothetical test systems and a case study with some data from Swedish power systems has been described. Typical results for these test systems are presented and the computational requirements discussed. Comparisons between the different algorithms are also given together with the suggestions for future work.

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STATEMENT OF COPYRIGHT

DECLARATION

The work contained in this thesis has not been submitted elsewhere for any other degree or qualification and that unless otherwise referenced it is the author's own work.

LIST OF DIAGRAMS

LIST OF FIGURES

LIST OF SYMBOLS

$P_H(j,k)$ . . . . the power output for unit j at time k in MW
$H(j,k)$ . . . . the effective head for unit j at time k in meters
$Q(j,k)$ . . . . the discharge rates for unit j at time k in (m^3/s)
K . . . . the total scheduling period in hours
J . . . . the total number of hydroelectric generating units
η_{Tj} . . . . the turbine efficiency in $MW * s/m^3$
η_{Gj} . . . . the generator efficiency in $MW * s/m^3$
$V(j,k)$ . . . . . the reservoir storage volume for unit j at time k in m^3
η_{jn} . . . . the unit generating efficiency in $MW * s/m^3$
$Q_n(j,k)$ . . . . the unit discharge rate at time k in m^3/s
$INF(j,k)$ . . . . . . the natural inflow into reservoir j at time k in m^3/s
$S(j,k)$ . . . . the spillage of reservoir j at time k in m^3/s
$\sum_i^M Q(i,k-t_{ij})$ the discharge of the j's upstream reservoirs at time k in m^3/s
$\sum_i^M S(i,k-t_{ij})$ the spillage of the j's upstream reservoirs at time k in m^3/s
t_{ij} the water transport delay from upstream reservoir to reservoir j in hours
$i \in M$, M . . . . the total upstream reservoirs of reservoir j
$j \in J$ . . . . the hydro unit number
$k \in K$ . . . . the time interval number
$V_{min}(j,k)$ . . . . the minimum reservoir storage of unit j at time k (m^3)
$V_{max}(j,k)$ . . . . the maximum reservoir storage of unit j at time k (m^3)
$Q_{min}(j,k)$ . . . . the minimum discharge rate of unit j at time k (m^3/s)
$Q_{max}(j,k)$ . . . . the maximum discharge rate of unit j at time k (m^3/s)
$S_{min}(j,k)$ . . . . the minimum spillage of unit j at time k (m^3/s)
$S_{max}(j,k)$ . . . . the maximum spillage of unit j at time k (m^3/s)
$P_T(i,k)$ . . . . . . the power output of thermal unit i at time k in P.U.
P_{imin} . . . . . the minimum power output for unit i at time k in P.U.
P_{imax} . . . . . the maximum power output for unit i at time k in P.U.
P_{iramp} . . . . the ramping rate of unit i in P.U. per hour
$P_D(k)$ . . . . the total electricity demand for time k in P.U.

$P_R(k)$ . . . . the total electricity reserve for time k in P.U.
T_{minup} . . . . the minimum up time for unit i in hours
$T_{mindown}$ . . . . the minimum down time for unit i in hours
$F_i(P_T(i,k))$ . . . . the fuel cost of thermal unit i at $P_T(i,k)$ (\$)
A_i, B_i, C_i . the coefficients of the fuel cost function for unit i (\$ per P.U.)
$X(i,k)$ . . . . the state variable for unit i at time k
$U(i,k)$ . . . the startup or shutdown decision variable for unit i at time k
$X_s(i,k)$ . . . . the state of unit i at time k
$ST_i(X(i,k), U(i,k))$ the startup and shutdown cost of unit i at time k (\$)
$C_{coldstart}(i)$ . . . . the cold-startup cost for unit i (\$)
$C_{shutdown}(i)$ . . . . the shutdown cost for unit i (\$)
$T_{down}(i)$ . . . . the shutdown time constant for unit i (hours)
$AINC$ . . . . the average incremental cost (\$ per P.U.)
$AFLC$ . . . . the average full-load cost (\$ per P.U.)
$\lambda(k)$ . . . . the Lagrangian multiplier at time k (\$/MW)
$\lambda_m(k)$ . . . . the marginal prices at time k (\$/MW)
P_{max} . . . . in P.U.
P_{min} . . . . in P.U.
$T_{startup}$ . . . . the startup time in hours
Status . . . . equivalent to $X(i,k)$
T_{change} . . . . the absolute time when a unit changes its status
I_{ramp} . . . . . . the ramping rate to increase for a unit in P.U. per hour
D_{ramp} . . . . . the ramping rate to decrease for a unit in P.U. per hour
G_{init} . . . . the initial power output of a unit which is on
$CCDP$ . . . . Composite Cost Dynamic Programming
MO . . . . Merit Order Scheme
LRD . . . . Lagrangian Relaxation Decomposition

CONTENTS

CHAPTER 1

GENERAL BACKGROUND

1.1 INTRODUCTION

This chapter describes conceptually the problem of short-term hydrothermal generation scheduling and other related areas of work. It briefly describes the problem of optimal economic operation of electric power systems and its importance in the operation and control of electric power systems. The requirement for the short-term generation scheduling in a hydrothermal electric power system is discussed. The main areas that the hydrothermal generation scheduling is related to are presented here to give a clear background knowledge of the problem. The unit commitment problem is then briefly described and the application of optimization techniques to this problem and hydrothermal generation scheduling problem is introduced.

This work is part of the Operational Control of Electric Power System (OCEPS) project which will also be briefly introduced in this chapter. The chapter therefore sets the background knowledge for the thesis and gives a brief description of the thesis layout.

1.2 INTRODUCTION TO ELECTRIC POWER SYSTEMS

As a very pure form of energy, electrical energy has gained an undoubted importance in the current century and is used in most domestic and industrial areas. The energy needs in our modern society are now supplied mainly in the form of electrical energy as it is preferred, in most applications, to other types of energy. More and more very complex power systems have been built to meet the ever increasing electrical energy requirement.

The objective of power system operation and control is to maintain a continuous balance between electricity generation and a constantly varying electricity demand while maintaining system frequency, voltage levels and security, and this must be achieved at a minimum production cost. It is the role of so-called energy management systems (EMS) to realise these principal objectives. These requirements can be stated more fully as follows:

- The continuity of the electricity supply (security).
- The quality or the constancy of the electricity supply.
- The optimality of the economic operation of electric power systems at all times provided the above two constraints are met.

The quality of the electricity supply implies that any variation of voltage and frequency of electric power systems should be maintained within a sufficiently small and acceptable margin.

The optimal economic operation of electric power systems must be carried out mainly for the benefit of power industries and is aimed to generate the electrical power to meet the load demand while achieving a minimum production cost.

A further difficulty arises out of the variable nature of the consumer power demand (load) coupled with the non-storable nature of electrical energy. This calls for continuous adjustment of electricity production, but in such a manner that the demand is met within the three constraints outlined above.

To conclude, in order to operate power systems in the best way, the fundamental problem that power system operation engineers always face is to ensure that the consumer's demand is met at the lowest possible cost compatible with adequate continuity in supply and sufficiently small frequency and voltage deviation in power systems. More recently, an additional constraint has been applied, namely, lower sufficiently the impact on the environment.

As modern electrical power systems have become more and more complicated and more heavily interconnected, the monitoring and control of an electrical power system has necessitated extremely sophisticated solutions. To ensure the optimality, security and reliability of electrical power generation, transmission and distribution systems, the appropriate mathematical modelling and powerful computer hardware are required, and most important of all, an integrated approach to software implementation is demanded. As a result, the on-line computer-assisted control of large scale electrical power systems and networks represents a significant real time processing problem in the information technology area.

The availability of not too expensive computer hardware and the ever-growing importance of energy management systems has resulted in a widespread adoption and development of sophisticated monitoring and control systems throughout the power utilities. A research programme has been developed over a long period of 15 years by the research group in the University of Durham. This research has led to the production of a comprehensive software package, termed OCEPS software suite, for on-line analysis, simulation, operation and control of large scale generation, transmission and distribution systems.

The main objective of this software is to satisfy the three above mentioned requirements but it also can serve as an operator training vehicle or simulation facility.

Following many years of research experience on the implementation and development of the programs for individual elements of the control problem in power systems operation, it became apparent that some of the aspects of control function interaction could not be studied using individual control function modules. As a consequence, all the aspects of the simulation, monitoring and control functions are now coordinated in a real time control package by passing data between the modules or, wherever necessary, between computers. To verify the performance of this software implementation, the real time dynamic simulation of a typical power system including generators, other plant and a transmission network is used as a realistic testbed to analyze the performance of the control functions. The real time approach has numerous advantages since this integrated real time approach has enabled the development and the analysis of the monitoring and control algorithms in a realistic environment and has lead to a software package with high computational efficiency. The major software modules include the following:

- Power system simulation
- Topology determination and data validation
- Observability determination
- State estimation
- Load prediction (forecasting)
- Load flow studies
- Unit commitment
- Economic dispatch
- Automatic generation control (Load frequency control)
- Security assessment
- Emergency rescheduling
- Load shedding
- Fault studies

The OCEPS software development and hardware configuration are represented in Diagram 1.1-4. Diagram 1.1 shows the major functional elements of the OCEPS suite in detail both on simulation, analysis and control, whereas a simplified overall scheme is illustrated in Diagram 1.2. The hardware configuration on which the OCEPS programs reside is shown in Diagram 1.3, and the time scale of the functional elements in the real time operation and control of power systems is illustrated in Diagram 1.4.

The particular area investigated in this project is **short-term generation scheduling in hydrothermal power systems**. In the OCEPS scheme, the hydrothermal generation scheduling belongs to the area of generation control and load shedding subsystems as illustrated in Diagram 1.5, and the project is one of the topics of operational planning of power systems. At the start of this work, tools already existed in OCEPS for thermal unit commitment,

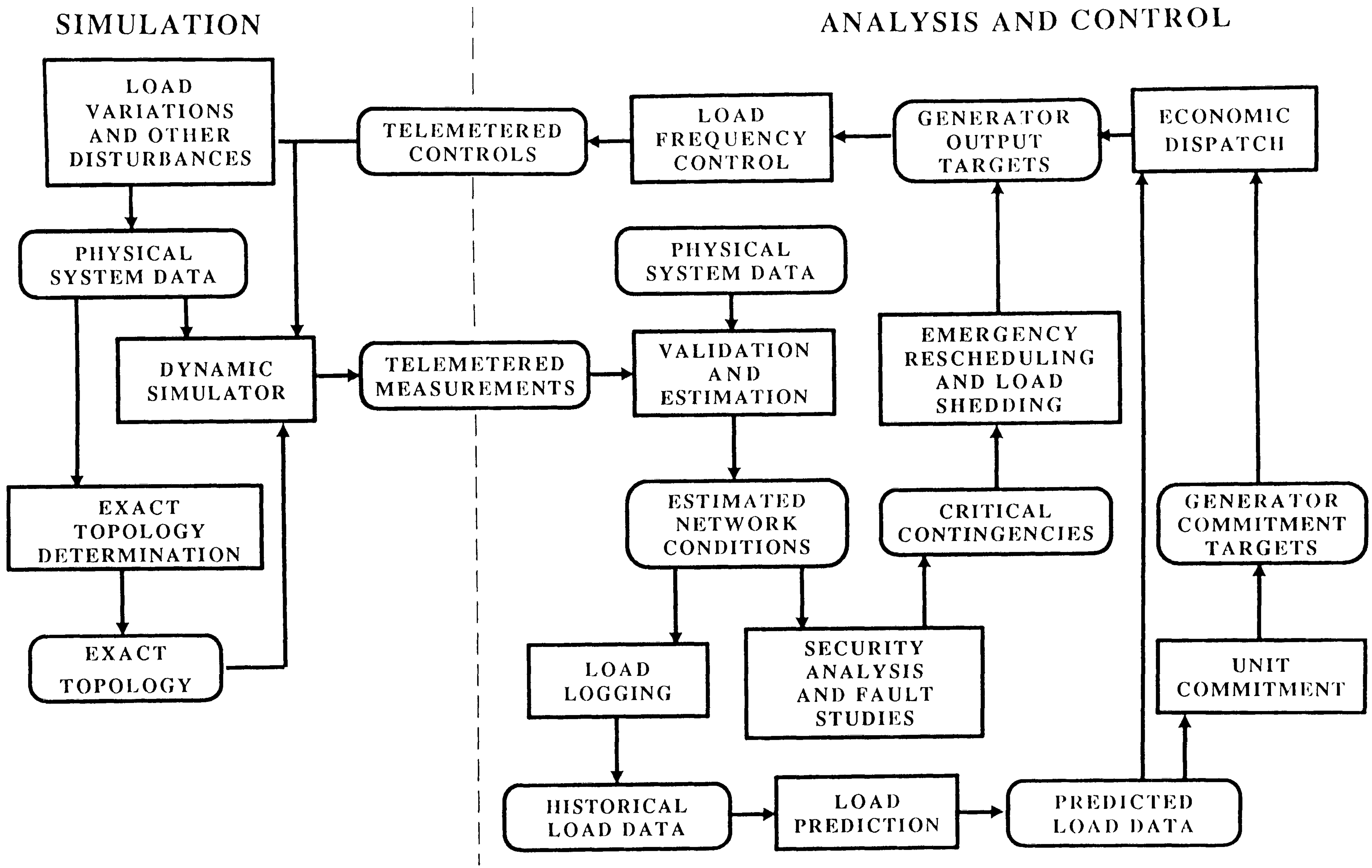


Diagram 1.1 Operational Control of Electrical Power Systems
Overview of Major Functional Elements

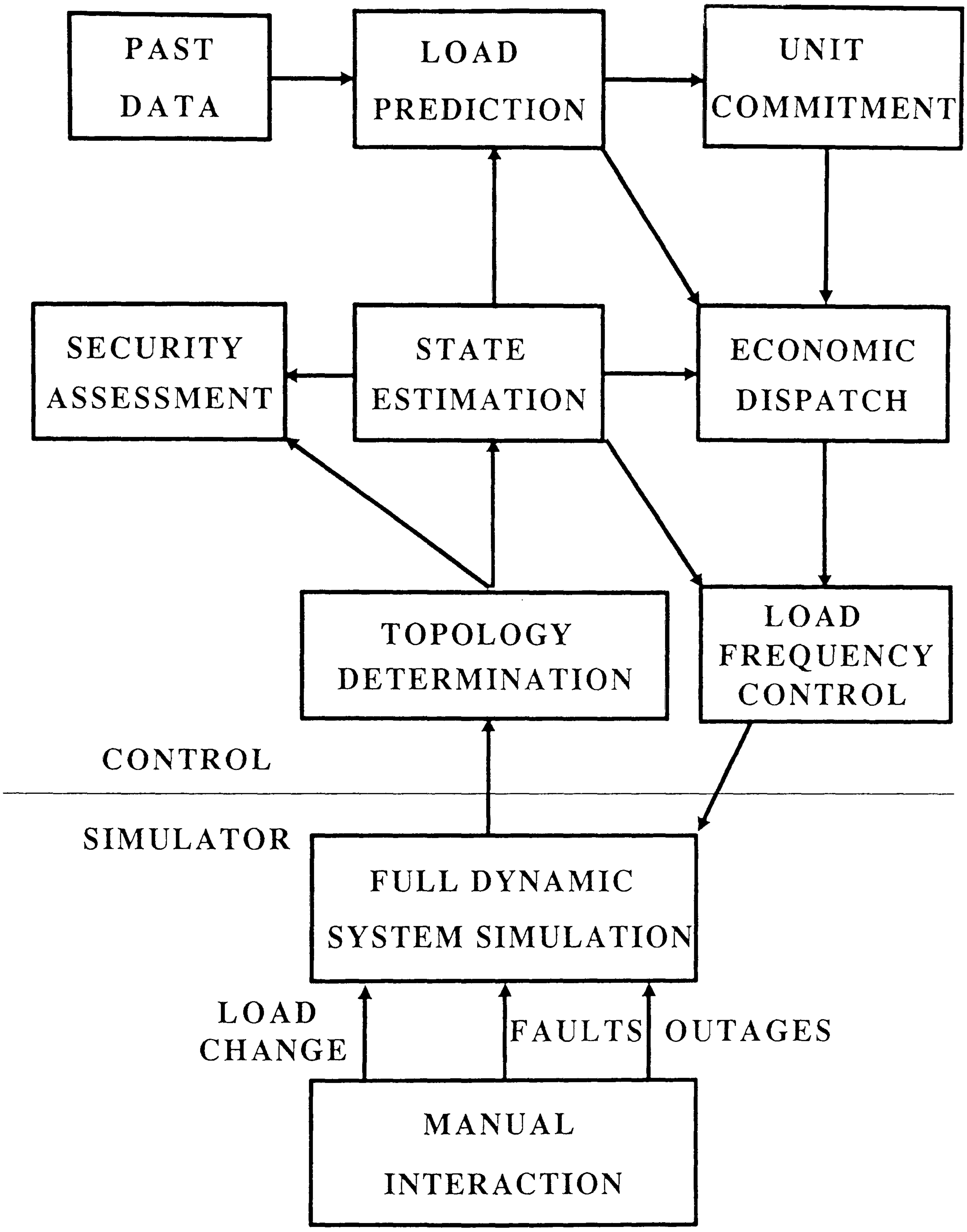


Diagram 1.2 OCEPS- Overall Scheme

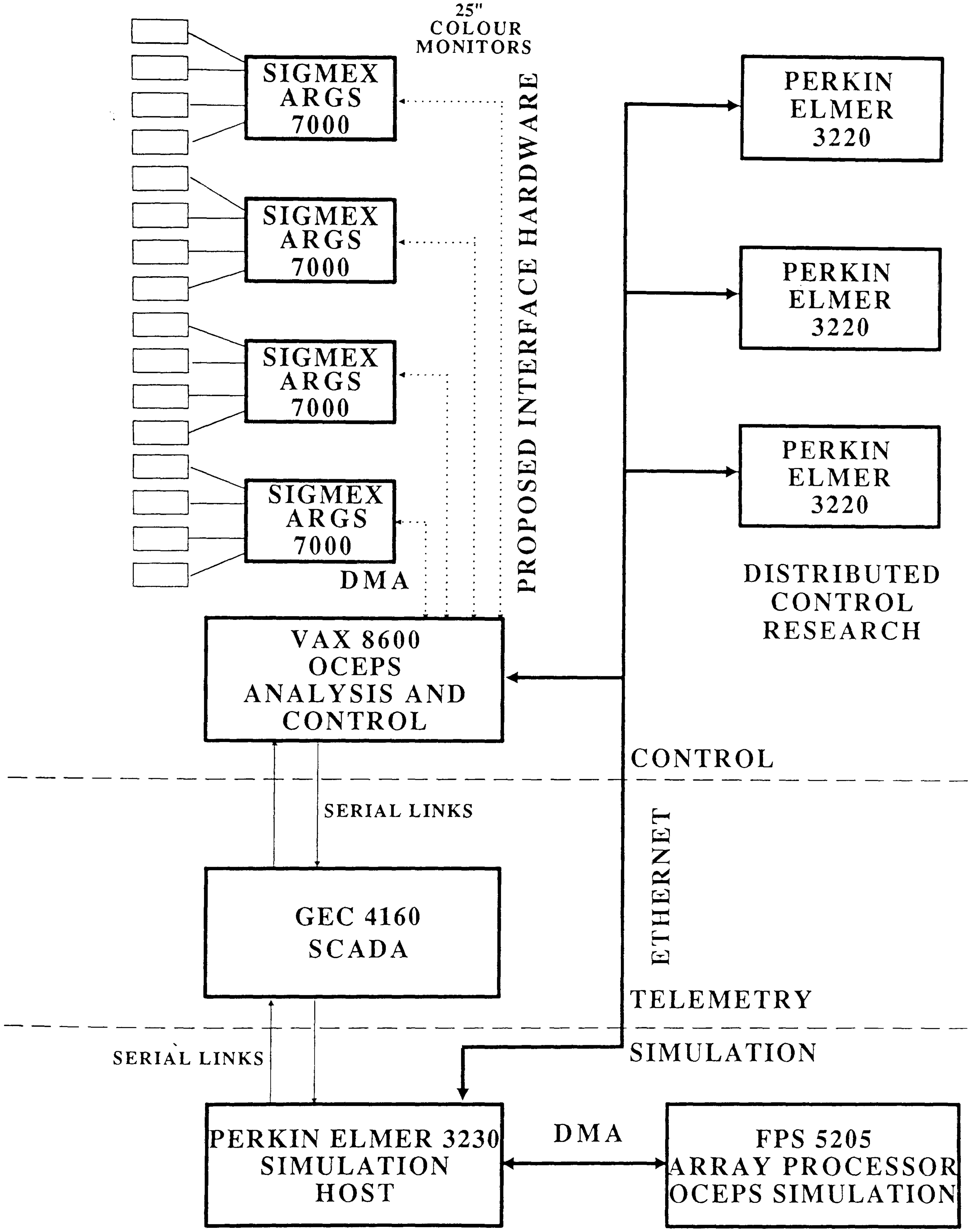


Diagram 1.3 OCEPS Hardware Configuration

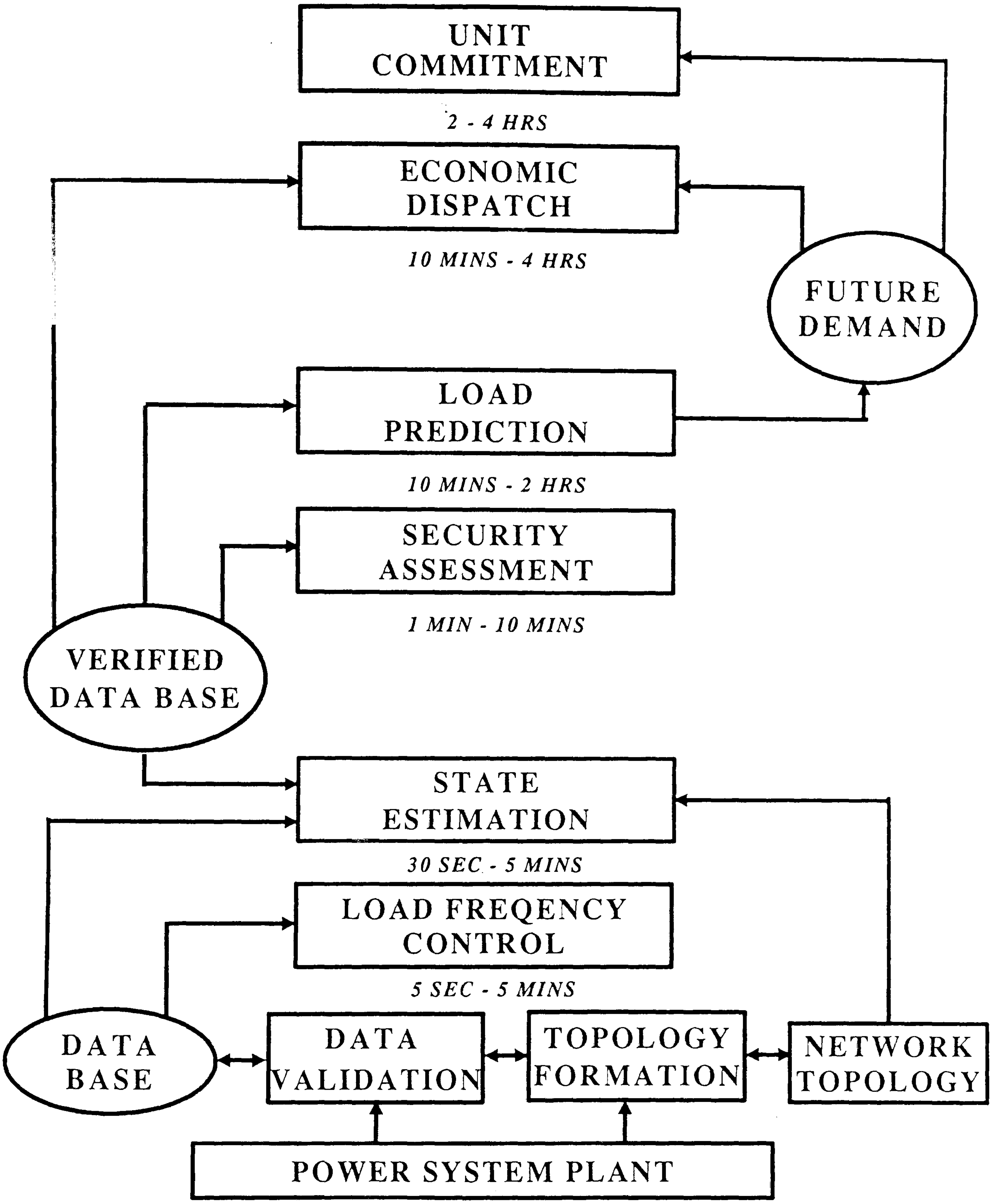


Diagram 1.4 Time Scale for the Operation of Functional Elements of Power System Control

economic dispatch and other optimal operational functions but the algorithms for hydrothermal unit commitment had not been previously implemented.

1.3 OPTIMAL OPERATION OF A POWER SYSTEM

The main objective, as mentioned above, during the operation of any electrical power system, is to supply the electrical energy to the consumers with maximum safety, maximum continuity and minimum production cost. The safety aspect is largely a matter of the electrical equipment design, supplemented by relay protection systems and electrical and mechanical interlocking arrangements, and is little affected by operational considerations. Maximum continuity and minimum production cost, however, are largely contradictory requirements. Maximum continuity implies having more generation and transmission capacity available than is required by system demand to ensure the "security" of the system, thereby preventing the collapse of the system due to sudden unforeseen events; whilst minimum production cost implies operating with the minimum amount of generating capacity and units as possible. The capital cost of the power plant has, of course, already been incurred during the construction and will not really affect the operation of power systems, as a result, the short-term future operational requirement is considered to involve only the operational production cost. Thus the economic generation schedule of units must be decided and this result serves as an auxiliary tool for the "real time" operation and control of electric power systems. This is exactly the task of the optimal operational planning and real time economic dispatch, i.e. the optimal operation of electrical power systems.

The complex problem of the overall control in which the maintenance of electricity supply quantity and quality is of more importance than economic factors to ensure the security and quality of power systems, has resulted in a division of the research into two main areas: network control and operational economics. Network control includes transmission switching, voltage, power factor, frequency and individual power plant control and is of prime importance over operational economics for without reliability in this field, the consideration of operational economics is of no value at all. However, once the requirement of

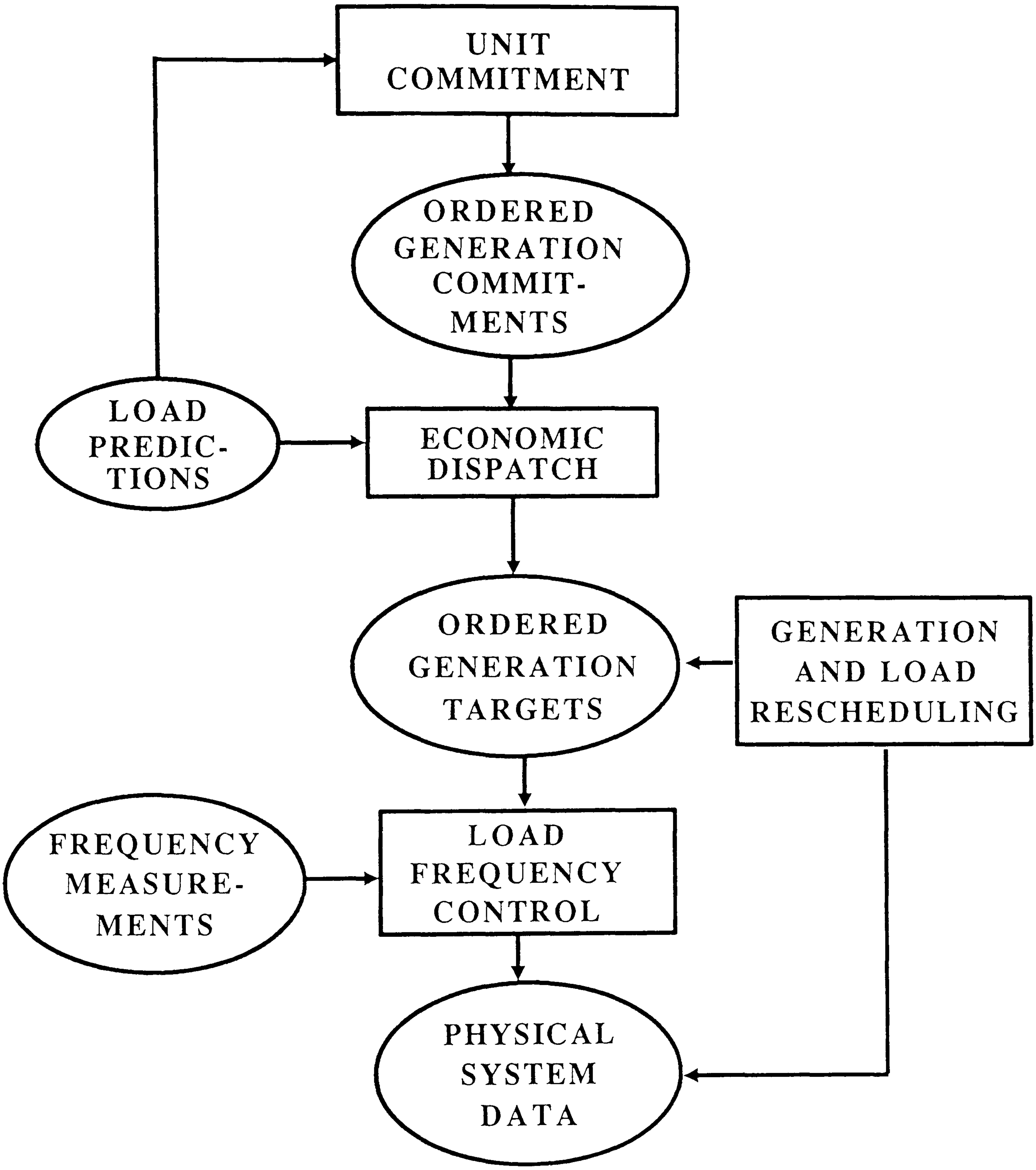


Diagram 1.5 Generation Control and

Load Shedding Subsystems

reliable system operation is satisfied and provided the power system has surplus generation and transmission capacity, there are large financial benefits which may be achieved from the economic allocation of the load to each power plant and generating unit and this can be achieved through the economic operation of the power system.

The problem of how to achieve the efficient and optimum economic operation of electric power systems has always occupied a very important position in an electrical power industry. Nowadays, more and more very complex power systems have been set up to meet the ever-growing electrical energy requirement, the increasing importance of the energy and the magnitude of the expenditure associated with the construction and operation of the power systems have created a very urgent necessity to operate the electrical energy systems in an optimal economic manner. The greater the capacity of the power system, the greater the potential for optimal economic operation. The ever-increasing size of power systems make it clear that a saving of a small percentage in the operation of a large sized system represents a significant reduction both in the operating cost as well as in the quantities of the fuel consumed, also the complexity and interconnectedness of power systems make it ever more necessary to have an accurate and efficient optimal operation solution.

1.4 OVERVIEW OF THE OPTIMIZATION METHODS

Optimization belongs to the branch of the applied mathematics. The research on the application of optimization initially began as early as in the eighteenth century, but it is only until 1940's, during the World War II, that with the development on digital computers, advanced optimization methods began to flourish. The simplex method by G.B.Dantzig in 1951 marks the beginning of numerous publications on optimization methods.

The economic operation and planning of electric power systems is one of the important application fields for optimization methods. The complex optimization problems associated with the operation and control of power systems have been the subject of considerable research for many years. Many mathematical programming techniques have been applied to the solution of operational planning problems in power systems, especially since the advent of high-speed digital computers. Recently, the rapid development in applied mathematical methods and the availability of high computational capacity and speed have made it possible to solve large scale optimization problems efficiently. Most of the complex optimization problems associated with the economic operation of electric power systems have been solved successfully. The solution for a large sized power system optimization problem becomes practical and "feasible". The methods employed in the field will be reviewed in the following chapters with respect to the particular application area.

1.5 TOPICS OF OPTIMAL ECONOMIC OPERATIONS

One of the most important aspects in optimal economic operation of power systems is the subject of selection of the combination of all the generating units to be committed for running or decommited for shutting down (or banking), and at the same time, to meet the load demand request, reserve requirement and other security constraints. The loading level of these committed units must then be scheduled to give a minimum total production cost subject to the physical and operating constraints imposed by the units and the system while satisfying the load demand level. This is known as the unit commitment and economic dispatch problem or in another words, generation scheduling.

For a particular load and a set of network conditions, an optimal combination of generators can be determined by examining the difference in their operational characteristics. Load variations consequently necessitate the calculation of new optimum generation dispatches. Since the load pattern displays regular daily and weekly cycles, demand forecasting can be very effective in predicting the new day's load curve. This can enable the approximate estimation of the required future generation schedule. A subdivision within the allocation problem is also necessary. The unit commitment must be made at least several hours in advance, in order to allow the sufficient time for plant to be run up and synchronized prior to being brought on-line. When a more accurate estimate of the immediate load demand is available, the generator output level must be decided. This allocation of committed generation is the task of economic dispatch program which will calculate each generator output so that all the operating constraints will be satisfied and the total production cost minimized.

Both the unit commitment and economic dispatch functions are subject to many operating constraints and have a common aim, i.e. to minimize the total production cost. Ideally, these two functions can be incorporated into the advanced computer control schemes as shown in Diagram 1.1-4 of the OCEPS project, but often they can be manually controlled in real time power system control schemes. The process may begin through the result of load prediction, if a large variation of the consumer load demand is detected, the unit commitment program is started to decide the startup and shutdown of the units to achieve a minimum production cost, while the economic dispatch decides the loading levels of available committed units to minimize the total operating cost of generation. The time scale of these optimization functions is implemented depending largely on optimization techniques applied for the allocation of unit power outputs, it can permit the separate consideration of these two problems. As can be seen from Diagram 1.4, the plant ordering or unit commitment schedule is decided typically 2 to 4 hours in advance, whereas the plant loading and dispatch decision is made normally as frequently as possible typically in every 5-10 to 30 minutes.

The relative importance of unit commitment and economic dispatch in the minimization of the total production cost depends, to a large extent, on the plant characteristics. In the case of multivalve prime-movers, the operation of the plant in a discontinuous mode may be avoided or reduced by variation of the loading on individual sets to correspond to the daily load curve thus reducing the unit startups-and-shutdowns. If the nonlinear generator fuel cost function is considered, this will result in economic partial-load operation, and consequently the control of startup and shutdown of the plant will be more crucial in daily operation.

A further complication in obtaining an optimal solution to the unit commitment problem is the requirement in a hydrothermal power system to coordinate the operation of hydro power stations with the operation of thermal generation system in order to achieve an overall optimal solution. In this context, the generation scheduling problem is either termed the hydrothermal unit commitment, hydrothermal generation scheduling or hydrothermal scheduling in short.

The hydrothermal scheduling problem is very different from the purely thermal power generation scheduling as it actually involves the planning of the usage of a limited resource (i.e. the water resource) that has a negligible operating cost over a specified optimization period. In the case of short-term operational planning, the scheduling period can be as long as a week or a day. The limited resource is the water available in reservoirs for hydro generation. The objective of short-term hydrothermal generation scheduling is to allocate the available hydraulic, thermal and pumped-storage resources (if any are available) of a hydro-thermal power system to the various time intervals of the period under consideration to meet the load demand so that the total system production cost (mainly from thermal) is minimized under the restriction of many constraints, representing reliability, environmental and other system requirements. Similar to any other economic operation problem in power systems, hydrothermal generation scheduling is always a highly constrained optimization problem that needs to incorporate the system operational limitations to ensure the security of the system.

In the following sections, the topics that are related to short-term generation scheduling problems will be discussed in more detail.

1.6 THE ENERGY RESOURCES MIX

There are several main energy resources exploited for the generation in electrical power systems, these resources can be classified broadly into renewable and nonrenewable resources. Hydrocarbon fossil fuels such as coal, oil, natural gas, and nuclear fuel are nonrenewable resources, while the most widely used renewable resource is hydraulic power. Other renewables include wind power, tidal power, solar power, etc. The former resources are used in thermal stations for electrical power generation, and hydraulic power is used in hydroelectric stations for the same purpose.

Each means of generation has its own advantages and disadvantages in the operation and control of power systems, also its own specific technical characteristics and operating constraints. With respect to the types of energy resources involved in an electrical power system, different mathematical programming approaches may be applied to tackle the economic operation problems involved in order to take care of various types of operating constraints and operating policies for the specified type of power systems.

The criterion for classification of different types of power systems is the generation resource mix that constitutes the whole generation system, mainly hydro resources and thermal resources. By evaluating the proportion of energy capacity from the hydro subsystem and the thermal subsystem, the power systems can be classified accordingly into the following types:

(1.) purely thermal power systems
(2.) purely hydroelectric power systems
(3.) hydrothermal power systems with a low proportion of hydroelectric capacity
(4.) hydrothermal power systems with a high proportion of hydroelectric capacity

Depending on the construction of different natural resources, the power systems throughout the world will belong to one of the above types. For example, Brazilian power systems and Norwegian power systems belong to the second type, while U.K. and other countries that are rich in fossil fuels contain nearly all thermal power stations, probably with only a few pumped-storage power stations, thus their power systems are in the first category. Power systems in France, Chinese regional power systems and power systems in many other countries will be the third type whereas the Swedish state power system is a good example of the fourth type of system.

1.7 LOAD DEMAND CHARACTERISTICS

Human activity follows cycles, so most of the service supplying systems, such as the water and electricity supply systems to serve a large population will experience cycles set by the human activity. As the electrical power supply system is not an exception, the total load demand in the system will vary quite a lot even during a day. The electricity demand is generally higher during the daytime and early in the evening because of heavy industrial requirement and lighting load; while during the late evening and early morning when the majority customers are asleep, the demand is obviously lower. The load demand also follows a weekly cycle, for people always enjoy their weekend offwork, the weekend load demand is usually lower than the weekday demand at the same hour. A load prediction profile over two days measured every half hour is shown in Figure 1.1 to illustrate the load demand cycle.

Due to the well-known non-storable nature of electrical energy, in order to keep the power and generation balance in the absence of bulk electrical energy storage facilities, it is necessary to have a permanent adjustment of generation production in response to load consumption. Since consumption varies constantly throughout the day, any electrical supply utility should forecast the load variations constantly and construct the correct load demand curve. As the optimum combination of units in operation may alter due to the load changes during any period of the day or a week, the utility must draw up an optimal schedule for its production facilities each day for the operation and control of the next day.

All these common considerations lead to the processing functions of optimal operation of power systems. This is an essential stage of the preparation of "real time" operation and control of the power systems. Among all these functions, one of the most important and difficult processing functions is the determination of unit commitment schedule.

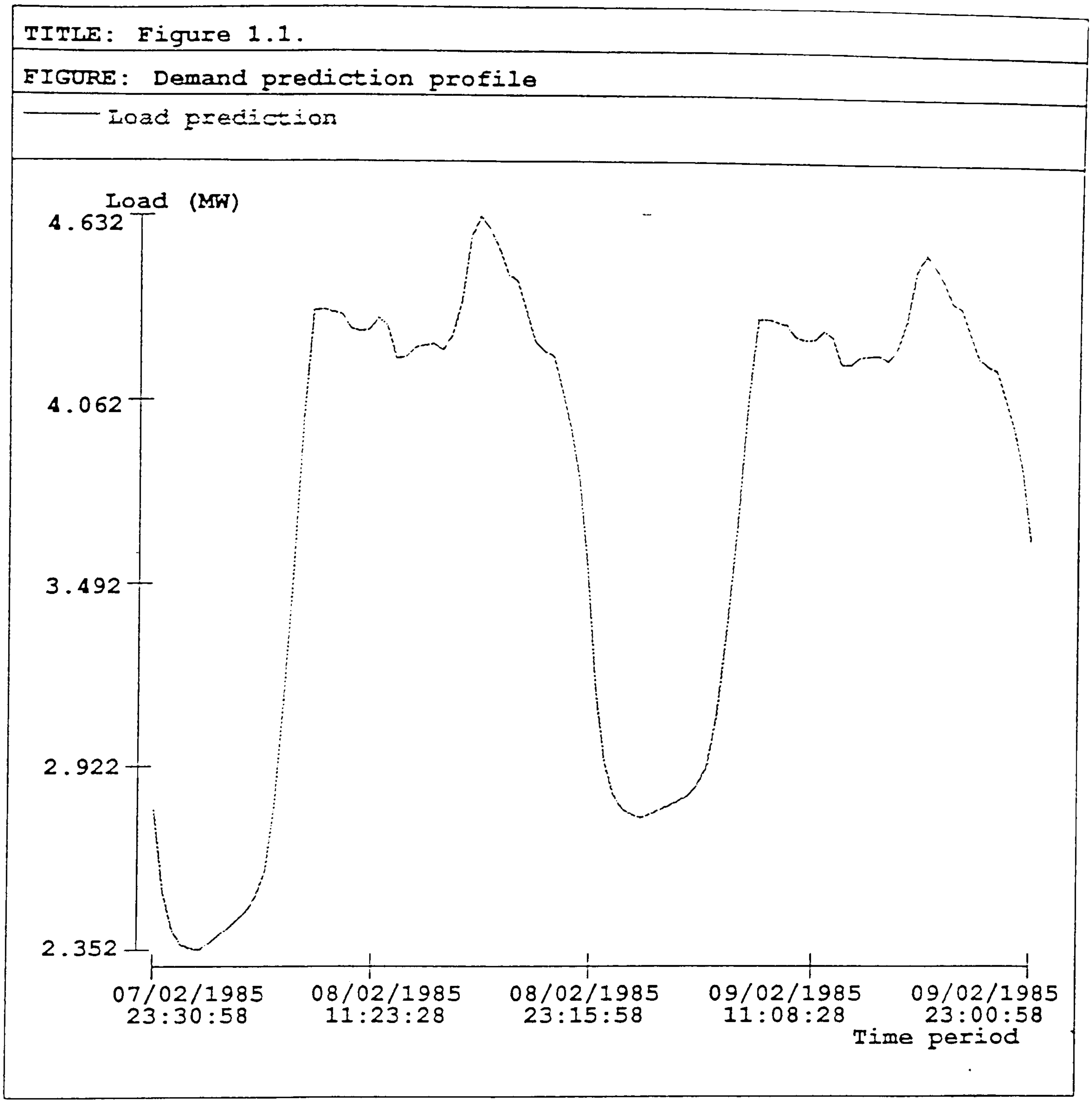

TITLE: Figure 1.1.
FIGURE: Demand prediction profile
Load prediction
Load (MW)
4.632
4.062
3.492
2.922
2.352
07/02/1985 23:30:58
08/02/1985 11:23:28
08/02/1985 23:15:58
09/02/1985 11:08:28
09/02/1985 23:00:58
Time period

Figure 1.1

1.8 UNIT COMMITMENT

"Commitment" of a generating unit is to "order the unit to run" i.e. to bring the unit up to its speed, synchronize it to the system and connect it to the electrical network so that it can generate electrical power.

The daily load demand pattern of an electric power system may exhibit a large difference between the minimum and maximum demand despite tariff adjustment which attempts to produce a more uniform profile, [183.] this results in a problem in the economic operation of electric power systems of "how to commit enough units and leave them on line". If we simply commit enough units to cover the maximum system load such that sufficient generation is ensured to safely meet the peak demand, and leave all the units running all day through, consequently, it will be very expensive for the generating system since the demand is only likely to be near the peak value for several hours, some units would be operating near their minimum generating limits during the offpeak periods, financial benefits may probably be achieved by decommiting them when it is not necessary to keep them on.

Given the operating costs of all generating units available it should be possible to allocate the available resources to satisfy the load such that for a particular load demand, the minimum total production cost is obtained while the peak demand will be safely satisfied by synchronizing enough generation prior to the occurrence of the load. The problem is therefore to decide which units, if any, can be decommited from service to achieve the maximum economy. So the task of the unit commitment program is to achieve this maximum economy as far as possible.

The task of a unit commitment program in a electric power system is to select appropriate generators to meet the forecasted load demand at various times during a period of one day or a week. The startup and shutdown schedule of units must be chosen so that during this period, the total operating cost for the generating system will be minimized.

Usually, the unit commitment decision indicates which generating units are going to be in use at each point in time. Since load varies continuously, the optimum combination of units may alter during any period, but in practice, one hour or half hour is the smallest time interval that needs to be considered as the startup and shutdown time of many units is of this order. While the economic dispatch decision indicates the allocation of system load among the generating units committed into operation at any point in time.

To solve the unit commitment problem, generally both the 'unit commitment' decision that decides the startup and shutdown schedule and the 'economic dispatch' decision which determines the loading level of committed units must be considered simultaneously in order to achieve the overall least cost schedule over the scheduling horizon. The nature of the power generation scheduling problem implies that the simultaneous consideration of unit commitment and economic dispatch decisions is necessary in order to achieve a minimum cost optimal solution.

A more detailed model for unit commitment and a review of the solution techniques will be presented in the later chapters.

1.9 ECONOMIC DISPATCH

Economic dispatch ranks in a very high position among the major economy-security functions of the operation of power systems. This function is concerned with the distribution of total generation requirement among all alternative sources for optimal system economy over a specified time interval while minimising system transmission losses or generating fuel costs within voltage, generation and transmission line constraints. The constraints imposed on this problem are mainly the requirements of reliable service and physical limitations of the equipment.

The difference in time scales between the unit commitment program and the economic dispatch program enable the plant mix schedule to be regarded as fixed in the very short-term economic operation studies, thus the unit commitment program will only decide the unit "on" and 'off' schedule and a preliminary generation dispatch for each hourly interval, and the optimum generator dispatch program is only concerned with the allocation of specific plant outputs at regular intervals subject to operational constraints. After the unit commitment schedule has been made, the set of units scheduled "on-line" have to be dispatched to give an economic choice of the loading level of the units in operation. The economic dispatch program is performed to allocate and dispatch preselected "on-line" generating units to its target active output power in order to satisfy the load demand and the spinning reserve requirement at the minimum operational and production cost while remaining within the operational constraints. This optimal control problem is essentially predictive and is based on a time scale up to approximately 30 minutes.

The economic dispatch problem can be expressed as a series of discrete optimization subproblems, the restrictions on power output and on the rates of change, together with spare capacity requirements form the interaction between each dispatch. The distribution of power flows throughout the network may be computed according to full load flow equations. However, in large scale dispatch problems, it is useful to simplify the problem by neglecting the load flow constraints.

In a similar manner to unit commitment problem, there has been a large amount of research effort devoted towards the application of mathematical programming methods to the solution of the active power economic dispatch problem. As unit commitment may include the preliminary dispatch of the generation among all the committed units to satisfy the load demand during each time interval, an economic dispatch decision may be actually involved in the whole unit commitment process.

In the OCEPS project, there are already merit-ordering methods, linear programming techniques and quadratic programming methods available for the economic dispatch. Details can be found in OCEPS documentation.

1.10 SHORT-TERM HYDROTHERMAL GENERATION SCHEDULING

A set of decision-making tools are needed for operational planning over an one-day to one-week time horizon. These software tools determine, one day or a week ahead of time, the optimal operational schedule for a mixed hydrothermal power system, which may include fossil-fired thermal plants, nuclear power stations and hydroelectric power stations. This is usually referred to as daily or weekly optimal operational planning in hydrothermal power systems or short-term hydrothermal generation scheduling. The solution of this problem is the main theme of this research work.

Short-term hydrothermal generation scheduling or unit commitment decides which generating unit (hydro or thermal) should be "on" or "off" during each time interval, usually one hour, in order to meet the load demand with a minimum total production cost over a scheduling period of 24 hours or a week while satisfying the reserve requirement within an adequate margin, and other physical and operating constraints from generation units and the system such as the transmission network, the river valley system and the reservoirs.

As far as the term "generation scheduling" is concerned, it means to schedule all the generating units in each time interval specified so that the various criteria are fulfilled such as the total production cost is minimized, a secure supply of electrical energy with required quality and quantity is ensured and all the physical, operational and legal constraints are satisfied.

The process to obtain the optimal loading of the committed hydro and thermal units over the planning period is usually termed "hydrothermal coordination" or "optimal hydrothermal scheduling". In this context, the hydrothermal coordination problem is itself an immense subproblem which is involved in the entire hydrothermal unit commitment process.

Unit commitment and generation loading in a mixed hydrothermal power system or a hydro dominated power system is a much more complicated problem compared with the purely thermal unit commitment or a thermal dominated system problem. The main differences are in the following aspects:

- Hydraulic energy forms a valuable energy resource because it is effectively free of charge and thus has a negligible operating cost.

- Hydraulic energy can be stored in the reservoirs of hydroelectric power stations and the natural inflows into reservoirs can be used as well when necessary, so hydroelectric power stations can ensure higher reliability in operation. Hydraulic energy becomes more valuable since it can be used at the most opportune time. The best economic benefit of hydraulic energy is obtained if the hydro power is used to replace the most expensive high-cost thermal generation plant and the importation of expensive non-contractual energy from neighbouring systems. Selling hydro energy to neighbouring systems may also be a good alternative objective when necessary.

- Although water stored in reservoirs can be utilized flexibly as required at one interval or another, it is not possible to produce a solution for one hour without considering the effects of the whole scheduling period. This is because the operation of a hydro power station at one instant will affect the operation at later instants through the reservoir storage that is available. The hydrothermal problem therefore has a nonseparable objective function and cannot be solved step by step as a static optimization problem. Instead, it has an essentially dynamic character, and falls into the dynamic optimization category.

- Because a large number of constraints may be involved in hydrothermal scheduling, the mathematical solution for determining an economically optimized operation of a combined hydro-thermal power system is tremendously complicated and much more difficult than generation scheduling for a purely thermal power system.

- The problem of generation scheduling in a hydro-thermal power system with substantial hydro power is even more complicated when these hydro power stations are hydraulically interconnected. The crucial point in this case is how to handle the constraints due to the hydraulic interconnection of hydro stations and reservoirs.

- If there is a long distance between two hydraulically coupled reservoirs, the water travelling from the upstream reservoir will not reach the downstream one immediately, so the water transport delay must be considered in the short-term hydrothermal scheduling model. This results in more difficulties, especially in applying dynamic programming algorithms.

All the above mentioned considerations will make the hydrothermal scheduling problem very difficult to solve. The multiplicity of various existing programs seem to indicate that a general optimization technique which is applicable to all hydrothermal power systems, does not exist. The choice of the solution method is very much dependent on the system characteristics of a hydrothermal power system, its technical constraints and operating policies, such as the balance between hydro and thermal generation capacity, the size of hydro reservoirs, and so on. The hydrothermal scheduling models may be in different mathematical programming forms; accordingly, different mathematical methods for finding the optimal operation schedule have to be devised.

The factors which may be considered in the solution of the short-term hydrothermal scheduling problem are wide-ranging. There is no simple way to put all the factors together to solve the problem practically. In fact, among all the related areas, only the following most important variables and factors can be taken into account:

- The generation resources mix
- Thermal unit commitment schedule
- The load forecast result
- The water inflow forecast result
- The water level of the regulating reservoirs

- The availability of the generating units.
- The availability of the fuel for thermal power stations.
- The planned changes in the topology of the electrical network.

1.11 HYDROTHERMAL SCHEDULING STRATEGY

In order to achieve overall optimality in the operation policies of hydrothermal power systems, a systematic strategy is required to solve the entire optimal hydrothermal scheduling problem; since it is not generally possible to formulate and solve the overall problem as a single mathematical programming model. To solve the subproblem of generation scheduling of thermal and hydraulic resources, the entire optimization problem is usually decomposed sequentially into three inter-related subproblems over different time horizons: long-term operational planning with a scheduling period of 2-3 years, medium-term (seasonal) operational planning with a period of consideration of up to one year, short-term (weekly or daily) operational planning and then real time operational scheduling. Each operational planning problem in this optimization chain has a different degree of detail of system representation and each problem is coupled with the other problems. The result from the longer term studies are fed into the shorter term studies as input data or constraints in order to take into account the longer term effects on the current decisions.

Firstly, the long-term operational planning problem must be solved, it may cover a scheduling period of one year, up to over 10 years with a monthly interval depending on the time cycle of water inflows. In this type of study, the hydroelectric generation system is usually represented simply by composite reservoirs, with each valley containing a simplified composite reservoir. The stochastic nature of water inflows must be taken into account, as must the load demand uncertainty. The problem is solved commonly by stochastic dynamic programming. The result obtained is in the form of operating policies and marginal water values. The results are passed on to the medium-term planning model together with the bounding constraints on reservoir release and storage to reflect the long-term operating conditions.

The medium-term operational planning model commonly has an optimization horizon of 1 to 2 years with weekly time intervals. It aims to optimize the utilization of water stored in reservoirs that are large enough to have cycles of greater than a week (ultra-week cycles). The medium-term study will take into account the effects from the long-term study and is performed in a deterministic manner, but the random nature of the thermal unit availability must be considered, both in the long-term study and the medium-term study. This effect will be neglected only in the short-term weekly and daily scheduling models.

The short-term hydrothermal scheduling model has a time horizon of one week or one day with an hourly time interval. The formulation and solution procedure of the short-term study are similar to those in a medium-term study except that the reservoirs with cycles of less than one week (intra-week cycles) must also be taken into consideration. The short-term operational planning study will produce an optimal generation schedule of reservoir release and storage. These results can then be used as a guideline for the preparation of real-time economic operation.

As a rule, in deriving an optimal unit commitment schedule for a mixed hydrothermal power system, the hydrothermal scheduling program must analyze all the generation resources in the system, including thermal generation, hydro generation and economic purchase and sales. The optimization approach employed must globally coordinate the use of system resources (rather than optimize the use of each individual resource separately). [127.]

As hydrothermal generation scheduling must take into account the coordination effects between the hydro generation schedule and the thermal generation schedule, it is obvious that the computation storage and time for optimization of large sized power systems is often excessive unless special solution techniques are employed. There are many ways of avoiding excessive optimization time by employing some heuristic rules to reduce the optimization problem. There are also mathematical decomposition techniques that can be applied to decompose the entire problem into a series of easier-to-solve subproblems. Details will be presented in later chapters.

1.12 THESIS LAYOUT

This thesis contains eight chapters in total. The layout of the thesis is organized as follows:

Firstly, this opening chapter gives a general background view of the problems that are going to be solved in this research project. The chapter provides a brief definition of the problems and a conceptual study of all the topics that are related to short-term hydrothermal scheduling. It also outlines the position of short-term hydrothermal generation scheduling as a economic function in the whole operational planning, operation and control process for electric power systems.

The next chapter will present a literature review of the previous research work on thermal unit commitment, short-term hydroelectric scheduling and short-term hydrothermal generation scheduling and other related areas.

Chapter 3 gives a detailed historical review on mathematical decomposition techniques, also applications to the short-term hydrothermal scheduling area such as the generalized Benders' decomposition and Lagrangian relaxation decomposition.

Chapter 4 describes the problem characteristics of short-term hydrothermal scheduling and presents a mathematical formulation of the problem. A comprehensive optimization model for the problem will be derived.

Chapter 5 is primarily concerned with the thermal unit commitment problem and contains the optimization techniques employed for the solution of large scale unit commitment problem in thermal power systems. The major achievement of this part of the work is the successful application of the Lagrangian relaxation techniques and the efficient way of generating the near-optimal while primal-feasible solution for thermal unit commitment.

All above chapters can be viewed as the first part of the work of the research project.

Chapter 6 presents the optimization methods applied for the solution of hydroelectric scheduling and hydro subproblems in hydrothermal scheduling respectively, including Lagrangian relaxation, network programming, Frank-Wolfe decomposition, etc. Many test results will also be presented. This chapter can be regarded as the second part of the thesis.

Chapter 7 is a key chapter of the thesis, as in this chapter the structural mathematical decomposition and coordination approach to short-term generation scheduling in hydrothermal power systems will be discussed in great detail. Finally, Chapter 7 will also demonstrate the theoretical and practical results of the test systems for short-term hydrothermal scheduling, and comparisons among the different optimization algorithms applied will be analyzed, major improvement will be discussed.

Chapter 8 concludes all the work and provides a brief summary and suggestions for future research.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter presents a brief literature review for the economic operation of electric power systems and its importance in the operation of electric power systems. A discussion on unit commitment problems is given. The mathematical programming methods employed previously for the solution of thermal unit commitment problems are outlined, followed by a literature review on short-term generation scheduling in a hydroelectric power system and in a mixed hydrothermal electric power system respectively. A detailed historical review of previous techniques to the solution of these problems is also presented.

2.2 PREVIOUS WORK ON THERMAL UNIT COMMITMENT

Unit commitment algorithms are defined to compute the solution of unit commitment problems. Such algorithms give the optimal economic startup and shutdown schedule of generators hour by hour over the specified horizon within the technical and operational constraints to meet the required power system security and operational quality criteria.

The unit commitment problem, even in its basic form, is a complex combinatorial problem. It is necessary to consider the no-load cost, incremental, startup and shutdown cost of each generating unit and other constraints on the units and the system. The solution of this problem has been the subject of intensive efforts over the last 25 years. From the mathematical viewpoint, the problem consists of optimizing an economic criterion simultaneously involving integer variables and fixed and variable costs, under multiple constraints.

A unit commitment problem for a large, realistic sized system will have an enormous number of combinations to search, although physical and operational constraints on the units and the system reduce the number of possible combinations. Necessary considerations, other than purely combinatorial, such as nonlinear fuel cost, time-dependent start-up and shut-down costs, minimum on and off time, minimum interval between synchronization and desynchronization events in a station, spinning reserve requirements, etc, add substantial complication to the problem. Precisely because of the nonlinear and complicated nature of all these constraints, no simple mathematical programming techniques may be easily applied to the unit commitment problem. Throughout the years, the electricity industry has developed various algorithmic approaches to the solution of the problem.

To summarize, thermal unit commitment is a long-standing problem and many intensive research efforts have been devoted to this topic. The traditional mathematical formulation of unit commitment problems naturally involves numerous variables and constraints and gives rise to a large scale, dynamic and mixed-integer programming problem. To date, numerous approaches have been proposed to solve the unit commitment problem. These approaches are linked to many kinds of optimization methods, the literature reported is abundant and extensive on mathematical programming methods for unit commitment. In this chapter, some of the previous research work will be surveyed. Generally, the available approaches can be categorised into two main groups. The first group consists of heuristic methods which are the main approaches actually used in practical situations, however, they give no guarantee that the schedule produced will be optimal or even close to optimal. The second group consists of rigorous optimization techniques that are able to guarantee the optimal solution theoretically, but may be impractical to be applied to realistic sized problems. The mathematical programming techniques applied previously to the unit commitment problem may be broadly classified into the following categories:

- Variational calculus methods
- Heuristic methods (Merit-order schemes or priority list methods)

- Mixed-integer linear programming (MILP) and partial enumeration methods (or branch and bound techniques) (BBT)
- Linear programming (LP)
- Methods based on the dynamic programming principle (DP)
- Benders' partitioning or decomposition methods
- Lagrangian relaxation techniques (LR)

A close examination of the existing published work on these applications will be described in the following subsections.

2.2.1 Heuristic Methods

The heuristic algorithms[208.] are based on the trial and error approach. Certain heuristic rules may be set up based on the system characteristics. For the unit commitment problem, the heuristic rules are set according to the priority list of the generating units. The merit-order or priority list methods are based on these heuristic rules, probably combined with other mathematical programming optimization principles to obtain better solutions. The approaches usually take the assumption that a predefined priority list or a merit-order list among the generating units is available, then use this order list to determine the startup and the shutdown of units and decide the commitment of the available units while satisfying all the operating constraints.

Of all the approaches to the solution of unit commitment problems, the merit-order schemes, or generating unit priority list methods, are the most popular ones in practice. The merit-order schemes are widely used in on-line real time optimal operation for simple implementation, fast speed and efficient solution. All kinds of constraints involved can be represented easily and as far as program development is concerned, it is quite straightforward to implement these programs on the computer even though the algorithms may appear huge.

Until recently, unit commitment for large scale realistic sized systems has been solved only by heuristic algorithms. Throughout the years, the electricity industry has developed various algorithmic approaches for the unit commitment problem, but large scale realistic sized problems generally make the application of rigorous optimization techniques nearly impossible. Instead, various heuristic approaches are used in practical scheduling for decision-making so that a reasonably good and acceptable solution for unit commitment can be generated within realistic computational requirements. These algorithms are very efficient to deal with practical large scale systems, especially when the program can be terminated at any time with a reasonably good and feasible solution.

Usually, the heuristic algorithms can provide satisfactory results for unit commitment problems with soft constraints, but they can not guarantee that the least cost solution can be obtained. The disadvantage of these methods is obviously that the solution is not optimal theoretically and it cannot be used as a general analytical tool for testing. The worst factor is that in some circumstances, the solution given by the merit-order schemes can differ drastically from the optimal solution. Hence more sophisticated scheduling techniques are needed for the improvement of the unit commitment schedule.

2.2.2 Mixed-integer Programming Techniques

As the mathematical formulation of the unit commitment problem derived from its physical conditions is often a mixed-integer programming problem, many attempts have been made to apply the mixed-integer programming techniques such as branch and bound techniques to tackle the difficulties that arise from the integer variables involved in the unit commitment formulation. The branch and bound techniques may be combined with a linear programming or a dynamic programming search routine in order to solve the unit commitment and generation dispatch problem.

The advantage of branch and bound techniques[54.] is that they can provide a sequence of solutions with estimates of their sub-optimality. Many branch and bound methods use a piecewise linear objective function as a reasonable approximation of a nonlinear function so that linear programming methods can be applied to solve the branch subproblem efficiently.

The branch and bound techniques can handle the integers involved in unit commitment problem satisfactorily from a theoretical point of view. They can be quite efficient provided there are only a few integer variables involved in the problem, but they are not widely used for a large scale system problem because of the difficulty of handling the high dimensionality involved with large sized problems and many integer variables. The branch and bound methods suffer from the "curse of high dimensionality"; in a similar manner to dynamic programming methods.

2.2.3 Dynamic Programming Methods

The greatest advantage of using dynamic programming techniques is that dynamic programming algorithms can handle the discontinuity and nonlinearity characteristics of the objective function and constraints very well and it is possible to include a great variety of constraints in the formulation with small computational effort, thus they are very suitable for the systems with significant nonlinearity. The algorithms can also be very adaptable.

As the unit commitment problem has a dynamic optimization feature, its optimal solution can be obtained by dynamic programming very efficiently. Dynamic programming algorithm are perceived to be a good possible alternative to merit-order schemes. A large number of research results have been published on various forms of dynamic programming approach to the solution of unit commitment problems.[36.],[49.],[98.],[102],[151.],[154.],[155.],[181.] The dynamic programming method can be viewed as a main theme for the solution of unit commitment.

Techniques based on dynamic programming usually achieve the computational feasibility by limiting the search range and can not give an estimate of the extent of sub-optimality until the final solution is obtained, thus no optimal solution is available until the end of the calculation, the premature termination is of no use. A further main disadvantage of the dynamic programming technique is that it generally suffers from the weakness of requiring large memory size and long CPU time when it deals with large scale problems or when a higher solution accuracy is required.

As the most severe problem with dynamic programming is that the computation time and memory storage requirement of a dynamic programming program appear to be too high to be practical for dealing with large scale problems (since they grow *exponentially* with the number of variables), various approximation techniques have been employed in order to overcome this high dimensionality problem. There are also proposals to employ heuristic approaches combined with dynamic programming rules and the principle of optimality to achieve a compromise between fast speed and the real optimal solution. These combinational approaches are more useful for practical reasons. There are also approaches to reduce the search-range of unit commitment lists so as to reduce the CPU time, and the dynamic programming successive approximations technique is often applied.[151.]

Both branch and bound techniques and dynamic programming methods are major options among all the unit commitment solution algorithms as they can always find a rigorous optimum given enough computational time. However, as a result of the "curse of dimensionality", any attempt to apply full branch and bound techniques or dynamic programming by considering all the combinations of units, becomes impractical even with extensive computational resources.

2.2.4 Mathematical Decomposition Techniques

More recently, the mathematical decomposition techniques for solving the large scale unit commitment problem have started to attract much attention and are gradually becoming competitive with the advanced dynamic programming routines for the solution of decomposed subproblems. Impressive results have been reported for schemes using Lagrangian relaxation and other decomposition techniques. Among the decomposition techniques employed, the generalized Benders' resource directive decomposition and the Lagrangian relaxation price directive decomposition [6.],[7.],[14.],[15.],[126.],[143.],[144.],[145.],[187.],[195.],[202.],[216.] are widely applied.

Lagrangian relaxation was firstly used as a solution technique for nonlinear constrained optimization problems, but here the Lagrangian relaxation technique is actually employed as a decomposition technique that is based on a pricing mechanism. By applying this price mechanism, large scale optimization problems can be decomposed through a unified price into a series of smaller problems, which can then be solved efficiently by a dual programming approach. Lagrangian relaxation is a quite popular decomposition technique which has been widely applied to various large scale dynamic scheduling and allocation problems involving integer variables.

Benders' decomposition technique has also been applied to the similar problems. For example, by applying the Benders decomposition method, the original thermal generation scheduling problem can be decomposed into a master problem that is concerned with the schedule of thermal unit commitments and a subproblem that considers the schedule of economic dispatch. Both Lagrangian relaxation and Benders' decomposition techniques were reported to achieve the optimal solution for large scale unit commitment problems efficiently. However, at present, in the area of power system generation scheduling, there are perhaps slightly more reports on the application of Lagrangian relaxation than the application of Benders' decomposition. A review of reported work and further details about mathematical decomposition techniques will be presented in the next chapter.

2.2.5 Linear Programming Techniques

A linear programming formulation can also be used as an alternative method for solving unit commitment problems or combined with the dynamic programming method as a solution technique for unit commitment.[159.],[187.],[196.] Linear programming is widely used for this problem because of its well-known robustness and simplicity of problem formulation. Other well-known virtues includes reliability and freedom from convergence problems, fast speed of solution and the sufficient accuracy of linearized power system models. The optimal solution can be achieved wherever it exists. The only limitation is, of course, that only linear relations can be treated. However, by linearizing around the best operating points or through successive approximation techniques, the result will still be acceptable within the required accuracy. The only concerns arises from the computational time that may be required for large scale problems, as standard linear programming algorithms still cannot solve realistic sized power system problems through a direct application of the algorithm. However, the basic ideas from linear programming methods are well known to be very helpful for the solution of nonlinear programming problems, and whenever applicable, linear programming techniques have been and will be favoured.

2.2.6 Variational Calculus

Methods based on variational calculus are good for solving the problems with a very small number of units. However, adding a new constraint into the problem means adding another Lagrangian multiplier, and this will not be acceptable for the solution of a realistic sized system. At present, the variational calculus method is used only as an analytical technique for justification, and very few algorithms of this kind are implemented on computers.

2.2.7 Post-Optimization Adjustments

Finally, in situations where the decision would violate a status or an operating constraint, various heuristic rules or post-optimal adjustments may be applied after the solution of a mathematical programming, if necessary, in order to obtain a feasible solution.

2.3 HYDROELECTRIC GENERATION SCHEDULING REVIEW

The main difficulties for determining the optimal operating policy of a multi-reservoir power system lie in the following aspects:

- The objective function can be a nonlinear function of both the reservoir discharge rates and reservoir storage.
- The production energy function of a hydro plant is a non-separable function of reservoir discharge rate and plant net head which is again a function of reservoir storage.
- For large realistic sized systems, thousands of variables and constraints may be involved.
- There are lower and upper bounds on both the state variables (reservoir storage) and the decision variables (reservoir discharge rates) in order to satisfy the multi-purpose stream management requirements such as

flood control, irrigation, fishing and other purposes. Thus, the hydro-electric generation system normally operates under a highly constrained environment.

- The problem is dynamic; the present decision on reservoir releases of one reservoir at one time interval may have an impact on future decisions of all the reservoirs in the same river valley at other time intervals.
- The optimal operating strategy for one reservoir depends not only on its own energy content and storage but also on the corresponding content and storage of each one of the remaining reservoirs in the valley.
- For the long-term operational planning problem, the stochastic nature of the problem is involved because neither the natural river flows nor the electricity load demand can be forecasted accurately over a long period in advance.

The long-lasting and almost irreversible implications of today's decisions imply that short-term decisions should be part of a long-term plan if electricity demand is to be met reliably and at a minimum production cost. This implies that the short-term decision making problem should be part of the chain of the overall operational planning problem. However, in this thesis, only the deterministic short-term scheduling problem is considered.

Stochastic dynamic programming (SDP) is in principle the optimization technique best suited to solve the long-term operational planning problem in the sense that, theoretically, it can handle both the stochastic nature and all other nonlinear characteristics very well. The only problem with stochastic dynamic programming is with the computation time and memory storage requirement as in the case of any dynamic programming algorithm. Hence full stochastic dynamic programming program turns out to become impractical for large scale problems. Instead, various approximation techniques are employed in order to cope with the high dimensionality problem, such as the deterministic approach to ignore the stochastic nature of the problem, the aggregation approach (AA), stochastic dynamic programming with successive approximation (SDPSA),[59.] and the aggregation-and-decomposition (AD) approach[63.] using the SDP to solve the subproblems, with a probabilistic production cost model[5.] or a composite representation of multi-reservoir hydroelectric power systems.[8.],[9.] Recently, there have been attempts to apply the functional analysis techniques to this problem as well.[51.],[128.],[212.]

The solution techniques proposed for medium-term operational planning and short-term operational planning are usually similar approaches. The only difference between these two operational planning problems is that in M.T. operational planning, only the reservoirs with ultra-week cycles are considered in the model, while the short-term operational planning needs to consider the reservoirs with intra-week cycles as well as the water transport delay and more detailed model, but both formulations are deterministic.

There has been considerable previous work on short-term hydroelectric generation scheduling, all this work was in some way attempting to consider the hydroelectric subsystem problem isolated from the total power system, or to consider the purely hydroelectric power system generation scheduling problem. Different objective criteria are also used depending on the different types of hydroelectric generation systems, their physical properties and different operating requirements or policies.

Generally, the optimal short-term scheduling of multi-reservoir power systems is aimed at maximizing the production benefits resulting from the hydroelectric generation with respect to the specified marginal prices under a highly constrained conditions. The optimization horizon will be chosen usually as one day to a week with the time interval as one hour. The short-term operational planning is usually formulated as a nonlinear programming problem with embedded network structure. The objective function is either to maximize the benefits of the hydro energy resource generation, in other words, to generate as much power as possible in accordance with changes in electricity demand; or in some other cases such as for a purely hydroelectric power system, to minimize the total amount of water discharged by the hydro system. These two targets are actually equivalent. Alternatively, in view of the dispersed nature of the hydroelectric generation, the objective function can be chosen as the minimization of the active power losses in the network.

The constraints are usually of two types: linear equations to ensure the water flow balance or conservation and simple bounds on the amount of water to be released from and stored in each of several interconnected reservoirs at each time interval. These bounds may be set by the flood control, navigational and recreational functions of the reservoir management system as discussed previously. Usually, the short-term operational planning problem must also take into account water head variations, spilling, and time delays between upstream and downstream reservoirs. The reservoir water losses due to seepage and evaporation are usually minor, and hence can be neglected.

Almost all the well-known optimization techniques have been applied or proposed to solve the short-term operational planning problem for hydroelectric power systems. Many solution algorithms have been proposed to exploit the network structure of the reservoir dynamics constraints and are generally very efficient.

The solution algorithms proposed for the solution of the short-term hydroelectric scheduling problem can be classified as follows:

- Variational calculus methods
- Heuristic techniques
- The maximum principle of Pontryagin
- General linear programming methods
- General nonlinear programming procedures
- Linear network flow algorithms based on the simplex method and graph theory
- Nonlinear network flow algorithms based on reduced gradient algorithms, which may be specialized for nonlinear network flow problems, or a conjugate gradient method
- Algorithms based on the principle of progressive optimality
- Dynamic programming with successive approximation or discrete differential dynamic programming
- Mathematical decomposition and coordination techniques

The maximization of the benefits of the hydro energy enables the programs to cope with the demand increases in peak hours, and this is ideal for reducing the total thermal production cost. Network flow algorithms have been claimed to be the most efficient of all.[86.],[87.],[88.],[89.],[104.],[142.],[167.],[204.],[210.] These algorithms usually have the ability to handle the network constraints and other major operating constraints much more easily as well as the necessary objective functions. Network optimization algorithms have been developed in the project, as presented in Chapter 6.

Heuristic techniques are very efficient for solving small sized problems, but may require excessive iterative calculations to reach a satisfactory solution, hence may be quite time-consuming. Even so, the solution obtained may not be really optimal, resulting in its inefficient usage of the potential energy of the river system. It is especially difficult to deal with large and complicated river systems.

Despite the fact that dynamic programming can handle the discontinuous objective function and constraints efficiently, standard dynamic programming or discrete-differential dynamic programming can not take into account the water travel time delays between upstream and downstream reservoirs very easily unless the problem is solved with respect to one variable at a time by dynamic-programming-successive-approximations (DPSA). In this case, the global optimal solution to the problem is not guaranteed. Furthermore, to apply dynamic programming algorithms, the state variables have to be discretized; consequently, to obtain the precise optimal solution of the problem, a finer grid for determining the optimal trajectory must be used in the later stage and this results in excessive computational time.[71.],[74.],[115.],[165.],[211.]

Linear programming methods can be used efficiently provided the production functions of the hydro power plant can be linearized or approximated by appropriate linear functions.[64.],[118.],[210.]

Direct applications of both linear programming and dynamic programming generally encounter the problems of excessive computational requirements and memory capacity, or difficulties in handling the complex operating constraints (linear programming).

Variational methods can not guarantee the algorithm will converge to the global optimum as discussed previously, also they are not practical for large scale systems.

Among the nonlinear optimization approaches,[4.],[76.],[101.],[134.],[135.] there is an unconstrained nonlinear optimization method using penalty functions that is worth mentioning; this method has been introduced to solve the weekly hydro generation scheduling problem for Pacific Northwest.[101.] However, although this nonlinear optimization method with penalty functions was claimed to be a very efficient algorithm, it still suffers the practical difficulty of deciding the proper weighting factors for penalty functions, which is a common drawback of penalty function methods.

The application of other nonlinear programming algorithms and the maximum principle of Pontryagin[41.],[96.] need to have the assumption of strict problem convexity in order to ensure optimality, which may not be the case in hydro scheduling problem where discontinuous functions may be involved. The maximum principle of Pontryagin can not easily take into account the bounds on state variables either. Besides, all these methods are to some degree incapable of solving large scale, complex problems, since the computation processing time tends to be unacceptable in practice.

The algorithm [129.],[146.],[147.],[190.] based on the principle of progressive optimality, as stated by Bellman,[21.],[22.],[23.] has been proposed by Turgeon[190.] to solve short-term hydro scheduling problems and was claimed to overcome the dimensionality problem of dynamic programming. However, many tests have also shown that the optimal solution is not guaranteed in spite of theoretical consideration to be optimal, versatile and efficient. Also the algorithm suffers from sensitivity to the initial trajectory since the number of iterations is a function of the selected initial trajectory.

2.4 SHORT-TERM HYDROTHERMAL SCHEDULING

The thermal unit commitment problem and hydrothermal coordination problem may be viewed as two separate problems. However, how should the thermal generation be coordinated with hydro generation operation to minimize the total system production costs? To answer this question will require a systematic strategy to perform the global optimization, including both thermal unit commitment and hydrothermal coordination. In the context of hydrothermal generation scheduling, hydrothermal coordination is a significant subproblem in the overall hydrothermal unit commitment process.

Short-term hydrothermal scheduling problem is usually treated to be deterministic. The energy generated by the hydraulic reservoirs during a week and a day is provided by an annual predictive management model and therefore is considered as input data of the short-term scheduling problem. The hydro scheduling is required to optimize all the hydro stations production with reference to the thermal cost for a week and then for a day. The constraint set for hydrothermal scheduling includes the active power balance equation, the cascaded and non-cascaded reservoirs system model, physical limits imposed on every generating unit and transmission limitations and transmission losses.

The idea of optimal short-term operation planning in hydrothermal power systems is not very recent, the work in this area can be traced back to as early as six decades ago. There has been a continuous flow of published papers on this subject from the time when electronic computers became commonly available. The program development is carried out all over the world. The literature published can be classified in two ways, one is with respect to the type of power systems as mentioned in Chapter 1, another with respect to the mathematical approaches applied to tackle the problem. The generation scheduling problem can be solved using very different strategies depending on different type of power systems such as purely thermal power systems, purely hydro power systems, hydrothermal power systems with a high percentage of hydro power and hydrothermal power systems with a low percentage of hydro power. With respect to the approaches that have been used to solve the short-term operation planning of all types of power systems, the methods are of a wide variety of types, the algorithms proposed for considering both hydro and thermal subsystems can be categorized into the following groups:

- Variational calculus methods
- Algorithms based on the maximum principle of Pontryagin
- Functional analytic optimization techniques
- Dynamic programming methods
- Heuristic methods
- Nonlinear programming methods
- Mixed-integer programming (branch and bound) methods
- Bender's decomposition techniques
- Lagrangian relaxation techniques

Most of the previous methods proposed for hydrothermal scheduling treated only the thermal generation or only the hydroelectric generation, whereas the coordination of hydro and thermal scheduling has been studied only for systems with a small percentage of hydroelectric capacity. From the 1960's to 1970's, there have been numerous mathematical approaches proposed to solve the short-term operation planning problem for various types of power systems, such as the variational calculus methods, the Pontryagin maximum principle, linear programming methods, general dynamic programming techniques and its variations such as incremental dynamic programming, dynamic programming successive approximation techniques, heuristic simulation methods, etc.[1.],[26.],[27.],[28.],[34.],[35.], [56.],[71.],[74.],[96.],[98.],[165.],[189.],[192.] In all these early works, the hydro plants are treated one after another to make sure that all of them are operated at the same incremental cost (a Lagrangian multiplier for each hour). Then following the thermal schedule the Lagrangian multipliers are updated, and a new iteration for hydro scheduling begins. This process is repeated until no more saving is incurred from thermal scheduling.

The principles of dynamic programming method have been widely used and there are many ways of using it. In Dahlin's paper,[56.] to avoid the excessive computer requirements, an iterative method of solution is developed using a series of grids or meshes with decreasing distances between points, this strategy is termed "incremental dynamic programming". Fucao[74.] used the successive approximation techniques in dynamic programming.

Prior to the application of Benders' decomposition, Lagrangian relaxation or Dantzig-Wolfe decomposition, the hydrothermal scheduling algorithms could only solve very small sized system problem with a small number of hydro and thermal plants. All the previous work during the period of 1950's to 1980's used the simplified model systems and only small sized power systems problems could be handled satisfactorily. To be more precise, in the past, the short-term hydrothermal scheduling problem was solved only by employing a simplified model both in the hydro subproblem and the thermal subproblem (which may contain the transmission network). These solutions neglected one or more of the following aspects of the hydro subsystem: the coupling between cascaded reservoirs, reservoir head variations and the water transport time delays. The transmission losses were ignored in the thermal subsystem model. Although this kind of result is useful under some specific circumstances, the methods have the common drawback that their solutions are not guaranteed to be practically feasible because some constraints associated with more detailed models have not been taken into account.

There were also some mathematical decomposition algorithms that were proposed and applied in the literature to solve the problem of hydrothermal scheduling before the 80's,[34.],[35.],[144.] but compared with the work at present, the programs for computing the optimal schedule for an integrated hydrothermal power system could only deal with very small scale and over simplified hydrothermal systems. Also many constraints were neglected, such as the hydro reservoir dynamics with variable water head, cascaded plants, spillage, and requirements such as navigation and flood control. The present decomposition approaches employed to solve the problem are much more advanced.

Since the 1980's, large scale optimization techniques began to be employed fully to deal with the optimization of realistic sized hydrothermal scheduling problems, including detailed models for hydrothermal power systems and nearly all the aspects of operating constraints and network constraints resulting from reservoir dynamics. Mathematical decomposition techniques have advantages over other approaches such as:

1. More complicated constraints and models can be included.
2. Large scale realistic sized problems can be solved.
3. Structurally and theoretically, the algorithms are more advanced and efficient.

Details of mathematical decomposition are presented in the next chapter.

CHAPTER 3

MATHEMATICAL DECOMPOSITION

3.1 INTRODUCTION

In this chapter, the importance and applications of mathematical decomposition techniques will be studied. This chapter together with the first two chapters provide the necessary background knowledge and introduce the purpose of this project study.

3.2 INTRODUCTION TO MATHEMATICAL DECOMPOSITION

Decomposition, hierarchical and multilevel system theories are all generic problem solving strategies that have been applied in a wide variety of contexts. Generally, they can provide a useful basis for the optimization and control of many large scale systems. These strategies can be described as dividing-and-then-solving procedures. Instead of directly addressing a very complex, hard-to-solve problem, the physical structure of this problem can be exploited and utilized by decomposition techniques. The entire problem can be broken down according to certain decomposition rules into a series of smaller and easier-to-solve ones, these subproblems can be solved individually, and their solutions recombined to achieve the solution of the overall problem.

In particular in the context of mathematical programming problems, direct solution of large scale mathematical programming problems may require excessive computation either in terms of time or storage or even both. Conversely, the application of mathematical decomposition such as Benders' decomposition, Lagrangian relaxation price decomposition and Dantzig-Wolfe decomposition can produce very efficiently the optimal solution to certain types of large scale optimization problems, especially in the case of many power system operation and control problems. If an on-line solution is needed, the decomposition becomes even more advantageous.

It is not necessary to assume that the problem could not be solved theoretically by the "brute force", or in other words, the direct solution for this problem. Although feasible, the direct solution might be computationally expensive and unreasonable as it may require excessive computation either in terms of memory storage or CPU time.

3.3 THE ADVANTAGES OF MATHEMATICAL DECOMPOSITION

There are numerous well-known advantages of mathematical decomposition techniques over the direct solution.

First of all, as mentioned above, decomposition can reduce considerably the computation time and storage requirement of the solution compared with a direct solution approach. The whole problem size can be substantially reduced to probably up to a few percent of the actual size.

Take the generation scheduling of a hydrothermal power system for example, if the system has more than 100 thermal units, and more than 50 hydro plants, for a 48 hour scheduling horizon with a hourly time interval, there will be easily more than 14400 variables and 21600 constraints involved in the problem formulation. This kind of problem size, even in its simplest linear programming problem formulation, will require the manipulation of matrices with thousands of rows and columns, the solution of such a large problem, even through a linear programming routine, will take nearly the full storage and CPU capacity for a modern mainframe computer, while mathematical decomposition may result in a dramatic reduction of the problem size. Optimal solution to the global problem can be achieved through the coordinated solution of substantially smaller problems (subproblems) in an acceptable CPU time such as twenty minutes.

Secondly, for some specific problems considered, fortunately, there will be some special features existing in these large scale problems that may be exploited. Also, because the matrices representing the constraint sets of the problems may have only a small percentage (e.g. 1-4%) of the elements that are non-zeros, the matrices will be very sparse. Moreover, the non-zero elements can be normally arranged into a special pattern that may result in a network flow problem structure as in the hydro subproblem scheduling. It is shown later that the hydro subproblem has a minimal cost network flow structure. By decomposing the entire problem into several subproblems, this special structure can be exploited and these subproblems can be solved much more efficiently by applying specially designed mathematical algorithms.

Furthermore, because of the recent rapid development in computer science in multi-processors and parallel processing techniques, decomposition techniques can be even more efficient by taking advantages of this computational development. In fact, the availability of multi-processors and the development of distributed computer systems can lead to a parallel processing capability that can often be compatible with a problem formulation using mathematical decomposition approaches. Also the use of parallel processors will make the decomposition structure much more attractive as the elapsed time needed for the solution can be consequently reduced by the division of the computational burden through parallel processing.

Finally, the decomposition techniques can provide a neat and efficient model. From the technical point of view, program development will be easier to handle. Models can be easily modified, updated and expanded and the algorithms for solving different parts of the model can be revised and developed independently.

3.4 THE DISADVANTAGES OF DECOMPOSITION TECHNIQUES

Unfortunately, mathematical decomposition algorithms are also likely to be theoretically and practically more complex than a direct approach in finding an overall optimal solution, as more iterations are needed to obtain the solution.

However, this shortcoming can generally be overcome if the computation time for each iteration of the mathematical decomposition is very short. Nevertheless, through the effective development of the coordination procedure, the algorithms can be developed to achieve an optimal or near optimal solution very efficiently.

3.5 THE PRINCIPLES OF MATHEMATICAL DECOMPOSITION

The basis of mathematical decomposition techniques to decompose the whole problem into many smaller subproblems, and to take into account the interaction between the subproblems so that the overall solution is achieved, by proceeding iteratively with information exchange between the subproblems and coordinating master problem. The manner in which coordination is achieved characterises the different decomposition techniques. "Hierarchical" decomposition refers to the vertical relationship between the master problem and the subproblems. If the entire problem is decomposed into a number of layers, some subproblems may be viewed as the master problems of the subproblems at a lower level, in this case, the term "multilevel" decomposition is used.

The earliest application of mathematical decomposition was Dantzig-Wolfe decomposition in the 60's.[57.],[124.],[171.] This decomposition technique is used for large scale linear programming problems. Since then, there have been many attempts to apply the various decomposition principles to solve the large scale problems in many fields. Much theoretical work has continued, together with investigations of the application of these methods to the decomposition of large scale problems.

There are also analogies between mathematical decomposition and organizational planning and control, which imply that mathematical decomposition rules can be applied to managerial decentralization. The mathematical decomposition process can be viewed as a hierarchical planning process in a utility organization with strategic planning and operational planning departments. The strategic planning department proposed different plans, and the operational planning department examines the technical feasibility of the plans, determines the best way that the proposed plans can be operated, and feeds back the marginal benefits or deficits to the strategic planning department. By transferring the information iteratively, these two types of departments work together until satisfactory plans have been found.

In mathematical decomposition, according to whether the decomposition techniques are applied to the original (*the primal*) problem or the Lagrangian (*the dual*) problem, the decomposition techniques can be classified into *primal decomposition* and *dual decomposition.*

Interest in the decomposition techniques was initiated by the examination of certain types of optimization problems that have a few sets of constraints which may complicate the problem in such a way that if these constraints were omitted, the problem could be readily decomposed into many independent subproblems. These constraints are usually termed to be "coupled constraints". Problems with this kind of structure can often arise from practical optimization problems. Notably many examples can be found in scheduling problems, in the case of the short-term hydrothermal generation scheduling problem.

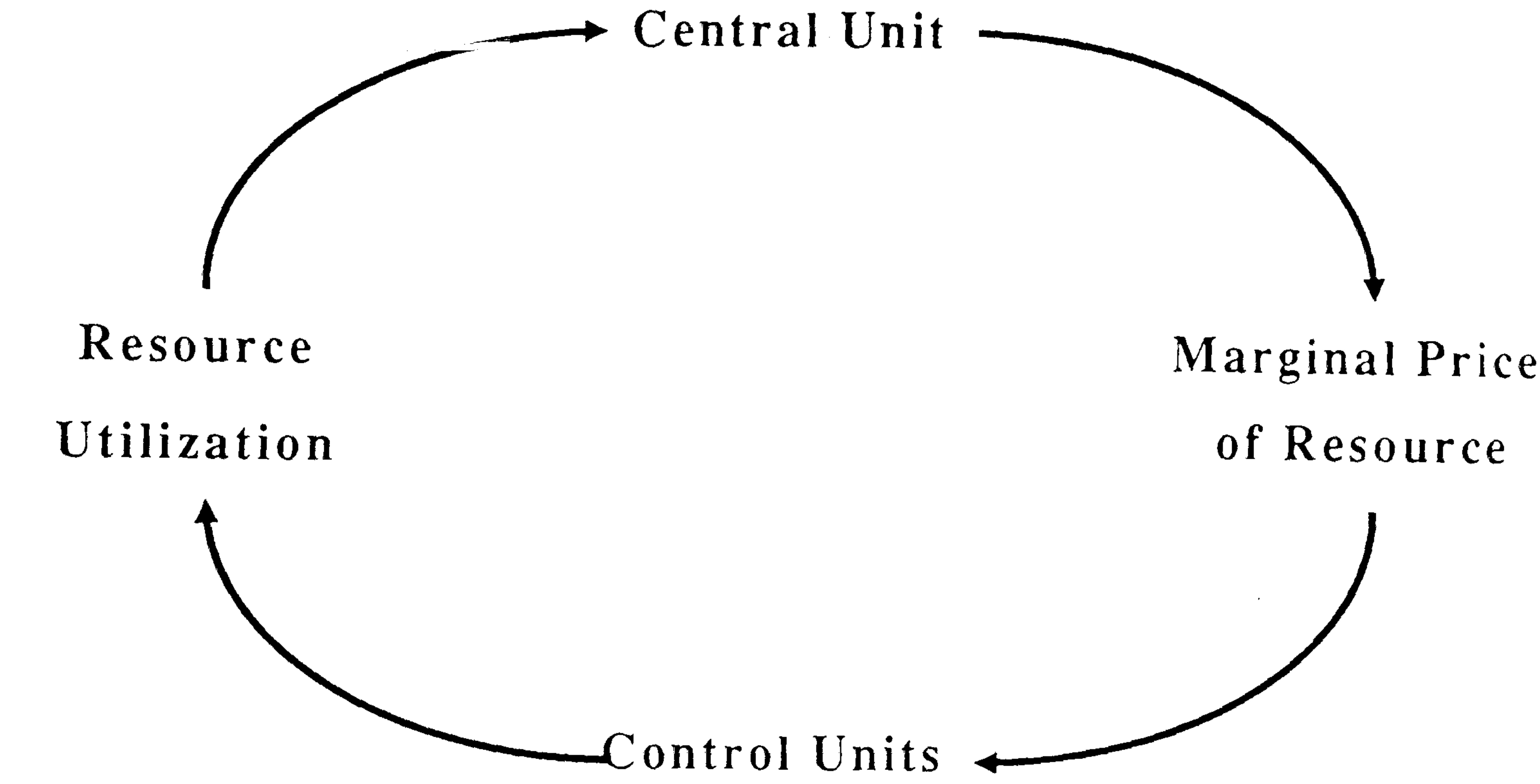
Various mathematical decomposition algorithms have been developed to take advantage of the special structures of large scale optimization problems. The most widely applied decomposition algorithms are from the following two categories:

1. Price directive decomposition scheme. Under this decomposition scheme, the central unit or the master coordination program will set the initial internal prices for all the shared resources of the subsystems, and let the control units of subproblems (i.e. the sub-programs) take over to determine the best plans independently and feed back the schedules and the amount of resources for this particular cost of resources. If some of the resources for particular subproblems are oversubscribed or under-subscribed, the headquarters will adjust the marginal operating prices of the resources according to the situation set by the sub-program. The price of the oversubscribed resources will be increased and the price of the under-subscribed resources consequently will be decreased in order

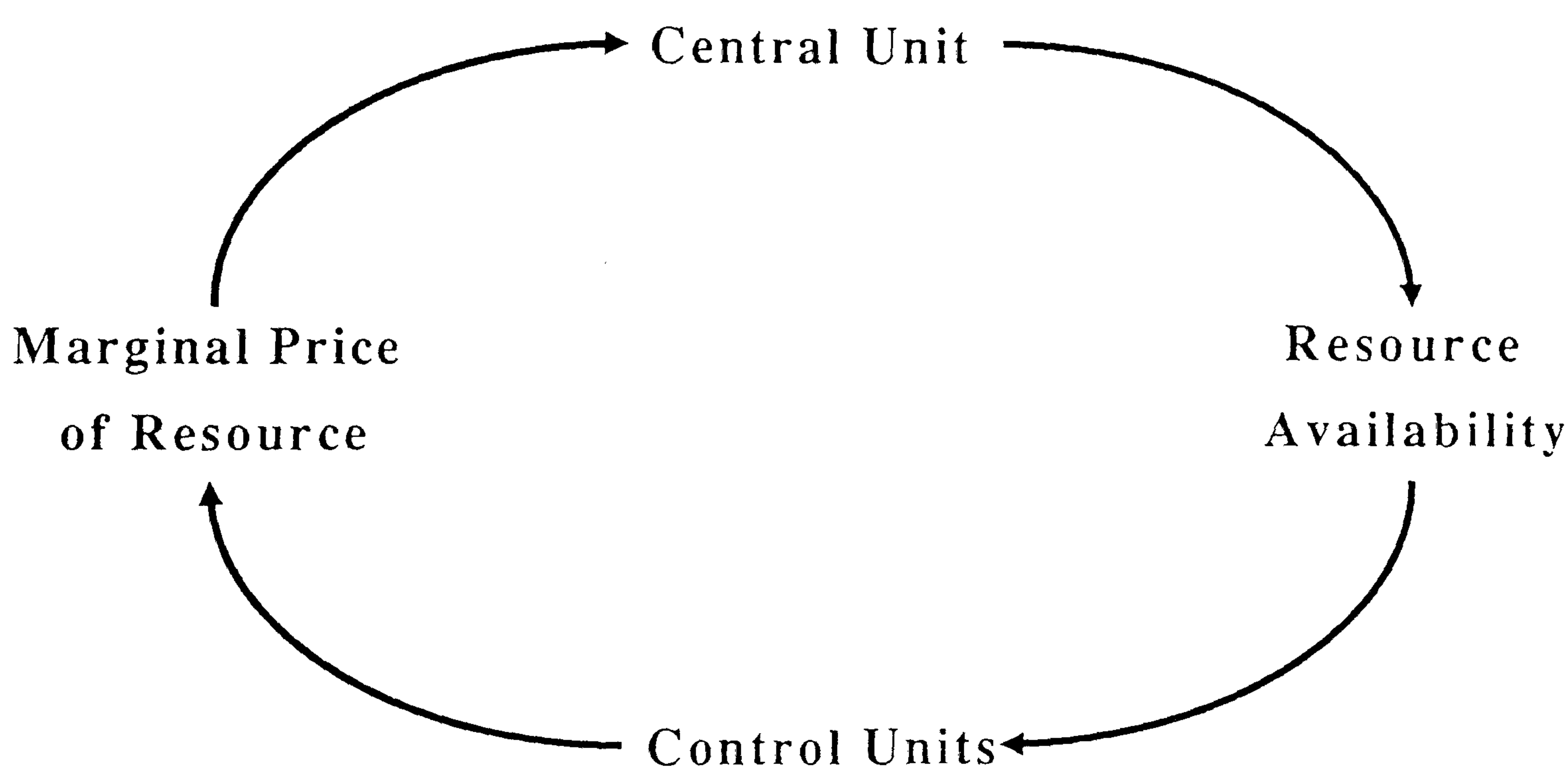
to ensure a feasible and balanced solution for certain limited resources. The central unit will then request the control units for submission of new schedules based on this new set of marginal prices, and so on. After several iterations, the headquarters can determine the best final plan which might be a combination of proposed schedules from the sub-control units. The Dantzig-Wolfe decomposition and the Lagrangian relaxation are typical approaches based on this principle.

2. Resource directive decomposition scheme. This type of decomposition scheme works quite differently from the first, in the coordination criterion: the first scheme uses an adjustment of the marginal prices, the latter uses an adjustment of resources. According to this scheme, the central unit or headquarters directly allocates the overall resources available to the different control units of the subproblems, and request the control units to send back a production schedule together with the marginal prices resulting from these resources in order to see the effect on marginal benefits from this adjustment. The central unit will then reallocate all the resources by processing this information; the control units with higher marginal benefits will receive more resources, while the control units with lower prices will be given less resources. The control units then adjust the production schedule according to their new resources and decide a new schedule based on the new resources allocation. The iterative process will continue until the overall expected profits are satisfactory. Benders decomposition is a well-known technique based on this principle.

The main difference between these two types of decomposition lies in the flow of information between the central unit (the master coordinator) and the control units (the optimization sub-programs). In the price directive decomposition the central unit sends down marginal prices of independent resources and receives the activity and resource utilization; while in the resource directive decomposition scheme, the central unit sends down the resource availability for independent units and receives the marginal benefits of resources allocated. The difference can be seen from Diagram 3.1.



Price Directive Decompsition



Resource Directive Decompsition

Diagram 3.1 Comparison of two Decomposition Techniques

3.6 OPTIMIZATION DUALITY THEOREMS

Duality theorems form the basis of the dual decomposition techniques. Duality theories are concerned with the global optimality conditions involving the dual variables (also called dual prices, shadow prices or generalized Lagrangian multipliers) for the optimal solution to a given primal optimization problem. The dual variables in these optimality conditions are optimal in a concave maximization problem that is known as the dual problem to the given primal problem.

Given a canonical primal problem of nonlinear constrained optimization such as:

$$(P) \qquad \min_{x \in X} \quad f(x) \quad subject \quad to \quad g(x) \leq 0, \tag{3.1}$$

where $g(x)$ is a vector of $\{g_1(x), g_2(x), ..., g_m(x)\}$, and f and each g_i are real valued functions defined on $X \in R^n$. It is assumed throughout that X is a non-empty set. There is a function

$$L(\lambda) = \min_{x \in X} \quad L(x, \lambda) \tag{3.2}$$

termed *the Lagrangian dual function* for this primal problem. Let

$$D = \{\lambda : \lambda \geq 0, \min_{x \in X} \quad L(x, \lambda) \quad exists\} \tag{3.3}$$

then *the Lagrangian dual problem* of (P) to the g-constraints is to maximize the Lagrangian function $L(x, \lambda)$ by associating with each constraint a real number $\lambda_i \geq 0, i \in I = (1, 2, ...m)$:

$$(D) \qquad \max_{\lambda \in D} \quad [\min_{x \in X} \quad \{f(x) + \lambda * g(x)\}], \tag{3.4}$$

where λ is an m-vector of dual variables. Dual variable λ is also called an optimal multiplier vector for (P) if (x, λ) satisfies the optimality conditions for some x.

To be more explicit, if a pair of points (x^*, λ^*) is a saddle point of $L(x^*, \lambda^*)$, then x^* is a global optimum of the primal problem (P). Thus, to find an optimal solution of the primal is equivalent to finding a saddle point of the dual and this condition applies to any mathematical program including non-convex programs or programs where X is a finite set. For convex differentiable problems the saddle point conditions are equivalent to the Kuhn-Tucker conditions.

A pair of point (x^*, λ^*) with $x^* \in X$ and $\lambda^* \geq 0$ is said to be a saddle point for $L(x, \lambda)$ if it satisfies the following:

$$(1) \quad L(x^*, \lambda^*) \leq L(x, \lambda^*), \quad i.e. \quad L(x^*, \lambda^*) = \min_{x \in X} \quad L(x, \lambda^*) \tag{3.5}$$

for all $x \in X$, and

$$(2) \quad L(x^*, \lambda^*) \geq L(x^*, \lambda) \tag{3.6}$$

for all $\lambda \geq 0$. The necessary and sufficient conditions for a saddle point of L is equivalent to satisfy the global optimality conditions for the primal problem, that is: let $x^* \in X$ and $\lambda^* \geq 0$, then (x^*, λ^*) is a saddle point for L or x^* is an optimal solution for primal if and only if the following equations hold:

$$x^* \quad minimize \quad L(x, \lambda^*) \quad over \quad X, \tag{3.7}$$

that is,

$$(1) \quad f(x^*) + \lambda^* * g(x^*) = \min_{x \in X} \quad \{f(x) + \lambda^* g(x)\} \tag{3.8}$$

$$(2) \quad g_i(x^*) \leq 0, \qquad i = 1, 2, ...m \tag{3.9}$$

$$(3) \quad \sum \lambda_i{}^* * g_i(x^*) = 0, \qquad i = 1, 2, ...m \tag{3.10}$$

If (x^*, λ^*) satisfies the global optimality conditions or in other words, is a saddle point for L, then x^* solves the primal problem. Suppose that $X \in R^n$ and real functions $f(x)$ and $g(x)$ are continuously differentiable, and that the problem satisfies a constraint qualification at $x^* \in X$, then a necessary condition for x^* to be a local minimum of the primal problem (P), i.e. to minimize $L(x, \lambda)$ for a fixed λ, is the existence of Lagrangian multipliers λ^* such that:

$$\Delta_x L(x^*, \lambda^*) = 0 \tag{3.11}$$

to ensure the stationarity and

$$\lambda^* * g_i(x^*) = 0 \tag{3.12}$$

for all $i \in I$ to ensure complementary slackness, these are called the Kuhn-Tucker necessary conditions. Thus, the optimality conditions for $x^* \in R^n$ and $\lambda^* \geq 0$ are the Kuhn-Tucker conditions as follows:

$$(4) \quad \Delta_x L(x^*, \lambda^*) = \Delta f_x(x) + \lambda^* * g_x(x) = 0 \tag{3.13}$$

for its minimality,

$$(5) \quad g_i(x^*) \leq 0, \qquad i = 1, 2, ...m \tag{3.14}$$

for its primal feasibility and

$$(6) \quad \sum \lambda_i{}^* * g_i(x^*) = 0, \qquad i = 1, 2, ...m \tag{3.15}$$

for its complementary slackness.

Furthermore, if $f(x)$ and $g(x)$ are convex functions, then the Kuhn-Tucker conditions above become necessary and sufficient conditions for global optimality of the primal problem.

There are some useful properties associated with the dual problem. The weak duality theorem states that for every $\lambda \geq 0$ the value of the dual function $L(\lambda)$ is a lower bound of the absolute optimum value of the primal $(P) = f(x^*)$, i.e. for all $\lambda \geq 0$ the following equation holds:

$$L(\lambda) \leq L(\lambda^*) \leq f(x^*) \tag{3.16}$$

where λ^* is the optimal solution for the dual problem, and $f(x^*)$ is the feasible and optimal solution of the primal. Thus, the dual problem to the primal problem is obtained by finding the greatest lower bound to the primal optimal solution $f(x^*)$, namely, $L(\lambda^*) = \max_{\lambda \geq 0} \quad L(\lambda)$. As $L(\lambda^*) \leq f(x^*)$, without further assumptions on the primal problem (P), it is possible that $L(\lambda^*) < f(x^*)$ in which case the difference between these two values is termed *the duality gap.*

The strong duality theorem states that if and only if $x^* \in X$, $\lambda^* \geq 0$ and (x^*, λ^*) satisfies the global optimality conditions, in another words, is a saddle point for $L(x, \lambda)$, then λ^* is optimal in the dual problem; moreover, the primal and dual problems have equal optimal objective function value, namely $L(\lambda^*) = f(x^*)$, i.e. there is no duality gap.

If the primal problem is convex and obeys some regularity conditions, then an optimal dual solution can be used to find an optimal solution to the primal. For an arbitrary primal problem, however, there is no guarantee that the optimal solution of the dual problem will yield an optimal solution for the primal problem. Nevertheless, the dual problem can be very useful in the solution of the problems such as hydrothermal unit commitment problems, which are non-convex as a result of the integer variables involved in unit commitment. Details will be shown in later chapters.

Very often the dual problem is much easier to solve than the original problem, mainly due to the absence of the dualized constraints. Even if sometimes a saddle point does not exist as is usually the case with mixed-integer or other non-convex programming problems, the duality theorems still give the means for evaluating the lower bound of the primal problem very efficiently, and this can be exploited easily in a branch and bound framework.

3.7 SOLUTION TECHNIQUES FOR THE MASTER PROGRAM

The implication of duality theorems is that we need only consider optimal dual solutions in seeking to establish the global optimality conditions for the primal. The indicated solution strategy is to find an optimal solution λ^* to the dual, and then attempt to find a complementary x^* which satisfies the global optimality conditions.

Concerning the algorithms employed for the solution of the master problem to maximize the Lagrangian dual function $L(x, \lambda)$, i.e. the coordination procedure to determine the proposals of the central unit, the solution techniques can be broadly classified into two main streams as follows.

1. The gradient approaches such as the steepest ascent (descent) algorithm.
2. Linearization and solution by generalized linear programming.

The Lagrangian function $L(x, \lambda)$ is finite and concave over any convex subset of its domain D. Generally, the dual function $L(\lambda)$ is not everywhere differentiable. The gradient approaches will adjust the current solution at each iteration by moving in the direction indicated by the gradient (the vector pointing in the direction of the maximum rate of improvement of the dual maximum objective function, or more generally a sub-gradient when the dual objective function is not differentiable at some points). Although a continuous, concave function may not have a gradient everywhere, it does have everywhere a generalization of the gradient. A vector $g(x_0)$ is called a sub-gradient of a concave function $f(x)$ at x_0 if

$$f(x) \leq f(x_0) + (x - x_0) * g(x_0) \tag{3.17}$$

for all x. If there is a unique sub-gradient of $f(x)$ at x_0, then the sub-gradient is the gradient. The sub-gradient of the dual function is defined as: for any $\lambda_k \geq 0$, let the corresponding $x(\lambda_k) = \{x \in X : L(x, \lambda_k) = L(\lambda_k)\}$, then for all $x \in x(\lambda_k)$, the vector $\{g_1(x), g_2(x), ...g_m(x)\}$ is a sub-gradient of L at λ_k. When the dual function is differentiable at λ_k, the set $x(\lambda_k)$ reduces to a single point x, and the vector $\{g_1(x), g_2(x), ...g_m(x)\}$ is the gradient of L at the point λ_k.

In generalized linear programming coordination, the solution space and the objective function are represented in the master program by linearization (inner or outer) in the neighbourhood of the current solution. The simplex method can be employed for the solution of the resulting linear programming problem. The generalized linear programming algorithm is a generalization of the primal simplex algorithm because its subproblems generate columns for a master problem that is iteratively resolved by the primal simplex method. Since the number of variables or constraints may be excessively high, the principles of generalized linear programming restriction and relaxation are usually utilised.

3.8 THE APPLICATIONS OF DECOMPOSITION

The unit commitment problem and hydrothermal generation scheduling problem, as formulated here, are large scale, dynamic and mixed-integer programming problems. Recently, there have been many applications of strategic mathematical decomposition approaches to solve the hydrothermal scheduling problem. Usually the mathematical decomposition technique is applied to divide the overall hydrothermal scheduling problem into a master problem, a hydro-based subproblem and a thermal-based subproblem. Various techniques for solving the subproblems and exploiting the special features of individual subproblems have been developed and various methods have been used to solve the master problem. The decomposition approach is the main theme of the project. So far, it is considered to be the most promising approach for solving large scale hydrothermal scheduling problems.

Depending on the different principles of decomposition, there are two main streams of decomposition techniques applied to hydrothermal generation scheduling problems: Lagrangian relaxation and Benders' decomposition. The processes for the two main approaches can be stated as follows:

1. Bender's decomposition technique is based on the resource directive decomposition scheme. In this decomposition scheme, for the hydrothermal scheduling problem, the master problem is the unit commitment of thermal units (integer variables only). The subproblem is the economic dispatch problem, which involves only continuous variables. The program will start with an initial unit commitment schedule, and different hydrothermal economic dispatch schedules with fixed thermal unit commitment decisions (resources) can be examined iteratively. For each iteration the operating cost of the resulting dispatch schedule is calculated along with the marginal price savings from this unit commitment decision. This information is then fed back to the "master program". The master program is the mechanism which generates new thermal unit commitment decisions. The next hydrothermal economic dispatch problem is then solved again according to the new unit commitment decision. In this way, the lower and upper bounds of the total thermal production cost are derived at each iteration, the program will stop when no further improvement in total production cost can be achieved and the convergence of this decomposition scheme is usually claimed to be fast.

2. Another type of decomposition is based on the price directive decomposition scheme, that is the Lagrangian relaxation. In this scheme, the coupled constraints (power balance, reserve, and other security constraints involved with units grouped together) are relaxed by assigning a Lagrangian multiplier and incorporated to the original (primal) objective function, Lagrangian dual function. The problem then becomes the dual problem of maximizing the Lagrangian function; that is, finding the saddle point of this Lagrangian dual function. This is actually a max-min dual problem that can be solved by examining different sets of Lagrangian multipliers or "marginal prices" and solving the resulting minimization problem. The minimization problem can be decomposed into many independent subproblems, according to the remaining constraints which can be solved individually. In the hydrothermal generation scheduling problem, this becomes a set of minimization problems each related to a single thermal unit, and a set of minimization problems related to each river valley for the hydro units. The master program here is the mechanism that generates a new set of multipliers according to the resource surplus or shortage at each iteration in order to achieve a higher value for the Lagrangian dual function.

A survey of the main decomposition strategies and approaches proposed in previous work on short-term hydrothermal scheduling can be broadly divided into the following groups:

1. There have been applications in hydrothermal generation scheduling using the primal decomposition.[38.],[39.] Actually this primal decomposition can be viewed to be a special case of the dual decomposition without an explicit coordination procedure.
2. The dual decomposition technique based on the Lagrangian relaxation and the master problem is solved by the sub-gradient optimization and tangential approximation technique.[90.],[91.],[92.]
3. The dual decomposition technique based on the Benders' decomposition. [90.] in which the continuous subproblems are solved using the Lagrangian relaxation approach.

4. A Lagrangian relaxation dual decomposition for both thermal unit commitment and hydrothermal generation scheduling, solved by penalty function methods.[29.]
5. A dual decomposition including a detailed model for the hydro and thermal systems and an optimal power flow model without thermal unit commitment, but with hydrothermal dispatch[138.]
6. Hydrothermal coordination based on the concept that the incremental value of the hydro generation is equal to the incremental cost of the displaced thermal generation.[62.],[170.]

The above approaches actually belong to one of the three main approaches which can be summarized as follows:

1. The heuristic marginal cost decomposition and coordination method. This comes from the concept discussed by Seymore, Larson and Waren that *"hydrothermal coordination is based on the concept that the incremental value of the hydro generation is equal to the incremental cost of the displaced thermal generation."* This statement is aimed to demonstrate that, supposing the power balance equation holds such that

$$\sum_{i}^{I} P_T(i,k) + \sum_{j}^{J} P_H(j,k) = P_D(k)$$

to ensure this load requirement is met with the least cost, the hydro generation should replace the most expensive thermal generation.
2. Lagrangian relaxation decomposition technique. This has been reported as one of the most efficient and suitable approach for the decomposition of a large scale hydrothermal scheduling problem. There are two approaches to applying the Lagrangian relaxation technique: the first approach uses Lagrangian relaxation only for solving the dual problem and finds its lower bound which may be infeasible for the original hydrothermal scheduling problem; the second approach combines the Lagrangian relaxation dual methodology with a branch and bound technique in order to find an optimal and feasible solution of the original (primal) unit commitment problem. There have also been many algorithms

applied to actually update the Lagrangian multipliers, such as the sub-gradient optimization algorithm, the tangential approximation technique, and the generalized linear programming algorithms. Among these, the sub-gradient optimization algorithm is the most popular.

3. The generalized Benders decomposition approach. In this approach, the entire generation scheduling problem is decomposed, transforming the solution of the overall problem into a master program that only involves integer variables and a subproblem with continuous variables only. The continuous subproblem with real variables may be further decomposed into many sub-subproblems. In the case of hydrothermal generation scheduling with thermal unit commitment, the master problem is the thermal unit commitment problem deciding the unit "on" and "off" schedule, and the subproblem corresponds to the hydrothermal economic dispatch problem. The subproblem can be further decomposed, by applying the Lagrangian relaxation technique, into a series of hydro subproblems for each river valley and thermal subproblems for each thermal unit.

The heuristic marginal price coordination has been proved to obtain the optimal solution or near-optimal solution through the successive application of a hydro scheduling program based on specified incremental costs and using the thermal unit commitment program to cover the remaining load in order to ensure that the load demand requirement is satisfied. It is quite a straightforward method, and can be very efficient in producing a near-optimal and feasible solution. Much work has been undertaken using this approach, or some variation of the concept, depending on the properties of different hydrothermal systems.

Algorithms based on Benders decomposition have been reported to have a slightly higher efficiency than algorithms based on Lagrangian relaxation where the Benders decomposition method has the advantage of coping with the integer variables. However, Lagrangian relaxation may be used to generate a near-optimal yet feasible solution in less time than the Benders decomposition method and this is more acceptable in practice. Also, program development will be simpler and easier to handle in the case of Lagrangian relaxation.

Within the context of branch and bound techniques, Lagrangian relaxation decomposition has been applied to solve the subproblems combined with branch and bound techniques to obtain lower bounds on the overall problems. [144.] However, due to the computational limitations of branch and bound techniques, only a few nodes of the accompanying tree could be examined. This difficulty is overcome by considering that the commitment schedule of a large part of the thermal plants can be identified without requiring the solution of the master problem (e.g. the nuclear power units, the 'must on' or 'must off' units). Consequently, the master problem must be solved only to obtain the unit commitment schedule for a small number of thermal units, in order to seek higher efficiency.

Both of these two methods were reported to be capable of producing better solutions than the heuristic techniques. Generally speaking, the heuristic methods require lower computation time, but they can not guarantee a near-optimal solution. If an exact optimal solution is not actually expected which is often the case under practical conditions, then this method will be very suitable. Even the Benders decomposition method or Lagrangian relaxation can not be guaranteed to reach the exact optimal solution, they can only guarantee the solution within 1-2% from the optimum. The disadvantage of Benders' decomposition and Lagrangian relaxation is that the solution of the dual problem may not be feasible for the primal because a duality gap may result from the application of these two decomposition approaches. This is undesirable whereas the heuristic methods can always ensure the load demand is satisfied by the generation and can rapidly provide a near-optimal solution for decision makers. Heuristic methods are therefore a good alternative approach to decomposition.

A global optimization approach has been proposed to coordinate the hydrothermal scheduling in order to achieve a better solution in an acceptable time through heuristic marginal prices coordination.[127.] This approach has been reported to be a quite appropriate strategy, because, for a hydrothermal power system with considerable hydro and thermal generation capacity, after the nuclear generating unit schedule is fixed to cover the base load, the small number of thermal generating units left to be committed may be solved through the first iteration of the hydrothermal coordination procedure. After the first hydrothermal coordination procedure, the thermal unit commitment schedule can actually be fixed, only the schedule of economic dispatch for the hydrothermal power system must be performed.

Previous work and tests have suggested that the hydrothermal generation scheduling problem, including thermal unit commitment, is an immensely complicated problem. The coordination procedures for hydrothermal generation scheduling may be enormous, the specific approaches used for hydrothermal coordination will differ from system to system depending on the system characteristics, the physical and operating constraints and the operating policies. The distribution of hydro generation capacity and thermal generation system capacity is an important factor for deciding the coordination procedure. If the thermal generation proportion is much higher than hydro, the thermal unit commitment schedule may change as a result of the coordination. If the proportion of thermal generation is much lower than the hydro, the thermal unit commitment can be fixed after the first hydrothermal coordination iteration.

Recently, in G.X.Luo, H.Habibollahzadeh and A.Semlyen's paper an efficient solution of short-term hydrothermal scheduling problems has been presented. This paper gives a fully detailed model of short-term hydrothermal scheduling including the main features such as head variations, the effects of cascaded multi-chained reservoirs, the time delays of water inflows, the effects imposed by load flow equations and other security constraints, etc. In this approach, the problem has been decomposed into a hydro subproblem and a thermal subproblem based on Kuhn-Tucker optimality conditions similar to the heuristic marginal cost coordination approach, and these two subproblems are solved iteratively. Network flow programming is used for solving the hydro subproblem. The equations of coordination such as the power balance equation and the optimal power flow equation are used for the thermal scheduling subproblem. However, in order to reduce the complexity of the hydrothermal scheduling problem, the approach assumes that the thermal unit-commitment has been solved separately such that the on-and-off schedule of the thermal generators is predefined before hydrothermal scheduling problem is solved. Thus only real variables were considered. An integer optimization process which is known to be much more complicated than the continuous optimization problem when Benders decomposition or the Lagrangian relaxation techniques are applied.

To summarize, as a price directive decomposition approach, the Lagrangian relaxation methodology has been applied to a broad class of large-scale dynamic scheduling problems and resource allocation problems. Even with mixed-integer programming problems, this technique has also been applied to realistic and practical thermal unit commitment problems and hydrothermal scheduling problems. Therefore, comprehensive work has been done in this project to apply this solution methodology to the solution of large scale thermal unit commitment problems and then to hydrothermal unit commitment problems. Details of the algorithms which have been developed will be given in later chapters.

CHAPTER 4

HYDROTHERMAL GENERATION SCHEDULING MODEL

4.1 INTRODUCTION

This chapter is intended to cover the fundamental aspects of modelling the various parts of hydrothermal power systems and the phenomena that are related to the short-term generation scheduling of hydrothermal systems.

A mathematical model must represent the real system problem as accurately as possible, but the level of detail in each model must rely on the availability of data. Even if complete data is eventually available, the model derived may result in too much complexity. As a result, practical modelling development can only be a compromise between the accuracy of the model and the necessity to limit the complexity of the problem.

Any system may be viewed as being made up of subsystems each of which involves a number of components. The degree of detail in the models of components and subsystems varies with the desired accuracy and relevance to the given problem. A hydrothermal power system is not an exception. It consists of three major parts: a thermal subsystem, a hydro subsystem, and an electrical transmission network, as can be seen from Diagram 4.1. A structural diagram of a typical hydrothermal power system is shown in Diagram 4.2.

The basic components that must be clearly modelled in a hydrothermal power system are individual thermal generating units, individual hydroelectric units and the transmission network system. These three parts are discussed respectively in the following sections.

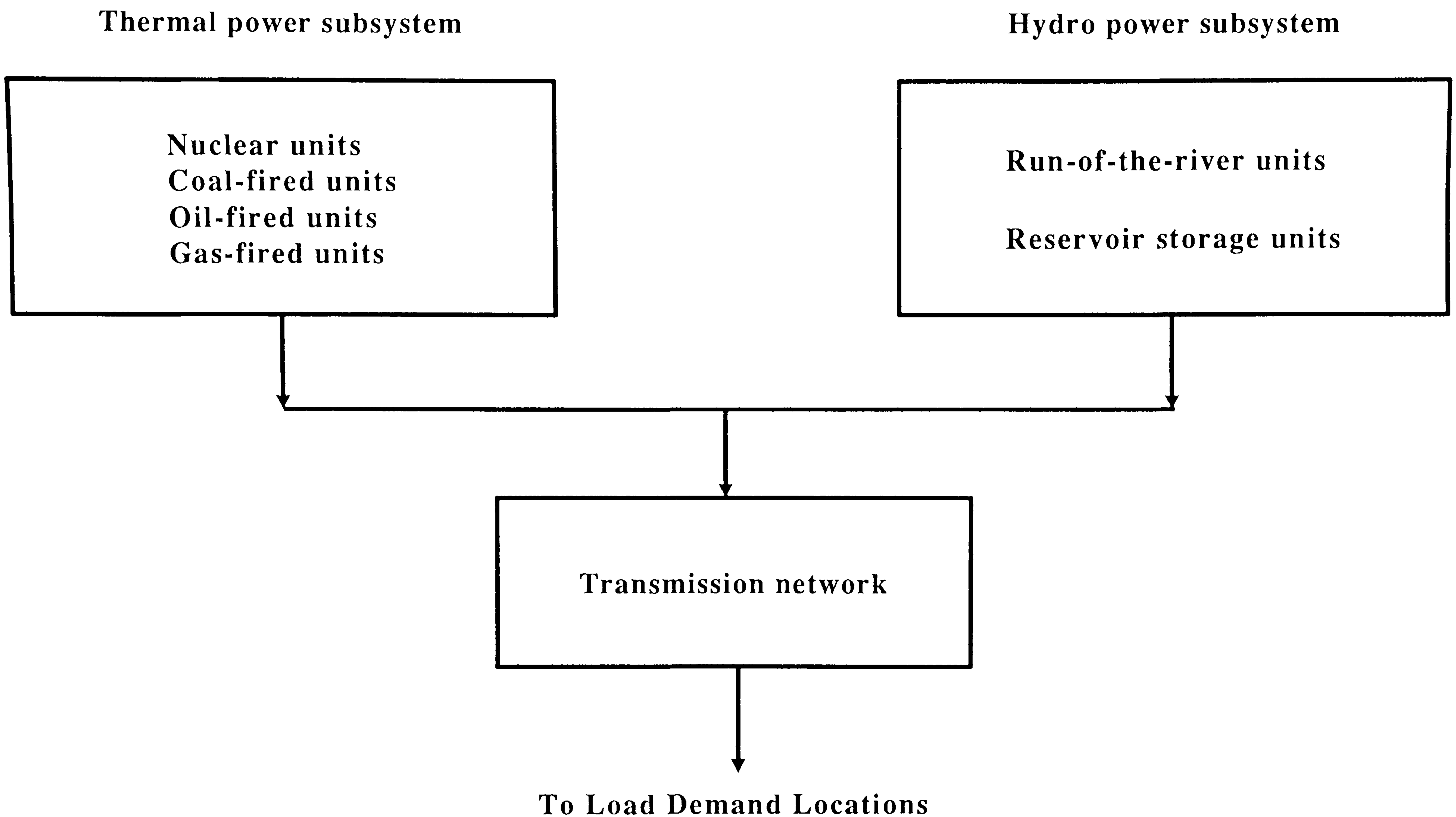


Diagram 4.1. A Hydrothermal Power Sytem and Its Components

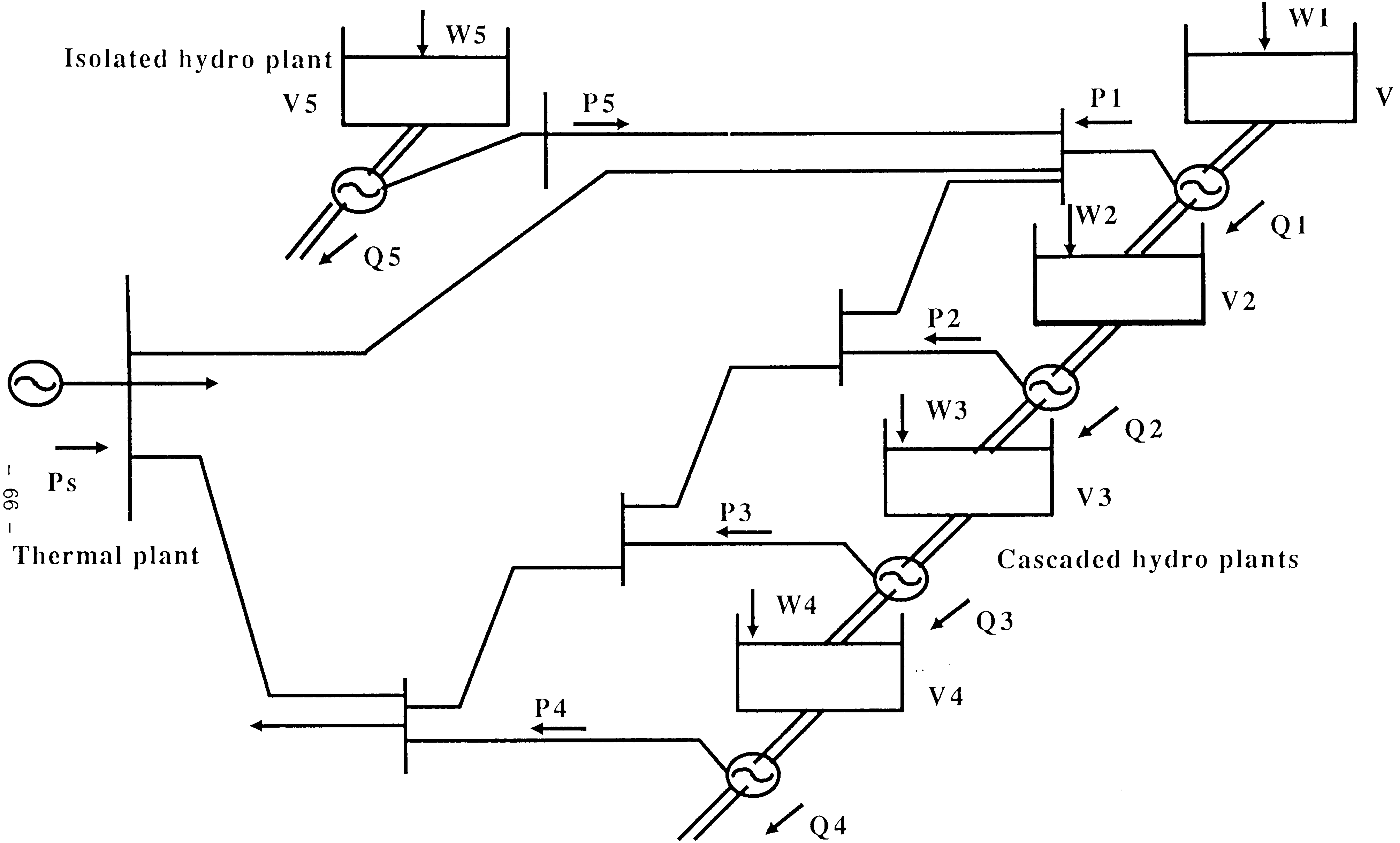


Diagram 4.2. A Typical Hydroelectric System in Series

The short-term generation scheduling of a hydrothermal power system, as modelled in this chapter, is a large scale, nonlinear, dynamic, mixed-integer programming problem. Due to the negligible marginal cost of hydroelectric generation, the objective of this optimization problem is to determine which generating units should be committed and what load should be placed on each unit to meet the forecast load demand while maximizing the hydro generation in order to minimize the total production cost of the thermal plants within all the operational constraints. The main objective of the hydro subproblem optimization procedure is to maximize the total utilization benefits of the stored water energy according to the water marginal values at each time interval.

In short-term generation scheduling, it is essential to model each unit and each reservoir separately and in great detail. It is necessary to point out here, that the fundamental difference between the long-term and the medium-term models compared with short-term models is that in the short-term models all data considered are deterministic, whereas in the long-term ones, the probabilistic characteristics of the problem must be viewed as a crucial point in the modelling procedure. The long-term model is usually solved by stochastic dynamic programming. The difference between the short-term model and the medium-term one is that the short-term model takes into account the intra-week cycle reservoirs, whereas the medium-term one only takes into account the reservoirs that have cycles more than a week or so.

Section 4.2 describes the hydro subsystem model including hydroelectric plants, reservoir dynamics and hydrological coupling between reservoirs in the same river. The network structure resulting from the majority of constraints involved in hydro subsystem is also discussed in this section. The hydroelectric plants are considered to have a negligible operating cost over the optimization period. Section 4.3 considers the thermal subsystem model. Different types of thermal plants are specified, such as nuclear plants and fossil fuelled plants. The operating cost of the unit and other constraints introduced in the overall model are discussed, followed by Section 4.4 where the transmission network model is described. Section 4.5 gives a complete model for short-term generation scheduling of a hydrothermal power system.

4.2 MODELLING OF HYDROELECTRIC SUBSYSTEM

4.2.1 Introduction

The requirements for the optimal operation of hydroelectric plants must be understood before one can go forward with the economic operation problems of hydroelectric plants. There are usually limitations imposed on operation of hydro resources by flood control, fisheries, navigation, recreation, water supply and other demands on the water bodies and streams, as well as the characteristics of the energy conversion from the potential energy of the stored water to the electrical energy.

4.2.2 Hydro Power Station Layout

The principle of a normal hydroelectric power station is to use the water energy falling from a high level source to drive a turbo-generator and produces the electricity. In a hydroelectric power plant, the hydro turbines convert the available water potential energy from the rivers into kinetic energy, which is in turn converted into the electrical energy through the generator units.

According to the installation characteristics, hydroelectric power stations can be classified into two types: conventional and pumped-storage hydroelectric plants. A pumped-storage scheme differs from the conventional hydroelectric plants in that it consists of two reservoirs, an upper reservoir and a lower reservoir. During the peak load period, water stored in the upper reservoir is released to drive the turbo-generators in order to generate electrical energy when it has high merit value to the network (i.e. displacing the high cost fossil-fuel generation). During the light load period, the water which has been collected in the lower reservoir is pumped back into the upper reservoir using the most economic energy available as a surplus from other sources in the system and at a time when marginal cost of the electrical energy in the network is much lower.

By considering the station net head and storage capability of the hydro-electric power stations, the conventional hydro power plants can be subdivided into two main categories: run-of-the-river power stations and storage power stations.

The run-of-the-river power plants have a very small and limited or no storage capacity at all and use water energy as soon as it becomes available. Water energy that can not be utilized at the time is spilled over. These stations are usually built on sites where a river flow is maintained. The power plants can be located in the stream or alongside. Sometimes if necessary, only a small dam is built for water regulation, navigation, or irrigation purposes. Normally they are operated at a constant forebay elevation and their power outputs only depend on the corresponding water inflow into the reservoirs. It is therefore obvious that in the case of the unregulated water inflows, the generated electrical power by this kind of hydro power plants is not controllable, so inside the optimal generation scheduling problem, its generation is usually taken as a negative customer load and subtracted from the total electricity load demand.

The storage power plants, on the other hand, have relatively large reservoirs or significant storage capacities which are generally built on natural sites. They are capable of permitting carry-over storage from the wet season to the dry season. Normally, they are operated at a variable gross head with a variable power output. During low electrical power requirement periods, the water energy can be stored and then utilized when the load demand is high.

Pump-storage plants can be treated as a special case of the second category because they have spare water capacity at the tailrace level and pumping facilities attached to the stations in order to store the water back in the upper reservoir. This type of generation source can be used for peak load generation, thereby replacing less economic sources such as thermal power plants. However, there is a trade-off problem due to the cost of the pumping. The generating cycle of pumped-storage plants must also be considered in the process of generation replacement. The optimal operation of pumped-storage plants forms a rather special problem and is usually treated independently. Generation scheduling with pumped-storage power stations forms a quite special problem and is very different from the conventional hydrothermal scheduling problem, it is therefore not considered in this research work nor in this thesis.

The design of hydroelectric plants very much depends on some practical factors, due to the geographical and hydraulic limitations. It is true to say that no hydroelectric systems will be the same as others. Diagram 4.3 and Diagram 4.4. show a typical hydroelectric power station layout and a schematic model. The main elements of a typical hydroelectric plant include an upper level reservoir or a forebay, a dam, a penstock, a plant house, turbines and the tailrace. The water from the river is carried through a passageway intake directly into the turbine in the low-head case or to a pressure conduit (called penstock) in the mid-head or high-head cases. For the purpose of pressure regulation, a surge tank is installed along the penstock preventing sudden pressure rises or drops during rapid load changes. The pressurized water then passes through the hydro turbine(s), turning the turbine(s) to move the generator, thereby producing electrical energy. Water flowing through the turbine(s) is passed through the draft tube and enters the tailrace, and is finally passed through the tailrace reservoir which may be one of the parts of the same river that has a lower elevation than the upper reservoir.

4.2.3 Hydroelectric Power Station Performance Model(P-Q,H)

The relationship between a hydroelectric plant output power with the water discharge rate of a turbine and the net head of the plant is nonlinear. In other words, the generation output function of a hydro power station is a nonlinear function of its effective net head and its turbine discharge rate. Generally this function can be represented by the following equation:

$$P_H(j,k) = f(H(j,k), Q(j,k)) \qquad k \in K, j \in J \tag{4.1}$$

Where

$P_H(j,k)$ is the power output for unit j at time k in MW.
$H(j,k)$ is the effective head for unit j at time k in meters.
$Q(j,k)$ is the discharge rates for unit j at time k in (m^3/second).
K is the total scheduling period.
J is the total number of hydro units.

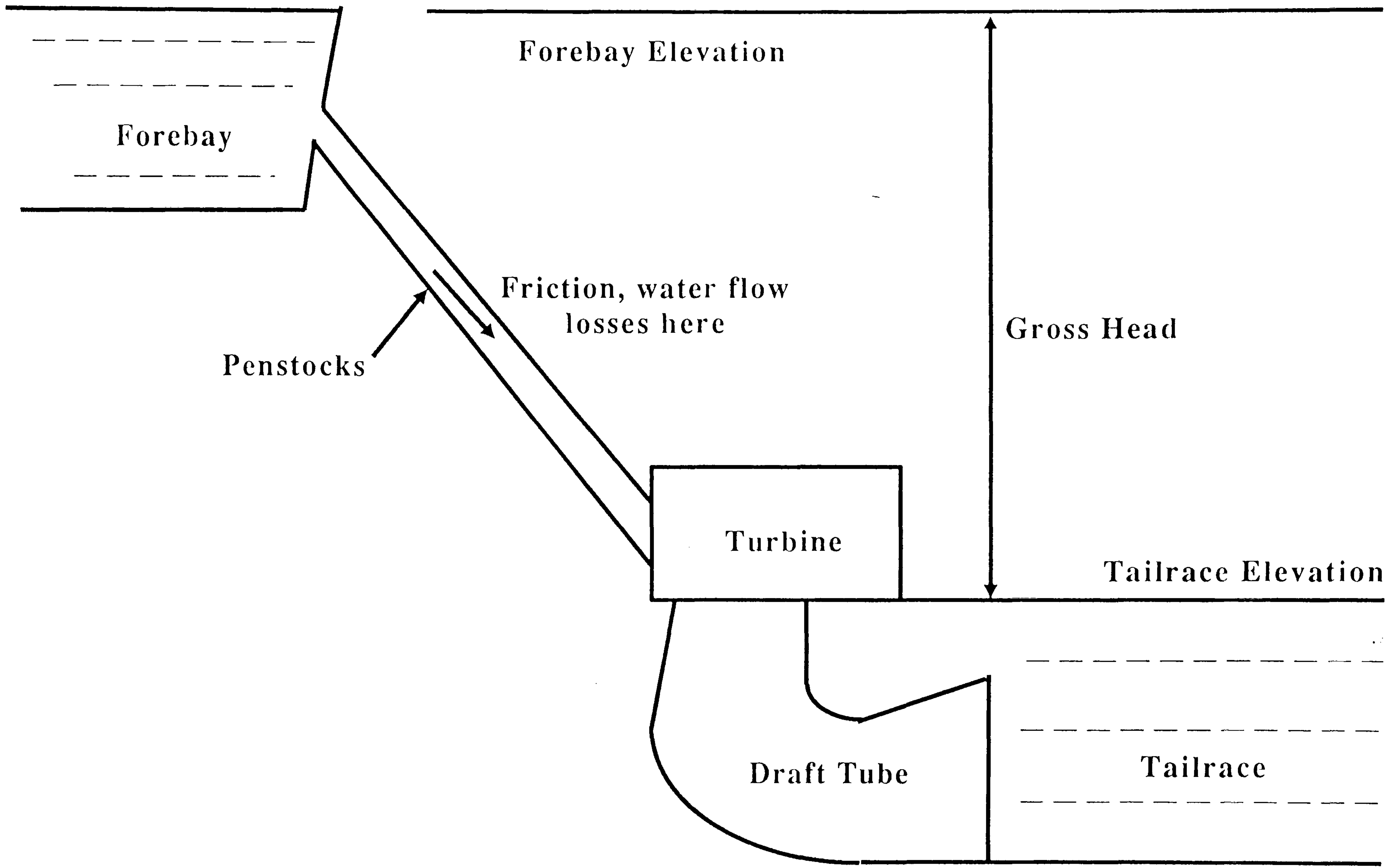


Diagram 4.3. Schematic Diagram of a Hydroelectric Power Station

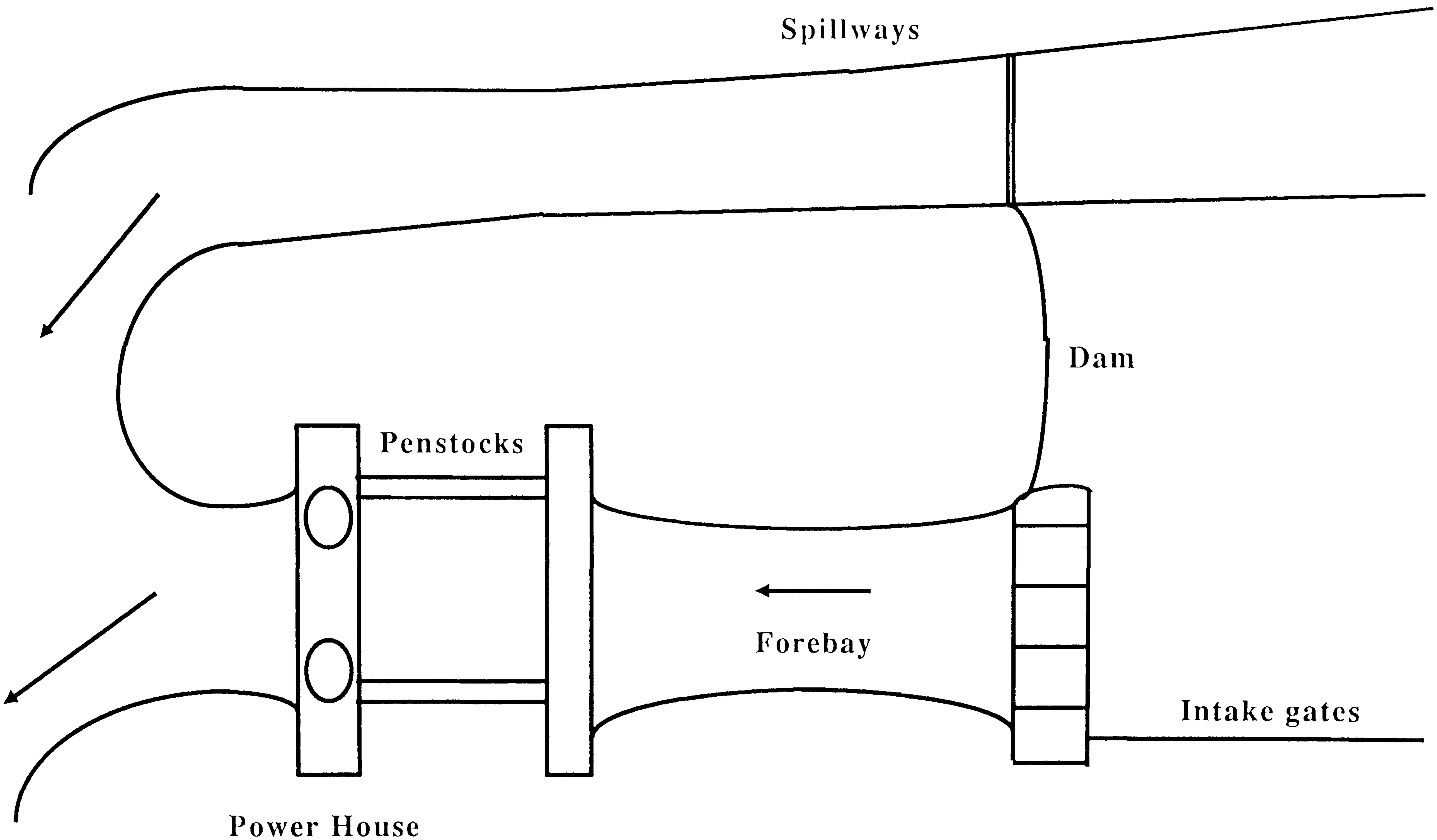


Diagram 4.4. Typical Conventional Hydroelectric Power Station Installation

To be precise, the output power (in MW) of a hydroelectric plant can be written as follows:[66.]

$$P_H(j,k) = [Q(j,k) * H(j,k)/(102.0 * \eta_{Tj} * \eta_{Gj})] \tag{4.2}$$

Where

η_{Tj} is the turbine efficiency

η_{Gj} is the generator efficiency.

102.0 is a constant coefficient.

If we define a total efficiency variable η_j given by

$$\eta_j = 1.0/102.0 * \eta_{Tj} * \eta_{Gj} \tag{4.3}$$

then an alternative form of hydroelectric power output function becomes:

$$P_H(j,k) = \eta_j * Q(j,k) * H(j,k) \tag{4.4}$$

There are many forms of model to represent the hydro plant performance function depending on how the plant efficiency is represented or the diversity of the installation characteristics. Since the plant efficiency depends on both the discharge rate and effective head, as seen from Diagram 4.5, when the discharge rate is low, the efficiency is low and the power output is low. When the discharge rate is higher than the best efficiency point, the efficiency again becomes low, resulting in a relatively lower power output. Only when the plant works at its optimal discharge rate can the plant achieve its maximum efficiency. Therefore the plant usually works at the best efficiency point or in other words, the best operating point. Also the discharge rate at the operating point increases when the head increases. It is then better to maintain a high head so that the discharge rate at the best operating point will be of high value, resulting in increased power output. Whereas if the head is low, to produce the same amount of power, the discharge rates must be much larger than in the high head situation, because of low efficiency.

Different models resulting from the representation of the efficiency can be summarized as follows:

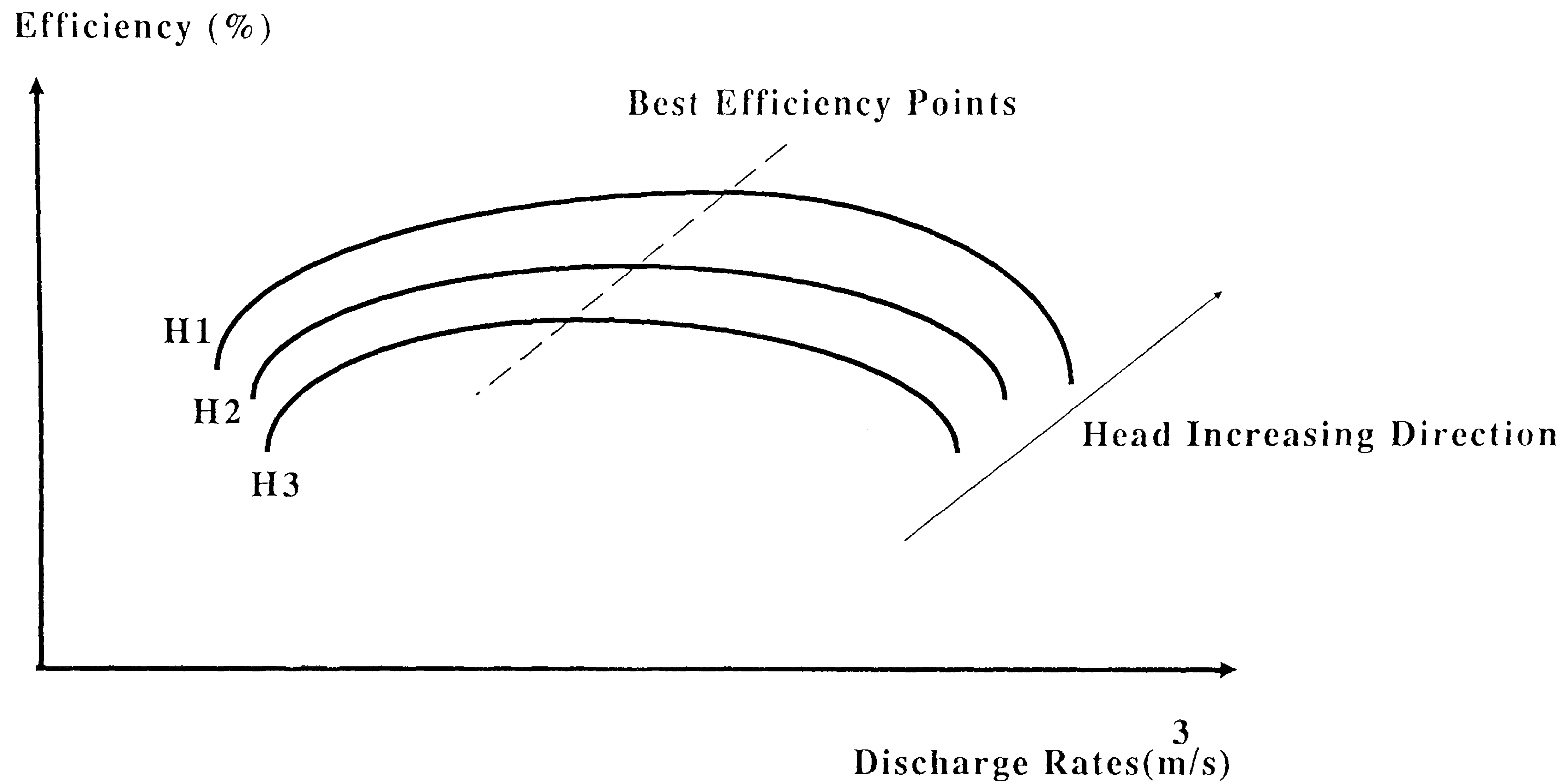


Diagram 4.5. Hydro Plant Efficiency vs. Discharge Rate and Net Head

1. The efficiency variation with the active power generation can be effectively modeled using the following quadratic expression:[66.]

$$A_1 * \eta_j^2 + A_2 * \eta_j + A_3 * P_H^2(j,k) + A_4 * P_H(j,k) + A_5 = 0 \qquad (4.5)$$

Where A_1, A_2, A_3, A_4 and A_5 are related coefficients.

2. The Glimn-Kirchmayer model[66.] describes the water discharge rates as a function of unit effective head and active power generation, it gives the variation of discharge rates $Q(j,k)$ as a bi-quadratic function of $H(j,k)$ and $P_H(j,k)$:

$$Q(j,k) = K_j * \psi_j(H(j,k)) * \phi_j(P_H(j,k)) \qquad (4.6)$$

Where K_j is a proportional constant and

$$\psi_j(H(j,k)) = a_{0j} + a_{1j} * H(j,k) + a_{2j} * H^2(j,k)$$

$$\phi_j(P_H(j,k)) = b_{0j} + b_{1j} * P_H(j,k) + b_{2j} * P_H^2(j,k)$$

3. The Hamilton-Lamont model[66.] also describes the water discharge rates as a function of effective head and active power generation, it gives the form as:

$$Q(j,k) = \upsilon_j(H(j,k)) * \omega_j(P_H(j,k))/H(j,k) \qquad (4.7)$$

Where a_{0j},a_{1j},a_{2j},b_{0j},b_{1j} and b_{2j} are coefficients.

$$\upsilon_j(H(j,k)) = a_{0j} + a_{1j} * H(j,k) + a_{2j} * H^2(j,k)$$

$$\omega_j(P_H(j,k)) = b_{0j} + b_{1j} * P_H(j,k) + b_{2j} * P_H^3(j,k)$$

4. The Arvanitidis-Rosing model[66.] takes into account the reservoir storage variable to show the change of efficiency rate.

$$P_H(j,k) = (\beta - \exp^{-\alpha * V(j,k)}) * H(j,k) * Q(j,k) \qquad (4.8)$$

Where $V(j,k)$ is the storage or volume of unit j at time k.

For the daily operational planning problem, the forebay elevation for reasonably large reservoirs will not change a lot, nor will the tailrace elevation. So the head variation is quite insignificant, and the model derived and used here is that the power generation is modeled as a polynomial function of discharge rates at its given head. If the head change does occur, the polynomials coefficients can be updated, hence, the head variation can be gradually taken into consideration. Thus we have:

$$\begin{aligned} P_H(j,k) &= a_{0j} + a_{1j} * Q(j,k) + a_{2j} * Q^2(j,k) \\ &\quad + a_{3j} * Q^3(j,k) + a_{4j} * Q^4(j,k) - Z(j,k) \end{aligned} \tag{4.9}$$

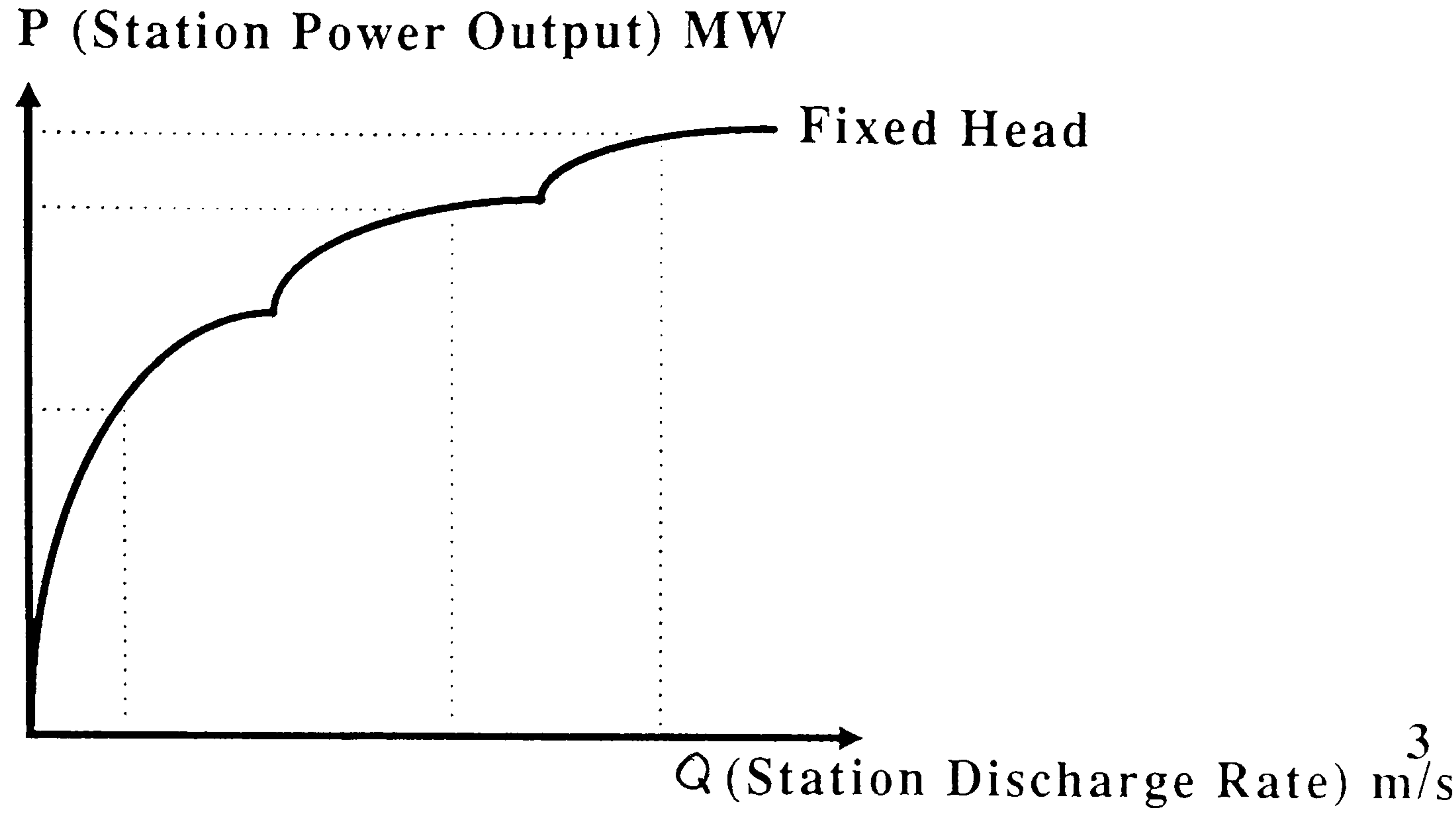
Where $Z(j,k)$ is due to the consideration of head losses.

Considering the power output as a function of discharge rate when the head is constant, as shown in Diagram 4.6, it is quite clear that the curve consists of N segments which corresponds to 1, 2, ... N units of the plant in operation. The best operating points for 1, 2, ... N units in operation are also shown by vertical dotted lines (Q_1, Q_2, ... Q_N) respectively. Under practical situations, the plant will operate within a reasonably small variation from these best operating points, but will never operate at the intersection of these segments because at these intersection points, the plant efficiencies will be very low.

With further piecewise linear approximation based on these unit best efficiency points (the operating points for each unit in the plant), and taking into account that the head variation will not change dramatically during daily operation and therefore can be ignored, the power output model for a hydro station at a given head during a time interval can be derived as a linear model. The linear power output function may be expressed as:

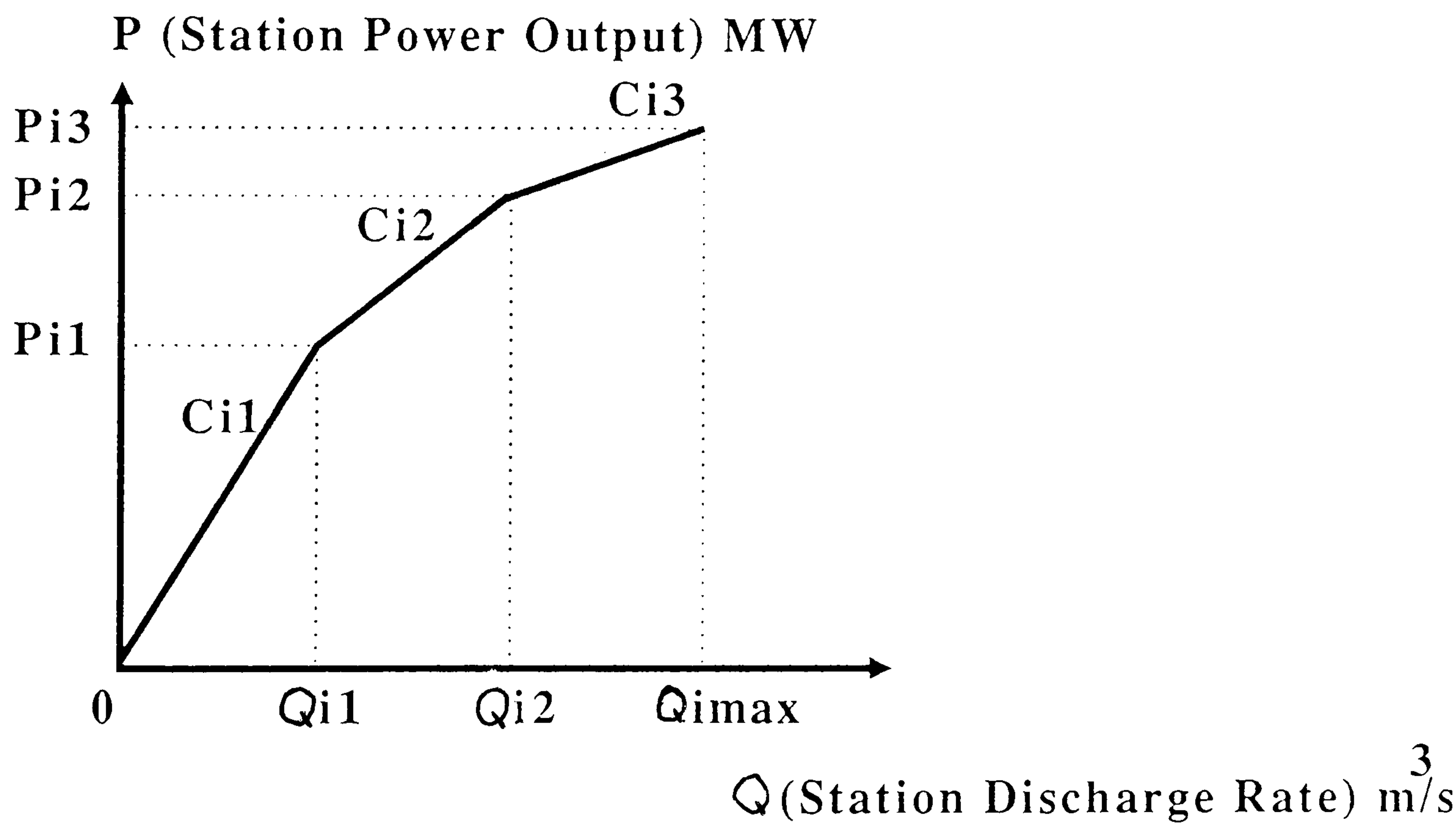
$$P_H(j,k) = \sum_{n}^{N} \eta_{jn} * Q_n(j,k) \tag{4.10}$$

Where j stands for plant number, and n stands for the unit or segment number of the piecewise linear function of hydro power production. $Q_n(j,k)$ stands for the unit discharge rate and is one of the components of the station discharge $Q(j,k)$, corresponding to a line segment n of the piecewise linear approximation of the power production function, and η_{jn} is the unit generating efficiency, corresponding to the line slope of the piecewise linear approximation of this power production function. Through connecting the minimum power output



Nonlinear Hydroelectric Station

Production Function



Piecewise Linear Hydroelectric

Production Function

Diagram 4.6. Nonlinear Production Function and Piecewise Linear Production Function

point, the maximum output point, and the best operating points by a piecewise linear curve derived from the original nonlinear curve, as shown in Diagram 4.6, a linear power function is obtained. The piecewise linear curve is illustrated in Diagram 4.6. So

$$Q(j,k) = \sum_{n}^{N} Q_n(j,k) \tag{4.11}$$

furthermore, by updating the unit efficiency points resulting from the head changes, the nonlinearity of the power output function can also be included through linear successive approximation techniques.

It is quite clear that the piecewise linear approximation of the hydro generation function is very close to the accurate nonlinear model near the best operating points, but may be quite inaccurate around the intersection points of different segments. This linear approximation is therefore very suitable for short-term studies with the best operating points policy, since, the plant will never be operated near the intersection points in practice.

4.2.4 Reservoir Storage Model (H-V or Forebay Elevation-V)

To obtain an operational model of a turbine it is necessary to know the head-storage characteristics of the reservoir and its appropriate mathematical model. The modelling of the reservoir storage forms a crucial part of the study of hydroelectric operations.

For man-made reservoirs, which have generally known geometric regular shapes, it is possible to calculate the volume contents of these reservoirs from their dimensions and to establish the head-storage characteristics directly. For natural reservoirs, topographical surveys must be carried out to find the surface area of the reservoirs at different levels and hence to determine the volume contents from various elevation levels. Usually, when the reservoir storage is high, the head variation rate is small; when the storage becomes lower, the head variation tends to be large. This results in a nonlinear head-storage relationship.

Once the head-storage characteristic curve has been found, it is possible to fit a quadratic or cubic curve for the head-storage relationship using the well-known least squares curve-fitting method to find the constant coefficients of this nonlinear function.

The model used here is a nonlinear function as proposed by El-Hawary[66.] et al, up to the second order. In this case, the reservoir is actually assumed to be a trapezoidal shape. See Diagram 4.7. The evaporation and seepage losses of water in reservoirs are ignored. So we have

$$V(j,k) = D_{0j} + D_{1j} * H(j,k) + D_{2j} * H^2(j,k) \tag{4.12}$$

Where D_{0j}, D_{1j} and D_{2j} are related coefficients.

Alternatively, the reservoir can be assumed to be vertical-sided, this assumption results in the linear reservoir model with

$$V(j,k) = D_{0j} + D_{1j} * H(j,k) \tag{4.13}$$

Where D_{1j} is the surface area of the reservoir. This linear relationship is used as the reservoir storage versus head model as shown also in Diagram 4.7.

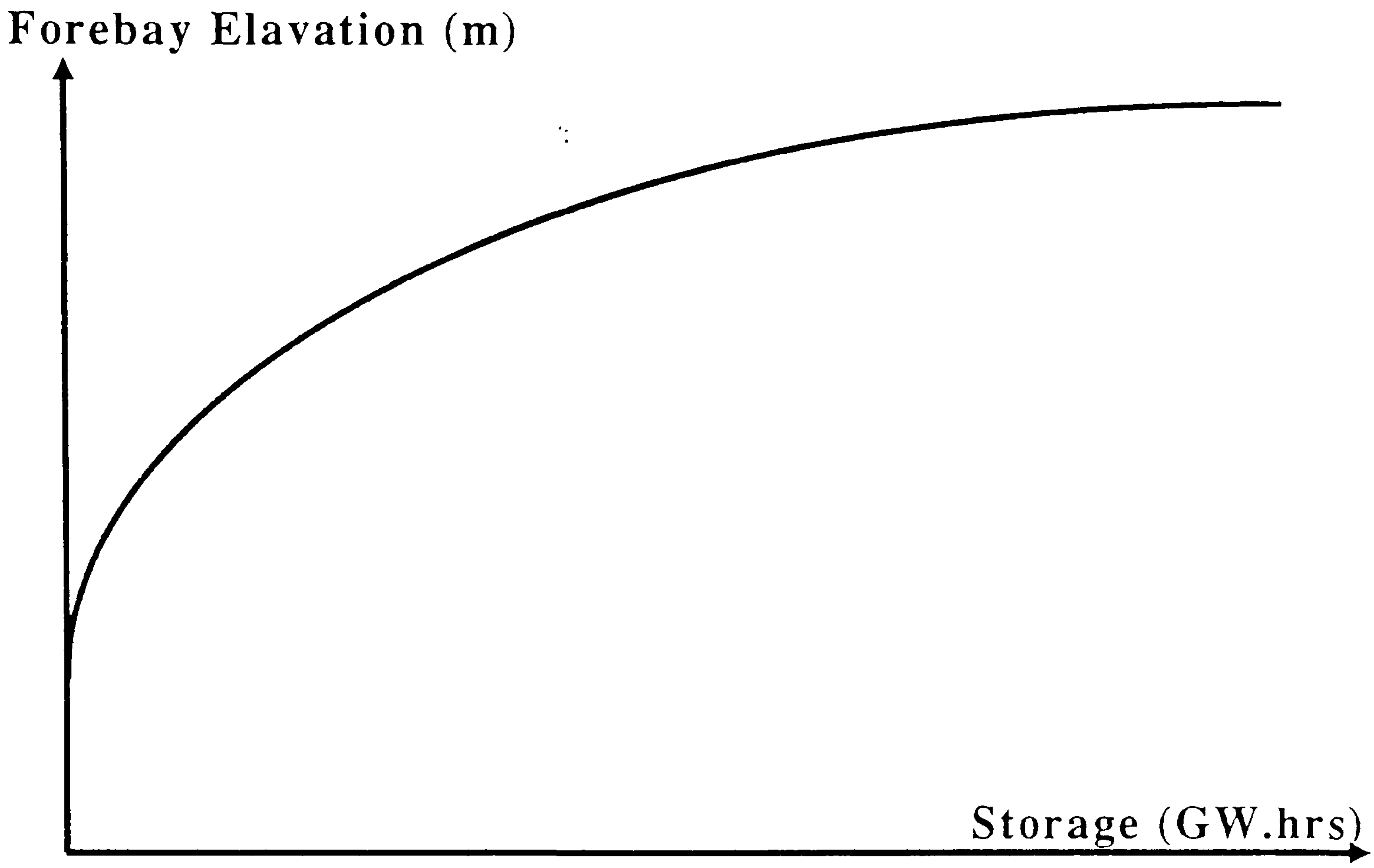
4.2.5 Effective Head Model (H)

The head of a hydroelectric plant is actually the elevation difference between the forebay level and the tailrace level. As there are always head losses when water passes through the penstock due to the friction in water flows, the effective head for a hydroelectric plant or the net head of the plant is actually the quantity that determines the power plant production that can be generated; it is obtained by subtracting the head losses from the gross head. See Diagram 4.8 for illustration. The head losses depend on the discharge rate as also shown in Diagram 4.8. Thus

$$H_{gross}(j,k) = E_{forebay}(j,k) - E_{tailrace}(j,k) \tag{4.14}$$

$$H_{effective}(j,k) = H_{gross}(j,k) - \Delta H(j,k) \tag{4.15}$$

$$\Delta H(j,k) = \Delta H_{0j} + A_j * Q(j,k) \tag{4.16}$$



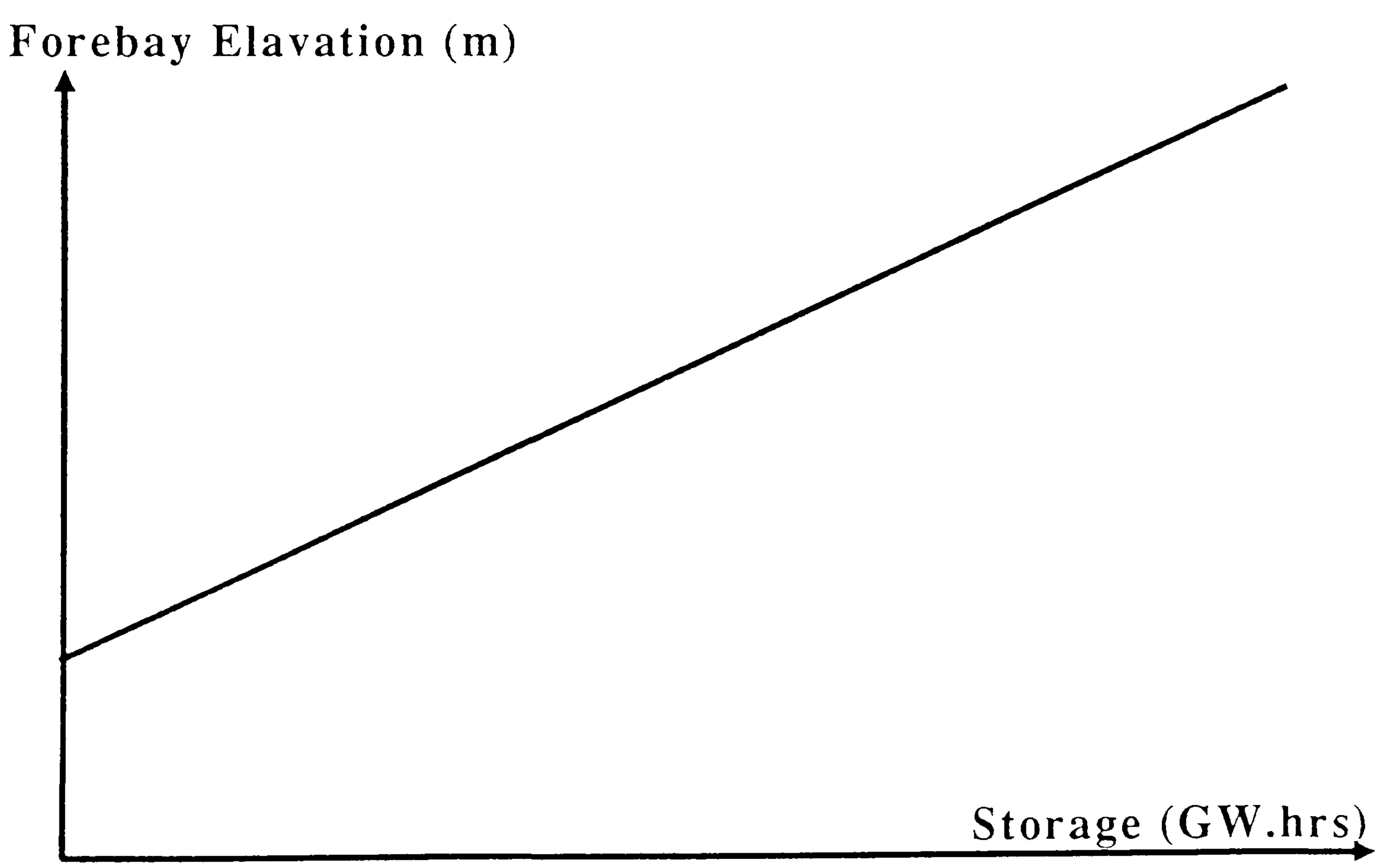


Diagram 4.7. Reservoir Elevation-Storage Curve

The tailrace elevation is dependent on water discharge rate and spillage. See Diagram 4.9.

$$E_{tailrace}(j,k) = E_{tailrace}0j + C_j * (Q(j,k) + S(j,k)) \tag{4.17}$$

Where $S(j,k)$ is the spillage in the case that water is spilled.

The forebay elevation is a function of reservoir geometry, natural inflow, water discharge rate and spillage. For variable head power plant, it is necessary to update the reservoir volume to obtain the actual forebay elevation $E_{forebay}(j,k)$. The entire model of the effective head then becomes:

$$\begin{aligned} H_{effective}(j,k) = E_{forebay}(j,k) - (E_{tailrace}0j + \Delta H_{0j}) \\ - C_j * [Q(j,k) + S(j,k)] - A_j * Q(j,k) \end{aligned} \tag{4.18}$$

4.2.6 Turbines

Generally, hydro turbines can be divided into two types, namely, reaction type and impulse type. In the reaction turbines, the potential energy of water under pressure is only partly converted into the kinetic energy of water at velocity before it enters the turbine runners, while in the impulse turbines, all the potential energy is converted into kinetic energy before it enters the turbine runner. The reaction type turbine is commonly used. Among this category are the Francis wheel turbine, the Kaplan wheel turbine and the propeller wheel turbine. The different types of turbine have different operating characteristics, see Diagram 4.10.

4.2.7 Reservoir Dynamics and Hydraulic Network Modelling

A hydroelectric system may consist of a number of power stations, possibly sited on different river valleys. The hydroelectric power plants on the same river have a hydrologically coupled relationship since the discharge rate from one plant will affect the operation of its downstream plants, i.e, the discharge rate of the upstream plant constitutes a part of the water inflows into the downstream plants. The coupled river system model can be as illustrated in Diagram 4.11.

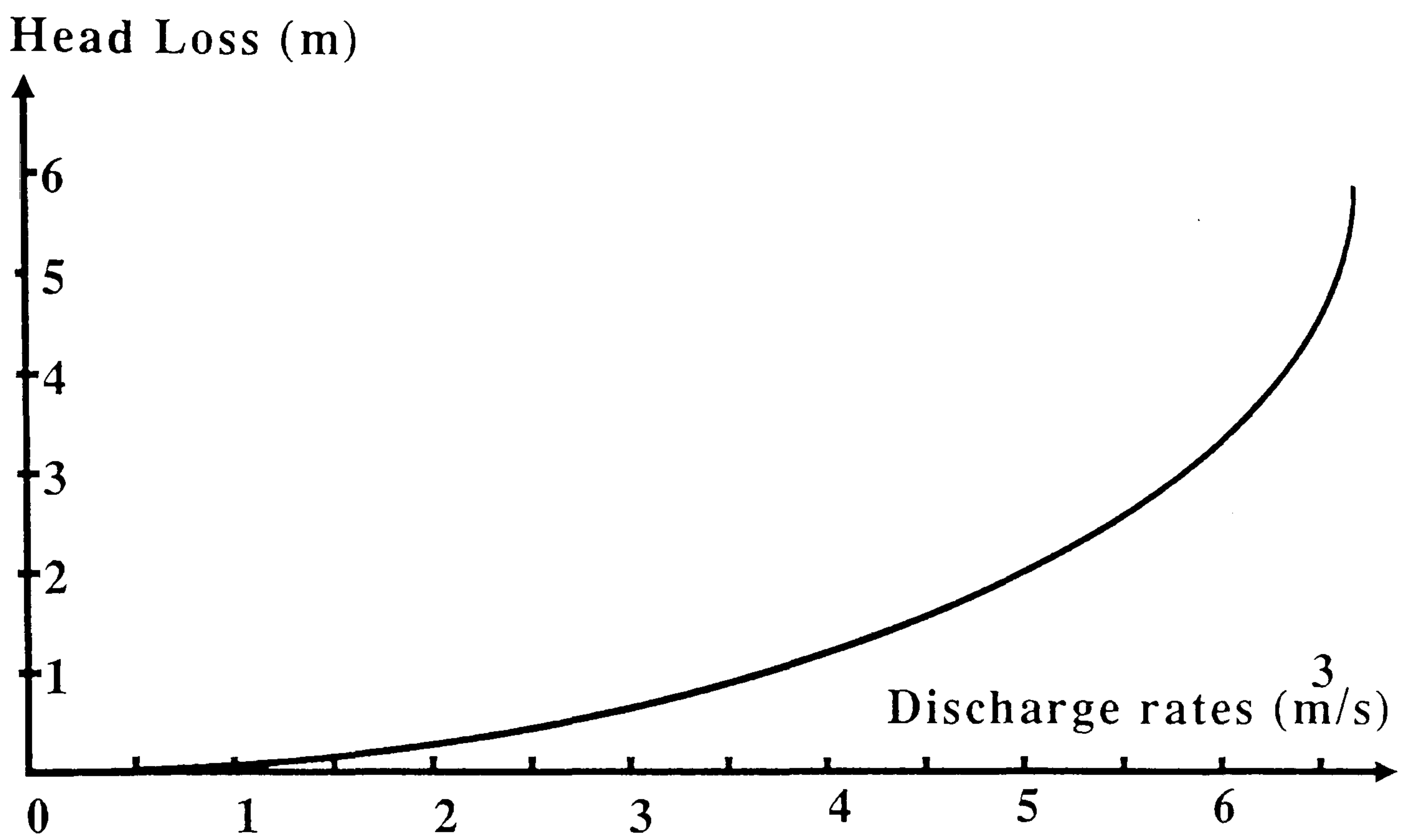


Diagram 4.8. Head Loss vs. Water Discharge

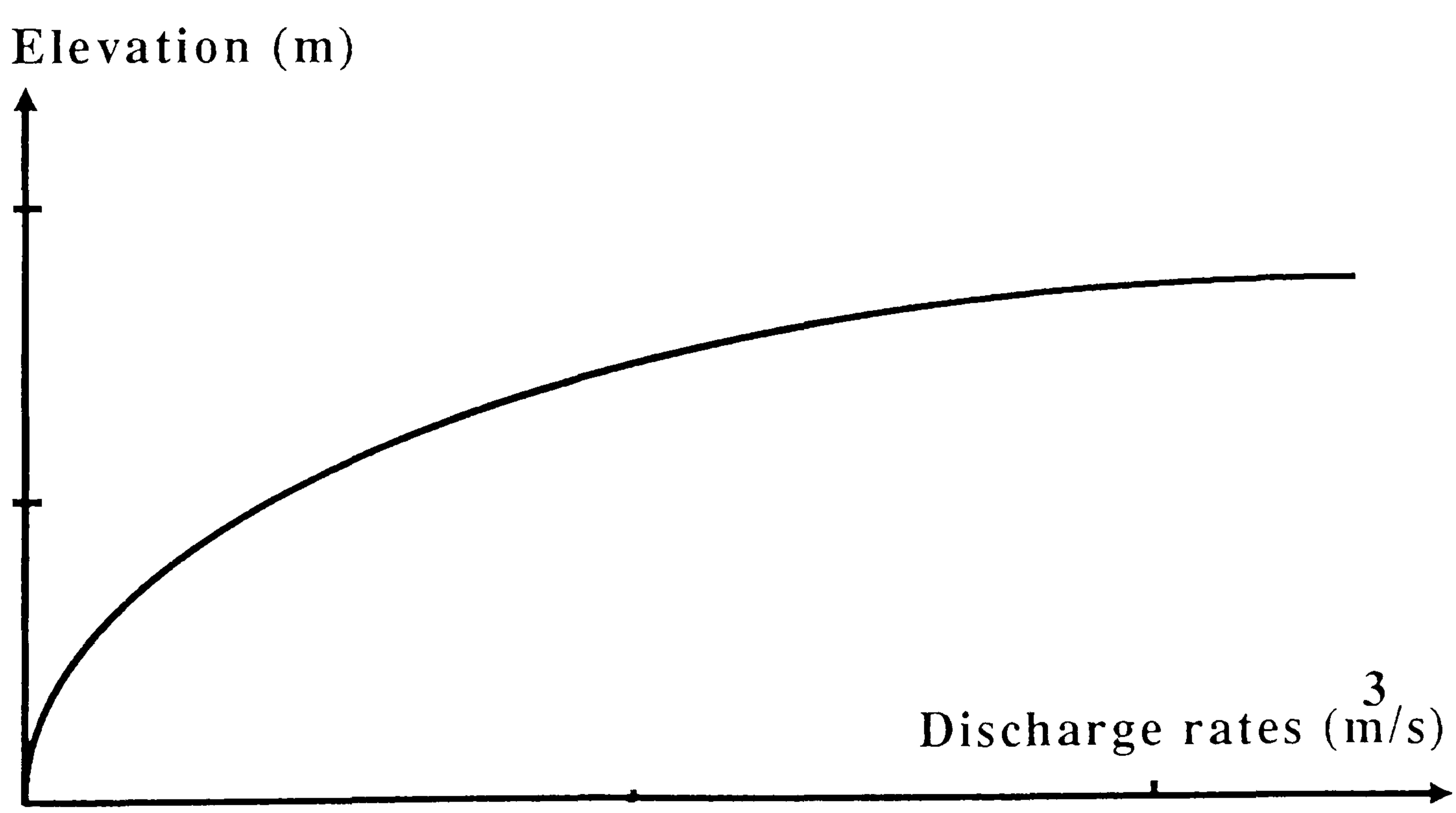


Diagram 4.9. Tailrace Elevation vs. Discharge

FRANCIS WHEEL KAPLAN WHEEL PROPELLER WHEEL

Diagram 4.10. Hydro Turbines Performance Characteristics

(Head values are per unit)

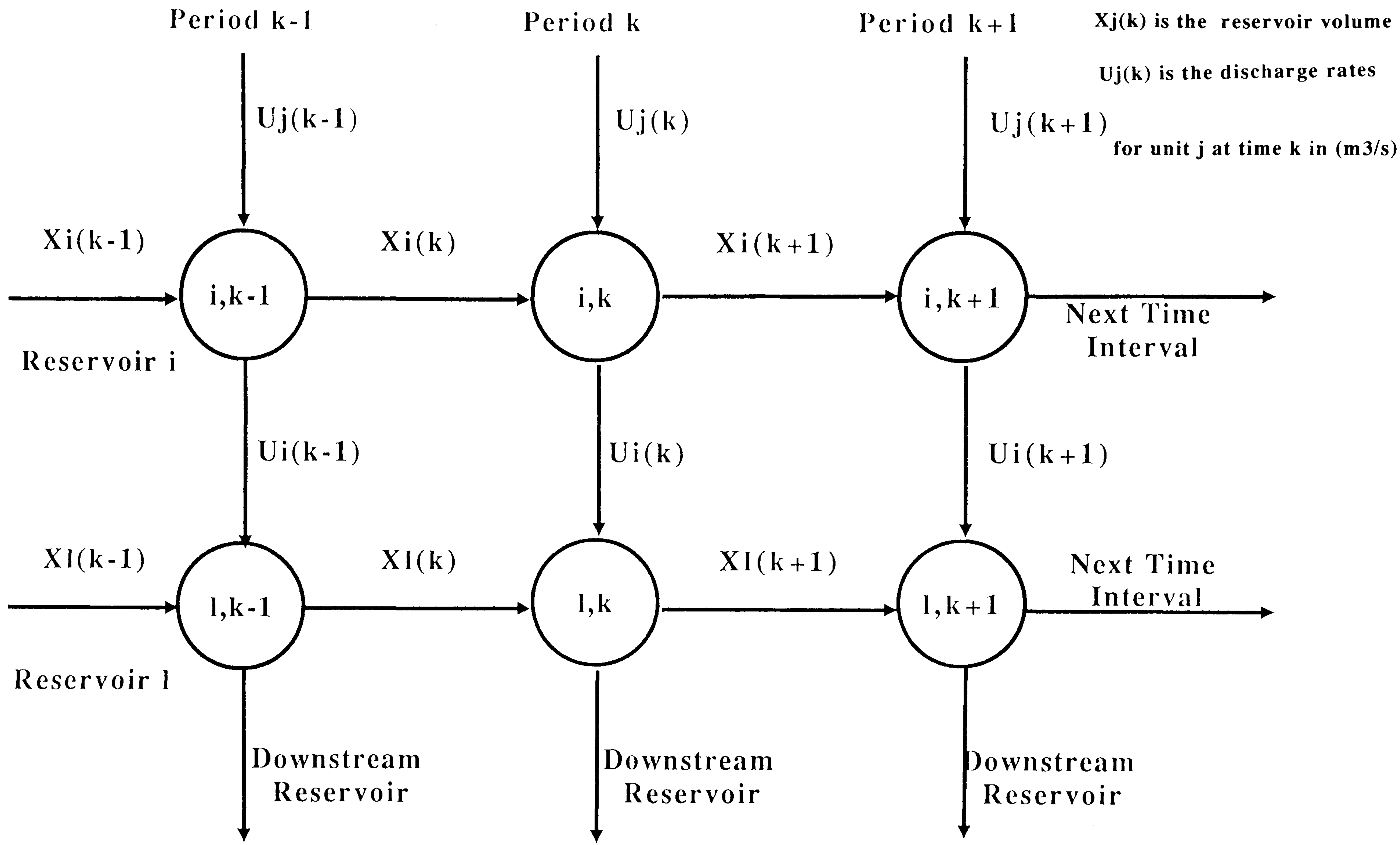


Diagram 4.11. Network Structure of Reservoir Dynamics

The hydraulic coupling between the plants located on the same river, and the effects of the upstream discharges on the downstream plants forebay is of great importance and necessitates river network modelling. This modelling is even more important and significant when the time delay of the water transport is greater than the scheduling time interval. This is the case when there is a very long distance interconnection between the cascaded plants in the same valley. The state-space approach is available for this kind of modelling, i.e.

$$dV/dt = INF(t) - Q(t) - S(t)$$

Where dV/dt stands for the reservoir volume changing rates, $INF(t)$ stands for the water inflow rate. However, for a large, highly interconnected hydroelectric system, it is not practically acceptable to apply this model because of the high dimensionality problem it will cause. Moreover, if reservoirs have quite long distances between each other, it is necessary to include the water transport delay time into the formulation of the reservoir dynamics model. To conclude, the approach which may be employed in this work for river network modelling is the transport delay model suggested by El-Hawary et al.[66.] The model is derived based on the water flow conservation equation, the dynamics of the hydro reservoirs can be represented by difference equations as follows:

$$\begin{aligned} V(j,k+1) = V(j,k) - Q(j,k) - S(j,k) + \sum_{i}^{M} Q(i,k-t_{ij}) \\ + \sum_{i}^{M} S(i,k-t_{ij}) + INF(j,k) \qquad j \in J, k \in J \end{aligned} \tag{4.19}$$

Where

$V(j,k)$ is the content of the j:th reservoir at the beginning of interval k

$Q(j,k)$ is the discharge rate of the j:th reservoir during interval k

$S(j,k)$ is the spillage of the j:th reservoir during interval k

$INF(j,k)$ is the natural inflow into the j:th reservoir during interval k

$\sum_i^M Q(i,k-t_{ij})$ is the discharge rates of the upstream reservoirs of reservoir j during interval k, if there is no such upstream reservoir exists, then $\sum_i^M Q(i,k-t_{ij})$ will be zero

$\sum_i^M S(i,k-t_{ij})$ is the spillage of the upstream reservoirs during interval k

t_{ij} is the water transport delay from the upstream reservoir i to downstream reservoir j

$k \in K$, K is the total time period considered

$j \in J$, J is the total hydro stations considered

$i \in M$, M is the total upstream reservoirs of reservoir i

4.2.8 The Operating Constraints

The operation of hydroelectric plants can not be isolated from other multi-purpose developments of the river system. The maximum and minimum limits on the reservoir contents and discharge rates must take into account time dependent constraints on environmental requirements and recreational considerations such as irrigation, flood control, water supply, navigation, etc and other hydro power station technical limitations. The upper and lower bound for reservoir volumes and discharges, or even spillage rates can be expressed as:

$$V_{min}(j,k) \leq V(j,k) \leq V_{max}(j,k) \qquad j \in J, k \in K \tag{4.20}$$

$$Q_{min}(j,k) \leq Q(j,k) \leq Q_{max}(j,k) \qquad j \in J, k \in K \tag{4.21}$$

$$S_{min}(j,k) \leq S(j,k) \leq S_{max}(j,k) \qquad j \in J, k \in K \tag{4.22}$$

Seasonal operational planning specifies the weekly initial and final values of the reservoir volume. The short-term scheduling problem in a hydrothermal power system is to find the best way to use the limited water energy available over one week or one day in order to achieve the minimum production cost of the whole system. The initial and final reservoir storage must be in the predefined range. We take the initial reservoir volume as a fixed value

$$V(j,1) = V(j,0) \tag{4.23}$$

and the final reservoir volume must be in the predefined range.

$$V(j,K+1) \geq V(j,K) \tag{4.24}$$

4.3 MODELLING OF THE THERMAL SUBSYSTEM

The modelling of the thermal subsystem for short-term hydrothermal generation scheduling is very similar to the model for thermal unit commitment.

A more detailed description of this part of the model will be presented in Chapter 6.

4.3.1 Thermal Plant Performance Modelling

Thermal plants can be divided into two categories depending on how steam is produced. There are conventional thermal plants and nuclear thermal plants. In the conventional thermal plants, or sometimes so-called hydrocarbon or fossil steam plants, hydrocarbon fossil fuels such as coal, oil, or natural gas are burnt in the boiler, and heat the water into steam. In nuclear thermal plants, the fuel assemblies containing nuclear fuels are loaded into the nuclear reactor core, and steam is produced through the heat resulting from the nuclear reaction. The steam produced has a high temperature and a high pressure, and carries an enormous heat energy which is converted to mechanical energy via turbines. The resulting rotational mechanical energy is converted into electrical energy via alternators. There are also combustion turbines that burn liquid or gaseous fuel directly producing mechanical energy without the intermediate step of steam production.

More detailed plant dynamics modelling is beyond the scope of the economic operational planning and consequently this thesis. A conventional fossil-fired power station layout is given in Diagram 4.12. Only conventional thermal plant performance modelling is considered here, which is the plant input (the fuel production cost) and output (the active power generation output) model. The model used here is the fuel cost curve for each generating unit, the nonlinearity of the fuel cost to the output power of an operating unit i at power level $P_T(i,k)$ over interval k is taken as a quadratic function of the active power generation $P_T(i,k)$. That is:

$$F_i(P_T(i,k)) = A_i + B_i * P_T(i,k) + C_i * {P_T}^2(i,k) \tag{4.25}$$

4.3.2 The Operating Constraints

The short-term hydrothermal generation scheduling formulation must take into the consideration the various operating constraints imposed on the

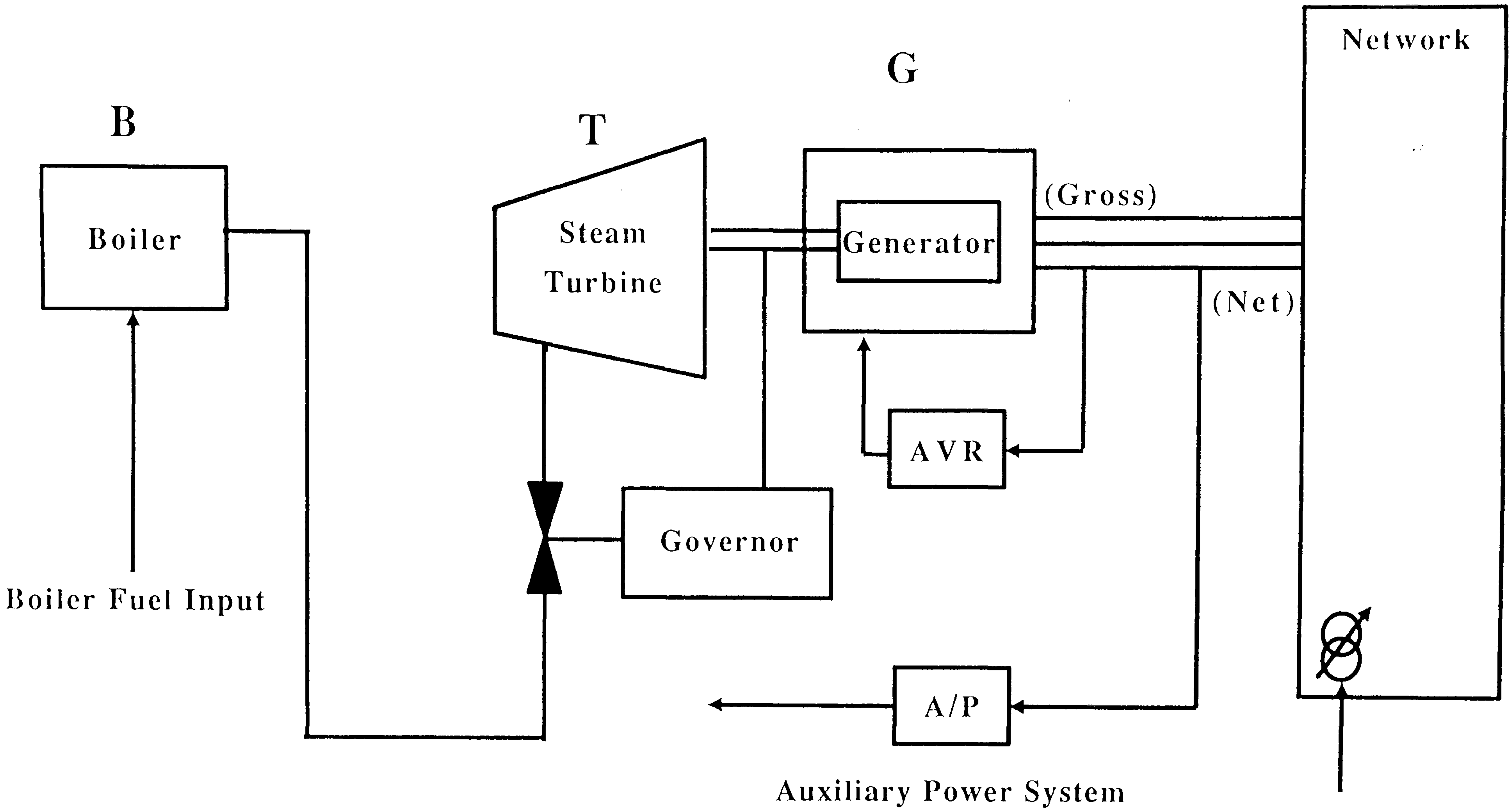


Diagram 4.12. Boiler-Turbine-Generator Thermal Unit

individual thermal unit and the system. All these constraints are discussed as follows:

1. Each thermal unit can be operated at one of two states: When the unit is up or committed into the operation, we denote the state by integer 1, if the unit is down or decommited, the state is denoted by integer 0. We have the state variable $X(i,k)$ for unit i at interval k:

$$X(i,k) = \begin{cases} 0, & \text{if unit } i \text{ is 'off'.} \\ 1, & \text{if unit } i \text{ is 'on'.} \end{cases} \tag{4.26}$$

The startup or shutdown decision variable for unit i at interval k is denoted as $U(i,k)$.

$$U(i,k) = \begin{cases} 0, & \text{if unit } i \text{ is decided to be 'off'.} \\ 1, & \text{if unit } i \text{ is decided to be 'on'.} \end{cases} \tag{4.27}$$

2. The unit minimum up time and minimum down time constraints such as T_{minup} and $T_{mindown}$ are considered.
3. We will show that each thermal unit has a total number of possible states of $T_{minup} + T_{mindown}$ and can be in any one of these states. see the state transition diagram of thermal unit i in Diagram 4.13 for details. So

$$X_S(i,k+1) = \begin{cases} X_S(i,k), & \text{if } X_S(i,k) = 1, \text{ and } U(i,k) = 0 \\ X_S(i,k) + T_{mindown}, & \text{if } X_S(i,k) = 1, \text{ and } U(i,k) = 1 \\ 1, & \text{if } X_S(i,k) = Maxstate, \text{ and } U(i,k) = 0 \\ Maxstate, & \text{if } X_S(i,k) = Maxstate, \text{ and } U(i,k) = 1 \\ X_S(i,k) + 1, & \text{if } 1 < X_S(i,k) < Maxstate \end{cases} \tag{4.28}$$

4. The startup and shut-down cost for unit i are denoted by $ST_i(X(i,k), U(i,k))$, this cost is dependent on the state $X(i,k)$ and decision $U(i,k)$. It is also possible to allow for the startup cost to be depended on the number of time periods that the unit has been shutdown prior to this operation. Thus

$$ST_i(X(i,k), U(i,k)) = C_{coldstart}(i) * \alpha(i) * T_{down}(i)/(1 + \alpha(i) * T_{down}(i)) \tag{4.29}$$

The shutdown cost of a thermal unit is normally much smaller compared with its startup cost, and is therefore considered here to be a fixed value.

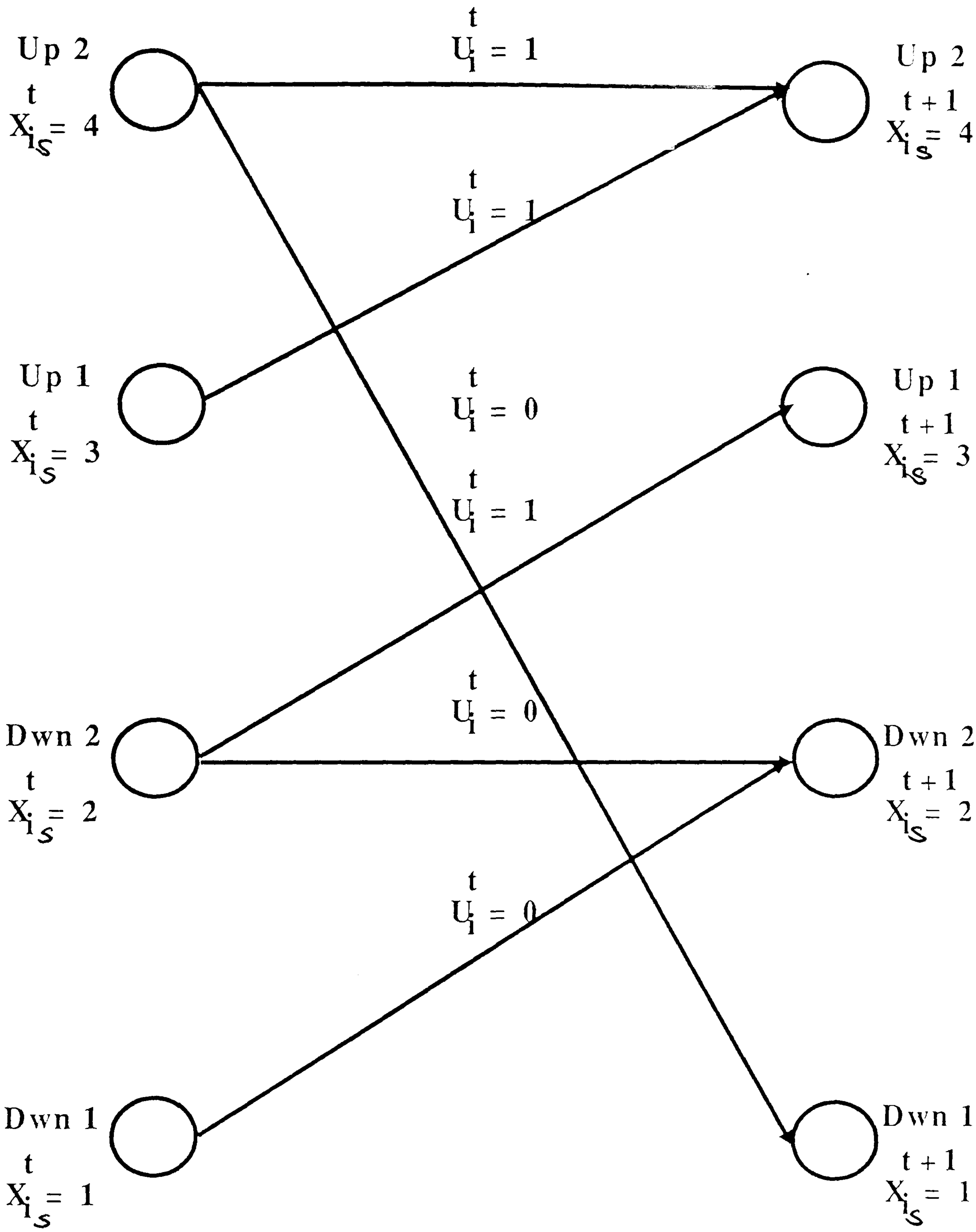


Diagram 4.13. State Transition Diagram

(Tminup=2 hours, Tmindown=2 hours)

5. The constraints on the power output are defined as the unit minimum and maximum output limits. This output constraint defines the allowable output power of the generating units. i.e.

$$P_{imin} \leq P_T(i,k) \leq P_{imax} \tag{4.30}$$

6. The nonlinear relationship between the fuel cost and the output power for an operating unit i at power level $P_T(i,k)$ over interval k is taken as a quadratic function.

$$F_i(P_T(i,k)) = \begin{cases} 0, & \text{if } U(i,k) = 0 \\ A_i + B_i * P_T(i,k) + C_i * {P_T}^2(i,k), & \text{if } U(i,k) = 1. \end{cases} \tag{4.31}$$

7. The unit ramping rates for decrease or increase are included as

$$\Delta P_T(i,k) \leq P_{iramp} \tag{4.32}$$

4.4 ELECTRIC TRANSMISSION NETWORK MODEL

For reasons of simplification, the detailed transmission network model is not considered in the problem formulation. From generation to transmission, the constraints involved are only represented by active power balance equations and active power transmission loss equations. The load demand and generation balance requirement and the reserve requirement for each time interval are the only coupling constraints between the hydro subsystem and the thermal subsystem.

$$P_D(k) - \sum_{i}^{I} P_T(i,k) - \sum_{j}^{J} P_H(j,k) < 0 \tag{4.33}$$

$$P_R(k) - \sum_{i}^{I} \Delta P_T(i,k) - \sum_{j}^{J} \Delta P_H(j,k) \leq 0 \tag{4.34}$$

Where $\Delta P_T(i,k)$ is the spare capacity of unit i at time interval k and $\Delta P_T(i,k) = P_{imax} - P_T(i,k)$ and $\Delta P_H(j,k)$ is the spare capacity of unit j at time interval k and $\Delta P_H(j,k) = P_{jmax} - P_H(j,k)$.

4.5 COMPLETE MODEL OF HYDROTHERMAL SCHEDULING

The complete model of short-term generation scheduling for a hydrothermal power system consists of an optimization objective function and various constraints from the hydro subsystem, thermal subsystem and transmission network. The variables include integer variables for thermal unit "on" and "off" decisions and real variables for thermal unit output levels at each time interval, and real variables for water discharge rates and reservoir volume contents at each time interval. The variables from the hydro subsystem and thermal subsystem respectively will be coupled through the transmission network, and this gives the rise to coupling constraints.

4.5.1 The Objective Function

As described above, because hydroelectric generation has negligible marginal costs, the overall objective of this optimization problem is to minimize the total production cost of thermal generation over the scheduling period. Suppose a hydrothermal power system contains I thermal units and J hydro units, the hydrothermal unit commitment problem is to schedule the startup, the shutdown and the unit generating level of all the units over the scheduling period K so that the total production cost (including thermal fuel cost, startup cost and shutdown cost) of the system will be minimized while satisfying the load demand and reserve requirement. The scheduling problem for hydrothermal generation becomes:

$$Min \qquad \sum_{k}^{K}\sum_{i}^{I}(F_i(P_T(i,k)) + ST_i(X(i,k), U(i,k))) \tag{4.35}$$

4.5.2 The Constraint Sets

There are constraints from different parts of the model: the thermal subsystem, the hydro subsystem, and the transmission network as stated previously. We present all these constraints in mathematical form as follows:

1. The hydro subsystem:

$$P_H(j,k) = \sum_{n}^{N} \eta_{jn} * Q_n(j,k) \tag{4.36}$$

$$Q(j,k) = \sum_{n}^{N} Q_n(j,k) \tag{4.37}$$

$$V(j,k) = D_{0j} + D_{1j} * H(j,k) \tag{4.38}$$

$$\begin{aligned} H_{effective}(j,k) &= E_{forebay}(j,k) - (E_{tailrace}0j + \Delta H_{0j}) \\ &- C_j * (Q(j,k) + S(j,k)) - A_j * Q(j,k) \end{aligned} \tag{4.39}$$

$$\begin{aligned} V(j,k+1) &= V(j,k) - Q(j,k) - S(j,k) + \sum_{i}^{M} Q(i,k-t_{ij}) \\ &+ \sum_{i}^{M} S(i,k-t_{ij}) + INF(j,k) \qquad j \in J, k \in J \end{aligned} \tag{4.40}$$

$$V_{min}(j,k) \le V(j,k) \le V_{max}(j,k) \qquad j \in J, k \in K \tag{4.41}$$

$$Q_{min}(j,k) \le Q(j,k) \le Q_{max}(j,k) \qquad j \in J, k \in K \tag{4.42}$$

$$S_{min}(j,k) \le S(j,k) \le S_{max}(j,k) \qquad j \in J, k \in K \tag{4.43}$$

$$V(j,1) = V(j,0) \tag{4.44}$$

$$V(j,K+1) \ge V(j,K) \tag{4.45}$$

2. The thermal subsystem:

$$X(i,k) = \begin{cases} 0, & \text{if unit } i \text{ is 'off'.} \\ 1, & \text{if unit } i \text{ is 'on'.} \end{cases} \tag{4.46}$$

$$U(i,k) = \begin{cases} 0, & \text{if unit } i \text{ is decided to be 'off'.} \\ 1, & \text{if unit } i \text{ is decided to be 'on'.} \end{cases} \tag{4.47}$$

$$X_S(i,k+1) = \begin{cases} X_S(i,k), & \text{if } X_S(i,k) = 1, \text{ and } U(i,k) = 0 \\ X_S(i,k) + T_{mindown}, & \text{if } X_S(i,k) = 1, \text{ and } U(i,k) = 1 \\ 1, & \text{if } X_S(i,k) = Maxstate, \text{ and } U(i,k) = 0 \\ Maxstate, & \text{if } X_S(i,k) = Maxstate, \text{ and } U(i,k) = 1 \\ X_S(i,k) + 1, & \text{if } 1 < X_S(i,k) < Maxstate \end{cases} \tag{4.48}$$

$$P_{imin} \le P_T(i,k) \le P_{imax} \tag{4.49}$$

$$ST_i(X(i,k), U(i,k)) = C_{coldstart}(i) * \alpha(i) * T_{down}(i)/(1 + \alpha(i) * T_{down}(i)) \tag{4.50}$$

$$F_i(P_T(i,k)) = \begin{cases} 0, & \text{if } U(i,k) = 0 \\ A_i + B_i * P_T(i,k) + C_i * {P_T}^2(i,k), & \text{if } U(i,k) = 1. \end{cases} \tag{4.51}$$

$$\Delta P_T(i,k) \le P_{ramp} \tag{4.52}$$

3. The electrical transmission network:

$$P_D(k) - \sum_i^I P_T(i,k) - \sum_j^J P_H(j,k) \leq 0 \tag{4.53}$$

$$P_R(k) - \sum_i^I \Delta P_T(i,k) - \sum_j^J \Delta P_H(j,k) \leq 0 \tag{4.54}$$

CHAPTER 5

UNIT COMMITMENT IN THERMAL POWER SYSTEMS

5.1 INTRODUCTION

The major issue of concern in this chapter is the long-standing problem of optimal unit commitment in a large scale thermal power system. The material contained in the sections of this chapter are arranged as follows: Firstly, an introduction to the need for unit commitment is presented, then a brief discussion of the previous optimization algorithms applied to the solution of thermal unit commitment problems is outlined, followed by the problem formulation in Section 5.3, where a detailed unit commitment mathematical model is formulated, including most of the important operating constraints. Then a presentation of efficient optimization methods employed in this project for solving the large scale thermal unit commitment problem is given in the following sections. All these solution algorithms employed will be described in full together with a detailed analysis of the results. Details of a merit-order heuristic scheme, a dynamic programming based composite cost function model and an algorithm based on the Lagrangian relaxation methodology are also given. Tests and comparisons are made in the later sections to demonstrate the advantages and disadvantages of each method. The test results have shown the efficiency and flexibility of both the dual Lagrangian relaxation methodology and the composite cost dynamic programming model. The results have also proved that the algorithm based on Lagrangian relaxation method gives very good numerical results for realistic large sized systems.

5.2 AN INTRODUCTION TO UNIT COMMITMENT

Cost savings can be achieved by proper startups and shutdowns of the available thermal units through a unit commitment program. A typical unit commitment problem is commonly defined as a problem that must be solved on a daily (24 hours) or weekly (168 hours) basis by a power utility of how to determine a generation schedule of units that will be used and their loading levels. The solution should meet the load demand anticipated over a 24 hour or 168 hour future period so that the total operating cost for this generating system in this period will be a minimum.

To solve this unit commitment problem, generally both the startup schedule, the shutdown schedule and the loading levels of committed units must be considered simultaneously in order to achieve an overall least cost schedule. In fact, there are two basic decisions to be made for a unit commitment problem: the "unit commitment" decision and the "economic dispatch" decision. The "unit commitment" decision-making problem will involve the determination of unit "on" and "off" schedules during each hour of the scheduling period, in other words, it provides the results of ordered generation commitments to the "economic dispatch" decision program. The "economic dispatch" decision involves the allocation of system load demand and spinning reserve among the committed units which are in operation during each hour of the scheduling horizon so that the allocation of the target active power outputs of generators can result in a minimum system operating cost.

It is well known that these two decisions actually are interrelated, i.e. when solving the unit commitment problem, these decisions must be considered together, so that an overall least cost function value for the operation of the power system considered can be attained over the scheduling period. As a result, the unit commitment problem becomes a very complex mathematical programming problem involving large numbers of both integer variables and continuous variables.

The solution of this large scale and complex unit commitment problem becomes nevertheless very difficult, since, from the mathematical viewpoint, it consists of optimizing an economic criterion simultaneously involving integers and real variables, fixed and variable cost and under multiple constraints. Therefore, the algorithms employed for the solution of the unit commitment problem must be correctly chosen in order to solve the problem efficiently.

As an example, consider a power system with 10 units and a predicted power demand curve for a 24 hour period, subdivided into 24 intervals. It is possible to follow a straightforward enumeration approach in order to establish the best unit commitment schedule, but the total number of possible unit combinations of "up" or "down" required to be examined in obtaining the optimal combination for the whole study period will be approximately $(2^{10})^{24}$, or 1.77×10^{72}. This would require many decades to complete the task on a moderate sized computer. Although the physical and operational constraints imposed on the units and the system largely reduce the number of possible unit combinations, there are other necessary considerations such as fuel cost, start-up and shut-down cost, minimum on and off time, minimum interval between synchronization and de-synchronization events in a station and spinning reserve requirements. All these constraints and considerations will make a substantial further complication to the unit commitment problem.

Precisely because of the nonlinear nature of some of these above-mentioned constraints, no simple mathematical programming techniques may be easily applied to the unit commitment problem. Throughout the years, the electricity industry has developed various algorithmic approaches to the solution of the problem. As discussed in Chapter 2 and Chapter 3, numerous programming approaches have been reported for the solution of the unit commitment problem, such as the partial enumeration method, branch-and-bound techniques, dynamic programming, Benders decomposition, other heuristic approaches, merit-order schemes, Lagrangian relaxation techniques, mixed-integer linear programming, etc.

The complexity of the problem of unit commitment is such a difficulty that until recently and for real size power systems, it has been solved only by heuristic methods. Theoretically, this dynamic optimization problem requires general dynamic programming principles for its solution. The vital weakness of dynamic programming is that it is difficult to cope with the high dimensionality problem. Hence, for a large scale generation system and a comprehensive model, the solution of the problem by general dynamic programming methods tends to become impractical.

The methods used here are tested to show the efficiency of a merit-order heuristic scheme, a composite cost dynamic programming algorithm that has been claimed to overcome the dimensionality problem of the general dynamic programming method and a Lagrangian relaxation dual decomposition method to achieve an optimal solution for realistic size systems in an acceptable computational time.

5.3 PROBLEM FORMULATION

The proposed unit commitment methods will take into consideration various operating constraints imposed on the individual thermal unit and the system. The constraints will be almost the same as those in the thermal subsystem formulation in hydrothermal generation scheduling model in Chapter 4. All these constraints are summarized in the following subsections below.

5.3.1 The Objective Function

A unit commitment model, to be described as a constrained optimization problem, requires the specification of an objective function and a set of associated constraints. It is desirable to operate the system at its minimum cost and the objective function is therefore chosen to approximate to the generator output dependent running costs together with the associated startup and shutdown costs.

Suppose a given power system contains I thermal units, the unit commitment problem is to schedule the startups, the shutdowns and the unit generating levels of all these I units over a scheduling period K so that the total production cost consisting of the fuel costs, the startup costs and the shutdown costs of the system will be minimized while satisfying the load demand and reserve requirement. The scheduling problem of unit commitment can be written in a mathematical optimization form such as:

$$\min_{P_T(i,k),X(i,k),U(i,k)} \sum_{k}^{K}\sum_{i}^{I}\{F_i(P_T(i,k)) + ST_i(X(i,k),U(i,k))\} \tag{5.1}$$

The production costs of a thermal unit include the running costs when the unit is "on", also the shutdown and the startup costs which will be involved should any changes be made in the "on" and "off" schedule of the unit. The running costs of a generating unit may be assessed in terms of the fuel input required to produce a certain power output together with a fixed cost which would be incurred even if the unit has a zero load. The startup and shutdown costs will further complicate the unit commitment as they are usually time-dependent costs.

There are subtle variations in modelling the generating cost function of thermal units, and the constraints imposed on the system and the units may vary according to different operating characteristics. The difference in the incremental costs of a unit at its minimum and maximum output depends mainly on the design of the turbine. For example, the incremental costs of a generator used in the U.S.A. will vary substantially between its minimum and maximum output, while the incremental costs of a generator used in the U.K. will have little change from its minimum to its maximum output, so the generators will have different running cost curves. Thus, generators used in the U.S.A. will have a nonlinear cost function curve (normally quadratic), while generators used in the U.K. will have a nearly linear cost function or piecewise linear function relation.

5.3.2 The Variables and Constraints Set

A number of equality and inequality constraints are included to represent the engineering limitations and physical laws of the generator, transmission and distribution system and consumer load demand. Further constraints may be considered to ensure the security of the network, termed security constraints, both in existing circumstances and in the event of unexpected loss of generation or transmission capabilities.

The mathematical model presented here for the unit commitment problem takes into account the generator maximum output, generator minimum output, generator ramping rate of increase and ramping rate of decrease, also the minimum shutdown time, minimum startup time, and all the costs involved both on-line and off-line, such as the shutdown cost, and the startup cost (the startup cost is a unit shutdown time-dependent function), etc, and nonlinearity of the fuel cost function. The unit commitment problem is then formulated as a large scale, dynamic, and mixed-integer programming problem, and solved here by three algorithms, a merit-order heuristic scheme, the composite cost dynamic programming approach and the methodology based on the Lagrangian relaxation. Details of these algorithms will be described in the following sections. A summary of all the constraints involved is given below:

- Each thermal unit can be in one of the unit states, represented by variable $X(i,k)$ and the startup or shutdown decision variable for unit i at time interval k is denoted as $U(i,k)$. When a unit is "up" or committed, the decision variable is denoted by integer 1, if the unit is "down" or de-committed, the decision variable is denoted by integer 0. As mentioned before, we have

$$U(i,k) = \begin{cases} 0, & \text{if unit } i \text{ is decided to be `off'.} \\ 1, & \text{if unit } i \text{ is decided to be `on'.} \end{cases} \tag{5.2}$$

- If unit i does not have its minimum up and down time constraint, then the state variable of unit i can be determined simply by

$$X(i,k) = \begin{cases} 0, & \text{if unit } i \text{ is `off'.} \\ 1, & \text{if unit } i \text{ is `on'.} \end{cases} \tag{5.3}$$

- The unit minimum up time and minimum down time constraints are specified for each unit as T_{minup} and $T_{mindown}$.

- If the unit ramping rates of increase and decrease are ignored, such as in the algorithm based on Lagrangian relaxation dual methodology, for each thermal unit a total number of states has been shown to be $T_{minup} + T_{mindown}$, and a unit can be any one of those states as shown in the state transition diagram for thermal unit i in Diagram 4.13. So the state of unit i at time interval $k+1$ can be determined by

$$X_S(i,k+1) = \begin{cases} X_S(i,k), & \text{If } X_S(i,k) = 1 \ \& \ U(i,k) = 0 \\ X_S(i,k) + T_{mindown}, & \text{If } X_S(i,k) = 1 \ \& \ U(i,k) = 1 \\ 1, & \text{If } X_S(i,k) = Maxstate \ \& \ U(i,k) = 0 \\ Maxstate, & \text{If } X_S(i,k) = Maxstate \ \& \ U(i,k) = 1 \\ X_S(i,k) + 1, & \text{If } 1 < X_S(i,k) < Maxstate \end{cases} \tag{5.4}$$

Where $Maxstate = T_{minup} + T_{mindown}$.

- The startup and shut-down costs for unit i are denoted together by $ST_i(X(i,k), U(i,k))$, usually both the startup and the shutdown costs are time-dependent functions that may depend on unit state $X(i,k)$ and the decision variable $U(i,k)$. The startup cost is modeled here to be dependent on the number of time periods that the unit has been shutdown prior to this startup. The shutdown cost of a thermal unit is normally not significantly large compared with its startup cost, hence it is considered to be a fixed value for each unit.

$$\begin{aligned} ST_i(X(i,k), U(i,k)) = {} & C_{coldstart}(i) * \frac{\alpha(i) * T_{down}(i)}{1 + \alpha(i) * T_{down}(i)} \\ & + C_{shutdown}(i) \end{aligned} \tag{5.5}$$

Where $\alpha(i)$ is the time elapsed for unit i to start up after it shut down.

- The constraints on the power output of committed thermal units are defined as unit minimum and maximum output allowable limitations. This output constraint defines the allowable output power of each generating unit when committed.

$$P_{imin} \leq P_T(i,k) \leq P_{imax} \tag{5.6}$$

- The nonlinearity of the fuel cost is considered, the fuel cost has been modeled to be a quadratic function of the output power for operating unit i at power level $P_T(i,k)$ over interval k.

$$F_i(P_T(i,k)) = \begin{cases} 0, & \text{if } U(i,k) = 0 \\ f(i,k), & \text{if } U(i,k) = 1 \end{cases} \tag{5.7}$$

where $f(i,k) = A_i + B_i * P_T(i,k) + C_i * {P_T}^2(i,k)$.

- The ramping rates for the power output decrease or increase are included as

$$\Delta P_T(i,k) \leq P_{ramp} \tag{5.8}$$

- The load demand and generation balance requirement at each time interval is a set of coupling constraints among all the units.

$$P_D(k) - \sum_{i}^{I} P_T(i,k) \leq 0 \tag{5.9}$$

Where $P_D(k)$ is the expected average demand during time interval k.

- The reserve requirement for each time interval is the other set of coupling constraints among all the units.

$$P_R(k) - \sum_{i}^{I} \Delta P_T(i,k) \leq 0 \tag{5.10}$$

Where $P_R(k)$ is the specified threshold reserve chosen to ensure that with a high probability, the demand will be covered even if some units fail to generate power or the actual demand varies from the expected demand during time interval k.

5.4 A MERIT-ORDER SCHEME

The simplest unit commitment solution methods utilize a merit-order list, or in other words, the priority list for the commitment of the units. The list could be obtained by ranking the average full-load production cost ($AFLC$) of each unit in the system in a descending order. The $AFLC$ of a unit is obtained by dividing the fuel cost at its maximum generation loading capacity by its megawatt output at full loading level as depicted in Diagram 5.1. A startup or shutdown schedule of the units is then constructed with the assistance of this priority list. The unit with the smallest $AFLC$ is usually committed first if other constraints on this unit are satisfied, then the unit with the second

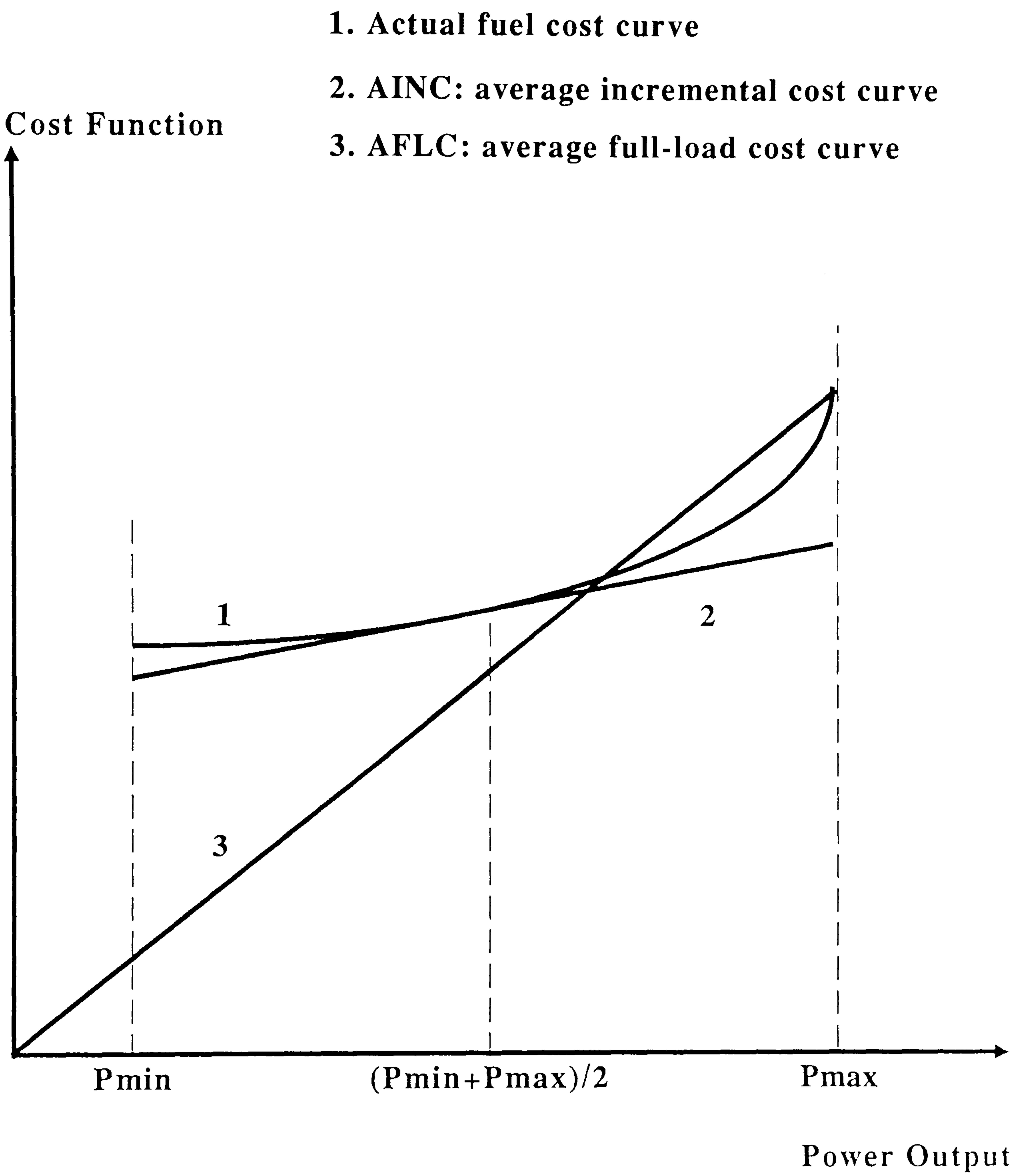


Diagram 5.1. Representation of a Unit's Average Full Load Cost and Average Incremental Cost

smallest $AFLC$, and so on until the power demand is satisfied. A merit-order scheme for unit commitment can be operated as follows:

1. Starting from the first time interval with $n \Leftarrow 0$.
2. At time interval n, compare the load demand of this interval with the load demand of the previous interval $n-1$; if the load decreases, continue, if it increases, go to step 6, otherwise keep the same commitment as the last interval and go to step 9.
3. The load decreases. Determine the most expensive committed unit that also satisfies its minimum up time constraint in the priority list, i.e. it can be shut down, check whether shutting down this unit will still leave enough generation to supply the load and satisfy the spinning reserve requirement, if not, keep the same commitment list as the last interval and go to step 9, otherwise continue.
4. Determine the number of intervals T (including the interval under consideration) before the load will return to the level that is equal to or exceeds the load of the last interval.
5. If T is less than the minimum down time of this unit, keep the same commitment list as the last interval and go to step 9, otherwise calculate and compare the operating cost for the next T intervals with this candidate unit and without this unit in the commitment list. For the case of taking the unit from the commitment list, it is assumed that when the load picks up after T intervals the unit will be required to generate again. Hence, in determining the operating cost for the T interval period without the unit, the shutdown or banking, and startup cost of this unit should be added to the total fuel cost. If there will be a saving in shutting down this candidate unit, de-commit this unit. Repeat step 5 for the next most expensive unit in the commitment list, otherwise keep the same commitment schedule as the last time interval and go to step 9.
6. The load increases. Check whether the committed units have sufficient capacity for the load and satisfy the spinning reserve requirement. If yes, keep the same schedule as the last interval, and go to step 9, otherwise determine the cheapest uncommitted unit from the $AFLC$ priority list

and commit this unit to the operation, repeat the process for step 6 until sufficient generation is committed.

7. Determine the number of intervals T (including the interval under consideration) before the load will go down to the level that is equal to or less than the load of the last interval. Determine the next cheapest uncommitted unit in the priority list and which satisfies minimum off time requirements.
8. If T is less than the minimum up time of the unit, continue; otherwise calculate and compare the operating cost for the next T intervals with this candidate unit and without the unit in the commitment list. It is assumed that when the load drops to the same level, the unit will be shut down. Hence, in determining the operating cost for the T interval period with the unit, the startup and shutdown cost of the unit should be added to the fuel cost. If there would be a saving in starting up this unit, commit the unit to the operation. Repeat step 8 for the next cheapest unit in the commitment list. Otherwise continue.
9. If all intervals have been considered, the optimal unit commitment schedule for the whole load forecast period is assumed to have been determined. Otherwise increase the interval number n by 1, $n \Leftarrow n + 1$ and go back to step 2.

Various enhancements and modifications to the scheme outlined above have been proposed. In this chapter, only the above version was examined. It is of particular interest because it is used to show the efficiency of merit-order methods, and at same time, provide a comparison with two other methods.

After the unit commitment schedule is determined, another merit-order list is employed to allocate the load demand to the committed units. The generation level of the committed units is decided by the $AINC$ priority list. The list could be obtained by ranking the average incremental production cost ($AINC$) of each unit in the system in a descending order. The $AINC$ of a unit is obtained by differentiating the fuel cost according to its average megawatt generation loading capacity as depicted in Diagram 5.1. The average incremental cost $AINC$ for unit i is determined by

$$AINC(i) = B_i + C_i * (P_{imin} + P_{imax})$$

An economic dispatch schedule of the committed units is then constructed with the assistance of this $AINC$ priority list. The unit with the smallest $AINC$ cost is usually loaded to its full loading capacity first, then the unit with the second smallest $AINC$, and so on. The merit-order scheme for loading the committed units according to their average incremental cost merit-order positions can be operated as follows:

1. Set all selected committed units to their minimum output.
2. Load the units to their maximum power output until the power demand at the considered time interval is satisfied. The incremental cost merit order list is used to select which unit is to be increased to its maximum feasible output first.

5.5 A LAGRANGIAN RELAXATION APPROACH

5.5.1 Introduction

In this section a new solution technique based on the Lagrangian relaxation dual methodology for optimal unit commitment and economic dispatch of a large scale thermal power system is presented, and the resulting problem is solved by a dual programming approach to achieve an optimal solution for the unit commitment problem. The resulting decomposed problems are reduced to solving a separable optimal control problem for each thermal unit. A discrete dynamic programming approach or a shortest path algorithm can be easily applied to solve this separable optimization problem, and non-differentiable optimization techniques are employed as the solution methods for the master problem of maximizing the lower bound produced by the relaxation. The important contribution in this section is the derivation of a simple and efficient approach to generate a sub-optimal yet feasible solution from the dual solution to the primal problem. Comparisons and tests have shown its efficiency and optimality.

It has been reported that as a complex, mixed-integer, dynamic, nonlinear programming problem, further complicated by a small set of constraints such as the generation, capacity and reserve requirements, (namely the coupling constraints,) the thermal unit commitment problem has a separable structure which is ideal for the application of mathematical decomposition techniques and the dual programming approach.[29.],[38.],[90.] The Lagrangian relaxation technique has been applied to employ this separable structure and to achieve an efficient optimal solution of the unit commitment problem.

Some earlier solution approaches have been considered for the solution of unit commitment problem such as the application of dynamic programming technique to decompose the multi-period decision problem of unit commitment into a sequence of interrelated single-period decision problems. The solution of each single-period problem involves the allocation of unit generation using an assumed merit-order list and the consideration of when the unit was last started or shut down in order to satisfy the unit minimum down time and minimum up time constraints. The dimensionality problem still exists in this approach and the computation time will increase drastically when the number of variables increases. Conversely, the decomposition approach in the Lagrangian relaxation methodology is different, the whole unit commitment problem is decomposed into a series of easy subproblems of deciding the unit commitment schedule for a single unit over the whole scheduling period. The unit commitment schedule of a unit is independent from the schedule of other units, and only through the master coordination procedure of updating the Lagrangian multipliers can the schedule of the units be related to each other, in order to satisfy the load demand and generation balance requirement and reserve constraints.

The main objective of this section is to show the efficiency of a Lagrangian relaxation methodology for the optimal solution of a large scale thermal unit commitment problem. The Lagrangian relaxation technique has been widely used for large scale optimal unit commitment problems in recent years. Muckstadt[144.] made the first application of Lagrangian relaxation in a fossil-fueled power generating system over a short-time horizon. The fundamental approach was first applied by Bertsekas[29.] to solve the optimal short-term scheduling problem in a large scale power systems. Much further work has been reported.[6.],[7.],[15.],[92.],[126.],[143.],[172.],[202.],[216.] Many contributions have been made in implementation aspects of trying to achieve a more practically acceptable feasible solution.

The Lagrangian relaxation approach applied has been claimed to be a new algorithmic approach to finding an optimal unit commitment and an economic dispatch schedule simultaneously. It is expected that this new approach will be a very efficient method to solve the complex unit commitment problem especially for large scale thermal power systems, because of two special features:

Firstly, because of Lagrangian relaxation, the scheduling problem of the units can be further decomposed and results in scheduling only a single unit over the time period. This is a single thermal unit dynamic optimization problem. As a result, the computational requirement normally grows only linearly with the number of generating units but may grow very nonlinearly with an increase in the scheduling period. This is very promising since it means large scale power system problems can now be solved very efficiently.

Secondly, the duality gap between the solution of the primal problem and the dual problem which affects the feasibility and optimality of the primal solution or in another words, measures the degree of relaxation of power balance equations, will decrease in relative terms as the number of generating units increases. This leads to the conclusion that the algorithm employed actually tends to perform better for larger sized problems.

Also, Lagrangian relaxation method can produce ϵ - optimal (i.e. near-optimal) solutions which are well acceptable in practical situations.

All the above mentioned advantages make it possible for the first time to solve the large scale practical unit commitment problem involving thousands of variables and associated constraints.

Lagrangian relaxation derived its name from the well-known variational calculus mathematical technique for the solution of constrained optimization problems. In this technique, by assigning a Lagrangian multiplier to each constraint, the constraint can be relaxed and adjoined to the original objective function. This leads to the solution of a Lagrangian augmented problem. Unconstrained optimization techniques can then be applied to solve this problem simply through determining each Lagrangian multiplier value.

However, this technique is used here as a mathematical decomposition method for the solution of large scale mathematical programming problems. Through relaxing a set of "complicating" constraints of a general mixed integer programming program into its objective function in a Lagrangian fashion (with fixed multipliers), the resulting dual problem will yield a "Lagrangian relaxation" of the original program. Essentially, as discussed in Chapter 3, the Lagrangian relaxation dual methodology is a price directive decomposition approach that involves solving the dual maximization problem of the original primal problem in order to exploit the special structure of the original problem.

The dual maximization problem can be decomposed into a sequence of master problems and easier-to-solve subproblems when the dual variables are specified. Theoretically, if the primal problem has a convex continuous objective function with convex constraints, the optimal solution of this dual will also be the optimal solution for the primal problem. That is to say, there will be no duality gap between the optimal solution of the primal problem and the dual problem. Practically, the solution of the dual problem will converge to an ϵ - optimal solution to the original problem depending on the convergence criteria.

The algorithm employed here uses Lagrangian relaxation in the usual way to decompose the unit commitment problem into a series of smaller problems. As unit commitment is a mixed-integer programming problem, it is well-known that the larger the problem size is, the more difficult it is to find the solution. When the Lagrangian relaxation technique is applied, it allows, for the first time, a consistently reliable solution of large scale practical problems involving several hundreds of generating units within realistic computational time requirements.

The difficulty with the application of Lagrangian relaxation techniques to the solution of the unit commitment problem is that as unit commitment involves many integer variables, the problem will not be a convex programming problem. Thus the optimal solution of the dual problem may not be the same as that of the primal problem. A certain amount of duality gap between the primal and the dual may exist. In fact, the Lagrangian relaxation dual methodology will only find a lower bound solution for the primal.

To overcome the potential infeasibility created by the application of Lagrangian relaxation to the solution of the unit commitment problem, there are approaches which combine Lagrangian relaxation with a branch-and-bound examination technique. Through examining sufficient branches, a feasible and optimal solution for the primal problem can be achieved. However, this will normally result in too much computation time to be practically acceptable, especially for large scale unit commitment problems. Other approaches try to ensure the feasibility of the primal problem by sacrificing some degrees of optimality through adjusting the Lagrangian multipliers and searching for an upper bound or by some heuristic post-dispatch adjustments. Here a simple yet efficient approach to generate a feasible and near-optimal solution of the primal is presented. The original idea comes from the characteristic of the power balance constraint and the availability of generation capacity.

All the practical aspects of the application of the Lagrangian relaxation methodology will be discussed in this section. The crucial points for practical and theoretical consideration as well as a good feasible solution generating approach are presented. The efficiency achieved by the program implementation is also shown in later sections.

5.5.2 Mathematical Formulation

The objective function of the unit commitment problem (termed the original problem or the primal problem) is to minimize the total thermal production cost,

$$\min_{P_T(i,k),X(i,k),U(i,k)} \quad \sum_k^K \sum_i^I \{F_i(P_T(i,k)) + ST_i(X(i,k),U(i,k))\}$$

The fuel cost function $F_i(P_T(i,k)) = A_i + B_i * P_T(i,k) + C_i * P_T^2(i,k)$ is assumed to be a quadratic function or a piecewise linear convex function and $ST_i(X(i,k), U(i,k))$ is the startup and shutdown cost as a function of the state variable $X(i,k)$ and the decision variable $U(i,k)$. The state variable is decided through the state transition diagram, it is defined as shown in the following example. If a unit has a minimum up time of 3 hours and a minimum down time of 3 hours, the total number of states will be 6 and $X_s(i,k) = 6$ indicates the unit has been up for 3 hours at the start of time interval k; $X_s(i,k) = 2$ means the unit has been shut down for 2 hours at the start of time interval k. The shutdown cost is assumed to be a fixed value.

If there are a total number of generating units I and time intervals K in the problem, the total number of variables including real variables $P_T(i,k)$, integer variables $X(i,k)$ and $U(i,k)$ will be $3 * I * K$. The constraints, as stated before, will include the unit minimum output limits and maximum limits, the unit minimum up time and minimum down time, etc. The most important constraints are the coupling constraints: The power balance equations:

$$P_D(k) - \sum_i^I P_T(i,k) \leq 0$$

and the reserve requirement equations

$$P_R(k) + \sum_i^I P_T(i,k) - \sum_i^I P_{imax} \leq 0$$

Since the solution for Lagrangian multipliers associated with the reserve constraints is similar to that associated with the power balance equations, the reserve constraints are not considered explicitly in this formulation.

The Lagrangian relaxation approach is based on a duality transformation of the original unit commitment problem, namely the primal problem, and the optimal solution of the primal problem is obtained through the solution of the associated non-differentiable dual problem. This approach coupled with the branch and bound technique is essential and common in integer programming problems. Assigning non-negative Lagrangian multipliers $\lambda(k) \geq 0$ to each power balance constraint respectively and adjoining this part to the primal cost function (note the power balance is the only coupling constraint in the system provided that spinning reserve is not considered), the corresponding Lagrangian relaxation dual function formulation is:

$$\begin{aligned} L(\lambda, U(i,k), X(i,k), P_T(i,k)) = \min_{P_T(i,k),X(i,k),U(i,k)} \quad & \{\sum_k^K \sum_i^I F_i(P_T(i,k)) \\ & + \sum_k^K \sum_i^I ST_i(X(i,k), U(i,k)) \\ & + \sum_k^K \lambda(k) * [P_D(k) - \sum_i^I P_T(i,k)]\} \end{aligned} \tag{5.11}$$

subject to the other constraints included.

The dual function $L(\lambda, U(i,k), X(i,k), P_T(i,k))$ can be rewritten as:

$$\begin{aligned} L(\lambda, U(i,k), X(i,k), P_T(i,k)) = \min_{P_T(i,k),X(i,k),U(i,k)} \quad & \{\sum_k^K \sum_i^I F_i(P_T(i,k)) \\ & + \sum_k^K \sum_i^I ST_i(X(i,k), U(i,k)) \\ & - \sum_k^K \lambda(k) \sum_i^I P_T(i,k)\} \\ & + \sum_k^K \lambda(k) P_D(k) \end{aligned} \tag{5.12}$$

By analyzing the primal and the dual function in the thought of the constraints, after the elimination of the coupling constraint by the introduction of the Lagrangian multipliers $\lambda(k)$s, the thermal unit commitment problem can be seen to be decomposable with respect to individual units. This results in solving separable subproblems for each single unit i and these can be solved easily by discrete dynamic programming, branch and bound or a shortest path routine. Discrete dynamic programming is adopted and combined with a set of heuristic rules which further simplifies the decomposed subproblems.

For each individual thermal unit i, there is a decomposed unit commitment subproblem such as:

$$\min_{P_T(i,k),X(i,k),U(i,k)} \quad \sum_k^K \{F_i(P_T(i,k)) + ST_i(X(i,k), U(i,k)) - \lambda(k) * P_T(i,k)\} \tag{5.13}$$

The overall dual optimization problem becomes:

$$\max_{\lambda(k)\geq 0,\quad k\in K} \sum_{i}^{I} \min_{P_T(i,k),X(i,k),U(i,k)} L(\lambda, P_T(i,k), X(i,k), U(i,k))$$

Where

$$\begin{aligned} L(\lambda, P_T(i,k), X(i,k), U(i,k)) = & \sum_{k}^{K}\{F_i(P_T(i,k)) \\ & + ST_i(X(i,k), U(i,k)) - \lambda(k) * P_T(i,k)\} \\ & + \sum_{k}^{K} \lambda(k) P_D(k) \end{aligned}$$

The overall problem is formulated as a min-max dual problem. The dual function is usually concave and not everywhere differentiable. The outer or master coordination problem is formulated as an unconstrained nonlinear optimization problem to maximize the dual function with respect to $\lambda(k)$. The sub-gradient optimization method is chosen for the maximization of the Lagrangian dual function in order to update the Lagrangian multipliers. For a specified set of Lagrangian multipliers, the dual function value is obtained by solving the inner problem of minimizing the thermal subproblems for each unit over the scheduling period.

The Lagrangian relaxation technique employed here for thermal unit commitment can be operated as follows:

1. Read in thermal system and load prediction data.
2. Start the solution of the discrete problem. The discrete problem is defined to decide the unit "on" and "off" schedule, i.e. to determine the values of integer variables. Obtain the values of $\lambda(k)$ for each time interval if there is a initial estimate available, otherwise initialize the values of $\lambda(k)$.
3. Start from the first unit with $n \Leftarrow 1$, $n \Leftarrow n+1$ as the process proceeds.
4. Use a discrete dynamic programming routine to schedule the single thermal unit problem according to the specified Lagrangian multipliers $\lambda(k)$.

5. Check whether all the thermal units are scheduled or not, if not, go back to step 3, otherwise continue.
6. Start the coordination procedure. Check whether the supply satisfies the load demand or not for all time intervals, if not, calculate the sub-gradient as

$$P_D(k) - \sum_{i}^{I} P_T(i,k)$$

which is the variation of the coupling constraint at each time interval, update Lagrangian multipliers $\lambda(k)$ by maximizing the dual function $L(\lambda)$, here the sub-gradient optimization algorithm is needed to update λ, then go back to step 2 using the updated $\lambda(k)$ until no improvement can be made to maximize the dual function, if all the coupling constraints are satisfied in the convergence criterion, continue.
7. Fix the unit commitment schedule ("on" and "off" schedule) and start the continuous problem. The continuous problem is defined to be the economic dispatch problem involving only real variables. Using Lagrangian multipliers obtained from the discrete problem as an initial estimate, dispatch for each committed unit according to these Lagrangian multipliers.
8. Check whether the supply satisfies the load demand for all the time intervals, if not, update $\lambda(k)$ by maximizing the dual function. Since without integer variables, the continuous problem is convex, a one-dimensional line search may be used to find the optimal step in order to maximize the dual function. Continue the program with updated $\lambda(k)$ until no improvement can be made to maximize the dual function, if all the coupling constraints are satisfied within the convergence criterion, stop with the current solution as an optimum.

The process of using a discrete dynamic programming routine for solving the decomposed subproblems for each individual unit commitment problem i, is as follows (the generator ramping rate constraints are not considered):

1. Decide the total number of states ($T_{minup} + T_{mindown}$). Establish the possible paths for each state to take according to the state transition diagram.

2. Initialise the optimal cost value for each state by $FSBEST \Leftarrow +\infty$ and initialise the state variable for the first interval with $k \Leftarrow 1$ and decide the initial feasible state.
3. Decide the optimal values and possible decisions for each state at the first stage.
4. Start the main loop with $k \Leftarrow 1$.
5. Initialise the best decision array for storing the new cost of each state.
6. For each present state S and each possible path of state S, determine the possible new decision and calculate the transition cost for each decision and from each state. If state S indicates the shutdown time of the unit is less than $T_{mindown}$ then no transition cost is incurred; if state S indicates the shutdown time of the unit is equal to $T_{mindown}$, and decision variable $U(i,k) = 0$, no transition cost occurs; otherwise if decision variable $U(i,k) = 1$, the unit power level can then be decided by equation:

$$P_T(i,k) = \begin{cases} P_{imin}, & \text{if } \frac{\lambda(k)-B_i}{2*C_i} < P_{imin}, \\ \frac{\lambda(k)-B_i}{2*C_i}, & \text{if } P_{imin} \leq \frac{\lambda(k)-B_i}{2*C_i} \leq P_{imax}, \\ P_{imax}, & \text{if } \frac{\lambda(k)-B_i}{2*C_i} > P_{imax} \end{cases} \tag{5.14}$$

   The process for determining the transition cost of other values of the present state S will be similar.
7. Compare the recursive cost plus the transition cost of each present state S and find out the new optimal cost for each state S, save this optimum decision of each state and update the recursive cost values for each state at this stage for use at the next stage.
8. If all the time intervals have been examined, end the main loop for each state, otherwise go to the next interval with $k \Leftarrow k+1$, go back to step 6.
9. Perform the backtracking process to find the best path and the decision for each time interval, evaluating the least production cost.

5.5.3 Solution Techniques for Coordination

As discussed, the single-thermal-unit optimization can be achieved by a dynamic programming routine, tests will show that these subproblems can be solved very efficiently as a result of the decomposition. However, the solution of the master problem of how to update the Lagrangian multipliers $\lambda(k)$ is much more difficult. In order to find an optimal schedule, while satisfying all the power balance equations, some set of Lagrangian multipliers must be found, this is equivalent to solving the overall dual problem of

$$\max_{\lambda(k)\geq 0,\quad k\in K} \qquad L(\lambda(k))$$

The dual function value for a specified set of Lagrangian multipliers is obtained through solving a minimization problem such as

$$L(\lambda(k)) = \min_{P_T(i,k),X(i,k),U(i,k)} \quad \sum_k^K \sum_i^I \{F_i(P_T(i,k)) + ST_i(X(i,k),U(i,k))$$
$$- \lambda(k) * P_T(i,k)\} + \sum_k^K P_D(k)$$

Since the maximization of $L(\lambda(k))$ indicates whenever $\lambda(k)$ is updated, the minimization of the inner problem must be carried out, the maximization of the dual objective is not easy. Furthermore, the dual function is not differentiable everywhere, and non-differential optimization techniques are therefore needed to solve this master problem.

The sub-gradient optimization method is a common solution method used for maximizing the Lagrangian dual function in order to solve the master problem. It is simple, very easy to implement, and usually gives a very good approximation of the steepest ascent gradient method. However, even if the maximization is eventually achieved, it may still be of limited value since the feasibility of the primal problem can not be guaranteed due to the non-convexity of the unit commitment problem. As a whole, the Lagrangian relaxation method can be very efficient for solving the large scale problems because of decomposition, but many tests have shown the difficulties associated with the application of this method. The most serious problem is from the coordination aspect, i.e. the solution of the master problem. The duality gap and the difficulty of convergence are well-known drawbacks of the Lagrangian relaxation dual approach.

For a large scale mixed-integer programming problem, it is difficult to obtain a feasible primal solution for the solution of the dual problem, and there may be a fairly large difference between the optimal solution value of the primal problem and the value of the dual problem. The magnitude of this duality gap may differ from one particular system to another, and sometimes it may be small. However, a method for generating a feasible yet near optimal solution must still be found.

The sub-gradient algorithm is very popular but there have been discoveries in using the conventional sub-gradient optimization method:[6.],[202.] it may depend very sensitively on the accuracy required and the optimal cost estimate, the convergence rate may be slow and for some complicated systems good convergence to a dual optimal solution may not be obtained.

There are alternative approaches to overcome these two problems by various degrees. One approach for generating a feasible solution is suggested such as in EDF's work,[64.],[65.] while another approach uses the variable metric methods.[7.] Both approaches are based on the sub-gradient optimization method while having some variations and modifications in order to accelerate the convergency rate or to avoid the instability problem created by the sub-gradient optimization, as well as to generate a feasible solution. All these aspects will be discussed in the next subsection.

5.5.4 Implementation Considerations

There is one Lagrangian multiplier λ for each time interval, so the total number of Lagrangian multipliers will be the same as the number of intervals in the scheduling period. This implies that the Lagrangian relaxation technique is more suitable for daily unit commitment scheduling with 24 hourly time intervals than for a long-term weekly unit commitment with one hour intervals, since the more Lagrangian multipliers are involved, the more difficult the λ updating process becomes, i.e. the more difficult it is to find the best shared marginal prices.

Tests using the Lagrangian relaxation methodology for the solution of thermal unit commitment have been made. The experience has shown the following sensitivities of the Lagrangian relaxation approach.

Firstly, Lagrangian relaxation is well-known for its sensitivity towards the initial value of Lagrangian multipliers. A good initial estimate of λ may lead to a much faster convergence of the problem as well as the stability of Lagrangian relaxation program. A good initial estimate also makes Lagrangian relaxation much more attractive and efficient than Benders decomposition. Thus the initial estimate of the Lagrangian multipliers is very important both in achieving the optimal solution and convergence speed. A simple incremental cost interpolation method without other operating constraints was used and proved to be near to the solution. In this scheme, each Lagrangian multiplier is assigned a value of the incremental cost as illustrated in the procedure below.

A good initial estimate of Lagrangian multipliers λ can be obtained by neglecting the time dependent constraints of all the units and determined by a heuristic rule as follows.

- Find the maximum load demand D_{max} over the scheduling period.
- Find the minimum load demand D_{min} over the scheduling period.
- Find the maximum incremental cost MC_{max} among all the units.
- Find the minimum incremental cost MC_{min} among all the units.

The Lagrangian multiplier $\lambda(k)$ at k time interval can be decided by the following equation:

$$\lambda(k) = MC_{min} + \frac{(P_D(k) - D_{min}) * (MC_{max} - MC_{min})}{D_{max} - D_{min}}$$

such that the Lagrangian multiplier will increase linearly according to the change of load demand.

Another procedure for generating an initial estimate is based on a merit-order scheme. For each time interval, according to the priority list, the units are loaded to their full capacity until the load demand at this time interval is satisfied, and the Lagrangian multiplier at this time interval will be assigned to be the average incremental cost of the last loaded unit (with the most expensive incremental cost). The first approach has proved in most cases to be better.

Secondly, depending on the tightness of the operating constraints, the feasible and near-optimal solution may vary a lot from the lower bound produced by the dual solution. The unit upper and lower limit constraints were found to affect the magnitude of the duality gap, as did the number of identical generating units. For example, take two systems with the same thermal system data except the generator lower limit constraints are ignored in one system while the other takes the generator lower limit into consideration. The duality gap created by considering the lower limits was found to be larger than in the other case. A certain amount of duality gap always exists and this creates a problem of infeasibility in the use of Lagrangian relaxation in solving mixed-integer programming problems.

Furthermore, the choice of termination criterion is important but difficult because it is system dependent. Since the primal problem can be represented as:

$$(P) \quad \min_{x \in X} \quad f(x) \quad subject \quad to \quad g(x) \leq 0$$

the terminated x^k at iteration k must be an optimal solution of the following approximation of the primal problem (P), that is,

$$(P^k) \quad \min_{x \in X} \quad f(x) \quad subject \quad to \quad g(x) \leq g(x^k)$$

The continuity of g and the fact that x^k will converge to an optimal solution x^* of (P) implies that the right-hand side of (P^k) will converge to $g(x^*)$ as $k \to \infty$, hence one may terminate when k reaches a value at which the right-hand side of (P^k) is *sufficiently near* to being equal or less than zero. But how near is "sufficiently" near depends on how precisely the g constraints of (P) really must be satisfied.

Another important point to be noted is that even though the cost difference between the primal problem and the dual problem will be very small at some iteration, the individual coupling constraint such as the power balance may not be satisfied for some time intervals. Some $g_i(x)$s will be large and positive, some may be highly negative. This result can not therefore be claimed to be satisfactory. Also because the dual problem solution is definitely a lower bound on the optimal solution of the primal problem, when the dual problem is maximized, some constraints relaxed by Lagrangian multipliers may not be satisfied according to the termination criterion. That is to say, for some time intervals, the power generated by all the committed units can not satisfy the load demand. This is the most severe problem of all when using Lagrangian relaxation.

All the difficulties imply that a more strategic approach for overcoming the duality gap and the infeasibility of Lagrangian relaxation dual solution should be developed. Through examining the reason why the duality gap exists and when the infeasibility situation occurs, a strategic method for adjusting the schedule of the Lagrangian relaxation solution can be applied. It has been proved to be very efficient and assures the feasibility of the unit commitment schedule while achieving a satisfactory near-optimal solution. Compared with EDF's feasible approach,[64.],[65.] the convergence of the approach adopted here is much faster and more efficient. In EDF's approach, the unit commitment schedule must be decided at each iteration, without mathematical decomposition, which is very time consuming, while the approach suggested here is based on the unit commitment characteristics. Moreover, it is a straightforward approach, simpler than the "variable metric methods", and takes less iterations than "variable metric methods" to converge to the optimal solution. It is also easier than the least squares fitting approximation method suggested by Aoki,[6.],[7.] in order to adjust the Lagrangian multipliers to ensure the feasibility of the unit commitment schedule. Furthermore, the proposed feasible approach always ensures the feasibility of the unit commitment schedule produced through the Lagrangian relaxation solution. As shown later, the results obtained via this approach are near optimal, and the overall production cost is usually about 0.1% cheaper compared with the results of the composite cost dynamic programming.

The idea of the feasible approach which has been developed comes from the fact that there is a special feature of unit commitment problem, that is, after the unit "on" and "off" schedule is fixed, the dispatching problem among the committed units does not involve any integer variables. The dispatching problem is also usually a convex minimization problem. Consequently, there will be no duality gap with respect to this dispatching problem when using

Lagrangian relaxation decomposition. However, this can only be achieved provided that the power balance constraints can be satisfied in some way. To ensure this, when solving the unit commitment using Lagrangian relaxation decomposition, the total committed generation capacity at each time interval should be larger than the load demand for the same time intervals. Similarly, the sum of total committed generator minimum outputs at each time interval should not be larger than the load demand in the same time interval. To illustrate the proposed approach, the whole optimization process, designed to achieve feasibility and minimize the duality gap, can be interpreted as follows.

1. Initialise the Lagrangian multipliers using the initialization procedure as mentioned before. Solve the unit commitment problem (i.e. decide the unit "on" and "off" schedule) using a solution method for maximizing the dual function based on the sub-gradient optimization method suggested by Heid.[100.] The convergence tolerance should not be too tight, and can be based on comparing the dual cost value between two iterations, usually an accuracy of 0.001 to 0.0001 will be sufficient depending on the particular system considered.

2. To avoid the infeasibility problem, check within each time interval whether the total maximum generator power is sufficient for the load demand and whether the total minimum generator power is less than or equal to the load demand (in case no adjustment can be made in the dispatching process). If these differences are greater than the predefined deviations, go back to step 1 with updated Lagrangian multipliers, otherwise continue.

3. Check the percentage difference between the load demand at each time interval with the total generation power
$$\frac{(P_D(k) - \sum_i^I P_T(i,k)) * 100.0}{P_D(k)}$$
If the difference is larger than the predefined deviation, go back to step 1 with updated Lagrangian multipliers, otherwise continue.

4. Having fixed the unit commitment ("on" and "off") schedule, solve the generation allocation problem such that the total generating power

satisfies the load demand for each time interval. This problem is termed the continuous problem since only real variables are involved. The convergence tolerance should be tight, for example in comparing two successive iteration dual cost values, an accuracy of 5.0×10^{-5} to 1.0×10^{-5} should be used.

5. If all the power balance constraints are satisfied, terminate the program with the current solution as a near-optimal solution, otherwise in the case that infeasibility has occurred, continue, go to step 6.

6. Start the heuristic unit commitment post-adjustment. Adjust the Lagrangian multipliers by a least squares fitting method which is similar to the one used by Aoki,[6.],[7.] go back to step 1 with updated and adjusted Lagrangian multipliers.

Theoretically, since there are no integer variables involved in the continuous problem, the primal continuous problem is a convex minimization problem, and there should be no duality gap between the dual problem and the primal problem. Tests have proved this to be true. Test results have also shown that the final heuristic unit commitment adjustment is usually unnecessary provided that the convergence criterion for the continuous problem is tight enough.

5.6 A DP APPROACH USING A COMPOSITE COST MODEL

5.6.1 Introduction

This section describes a composite operating cost model used for short-term generation scheduling of thermal power systems. This model simplifies the total system production cost as a nonlinear function of the total power generating level. A new computational algorithm based on the dynamic programming (DP) principle is developed to select the optimal generating unit combination and define the loading level of the committed generators so as to achieve an optimal commitment schedule. The algorithm applied here is a modified version of the method first introduced by Cheung,[49.] The special features of this method are mainly the following:

- The composite operating cost model gives a very close approximation to the actual system fuel cost function. It is very straightforward to construct, and is commonly used in power dispatching practice.

- The dynamic programming algorithm proposed here brings the dimensionality problem associated with dynamic programming techniques under control through using the composite cost model and by storing only an appropriate range of stages and states necessary to allow the computation to proceed to the optimum. The search range for generating units can be reduced as a result and the computer memory and computation time will be reduced substantially. Experience of using this algorithm has shown that the computation time required to obtain the optimal or near-optimal unit combination is nearly independent of the number of generating units in the system, but rather it depends on the total generating capacity and required accuracy. Therefore, this algorithm is very suitable for the solution of the unit commitment problem in medium-sized thermal power systems with reasonable accuracy requirements.

5.6.2 The Composite Cost Function Model

The composite cost model of each individual thermal unit at each time interval under consideration must be defined before the dynamic programming approach is applied. The composite cost function of unit i at interval k is defined as follows:

If unit i was 'off' at interval $k-1$ the composite cost is

$$COM_i(L) = F_i(L) + ST_i(X(i,k), U(i,k))/H_k \tag{5.15}$$

If unit i was 'on' at interval $k-1$ then the composite cost is

$$COM_i(L) = F_i(L) - C_{shutdown}(i) \tag{5.16}$$

Where

$COM_i(L)$ is the composite cost of unit i at power output level $L(MW)$

$F_i(L)$ is the fuel cost of unit i at output level $L(MW)$ given by the original fuel cost function

$ST_i(X(i,k),U(i,k))$ is the startup cost of unit i calculated by the startup cost function depending on the state $X(i,k)$ and decision $U(i,k)$

H_k is the estimated number of time intervals the unit will be up if it were started up in interval k

$C_{shutdown}(i)$ is the shutdown cost of unit i

The composite cost function of unit i is an artificial operating cost function combining the fuel cost, startup cost and shutdown cost of unit i together in a single cost description. The idea of the composite cost function comes from penalty and merit functions, the process of deriving this function can be illustrated as follows:

Suppose there are a total number of units I in the system which were in the "off" state at time interval $k-1$, the unit commitment problem is to determine which of these units if any, can be and should be started up at interval k so that the load demand can be satisfied and at the same time the overall production cost will be a minimum. It is assumed that any unit started up at interval k will usually be shut down at a later interval when the load demand returns to the same level or less than that at interval k. Then H_k for all the units can be represented as h, the total operating cost for any of these units supplying energy in these h intervals will be:

$$\sum COM_i = F_i^{k} + F_i^{k+1} + ... + F_i^{k+h-1} + ST_i(X(i,k),U(i,k)), \quad i \in I.$$

Where

i is the index of "off" units

F_i^k is the fuel cost of unit i at interval k

$k+h$ is the interval number at which the load returns to the same level as at interval $k-1$

It is clear that the "effective" operating cost of unit i at each interval between k and $k+h-1$ is the fuel cost of this unit plus an average startup cost as depicted by the above composite cost function. Since the estimated "up" time for all units started up at interval k is h, any unit pre-scheduled to be shut down before time interval $k + h$ will have a smaller expected "up" time and the contribution from its startup cost to the resultant composite cost function will be larger than it would have been if this pre-scheduled shutdown constraint was absent.

For a unit which was already "on" at time interval $k - 1$, there is no startup cost involved for this unit to operate at interval k; however, the unit could be shut down at interval k and a shutdown cost must be considered. Therefore, if the unit is to continue to operate at interval k, the cost to the system is "effectively" the fuel cost minus the shutdown cost of the unit as illustrated in the above composite cost function.

In a unit commitment program, using the composite cost function model means avoiding unnecessary startups and shutdowns among units in order to achieve a cost saving. The composite cost function model of an available unit at interval k can depicted in Diagram 5.2 and is essentially the fuel cost function plus a constant component. The shape of the fuel cost curve remains the same. The constant component added to the fuel cost will therefore affect the selection of units but not the optimal loading level of the selected units. With this composite cost function model, the composite cost dynamic programming algorithm will automatically select those units which optimize the fuel cost, startup and shutdown cost of the system.

5.6.3 Dynamic Programming Computational Algorithm

The principle of dynamic programming techniques, as discussed in the literature review, is widely used in the solution of unit commitment problems and has received much attention recently for its flexibility and ability to recognize nonlinearlity and time-dependency both in the objective function and constraints. However, the major disadvantage of dynamic programming techniques is the requirement of excessive computer storage and computational time for the solution of large scale problems. The algorithm proposed here is termed *a composite cost dynamic programming.* It largely overcomes the dimensionality problem and produces a unit commitment schedule with the required accuracy and within a reasonable time.

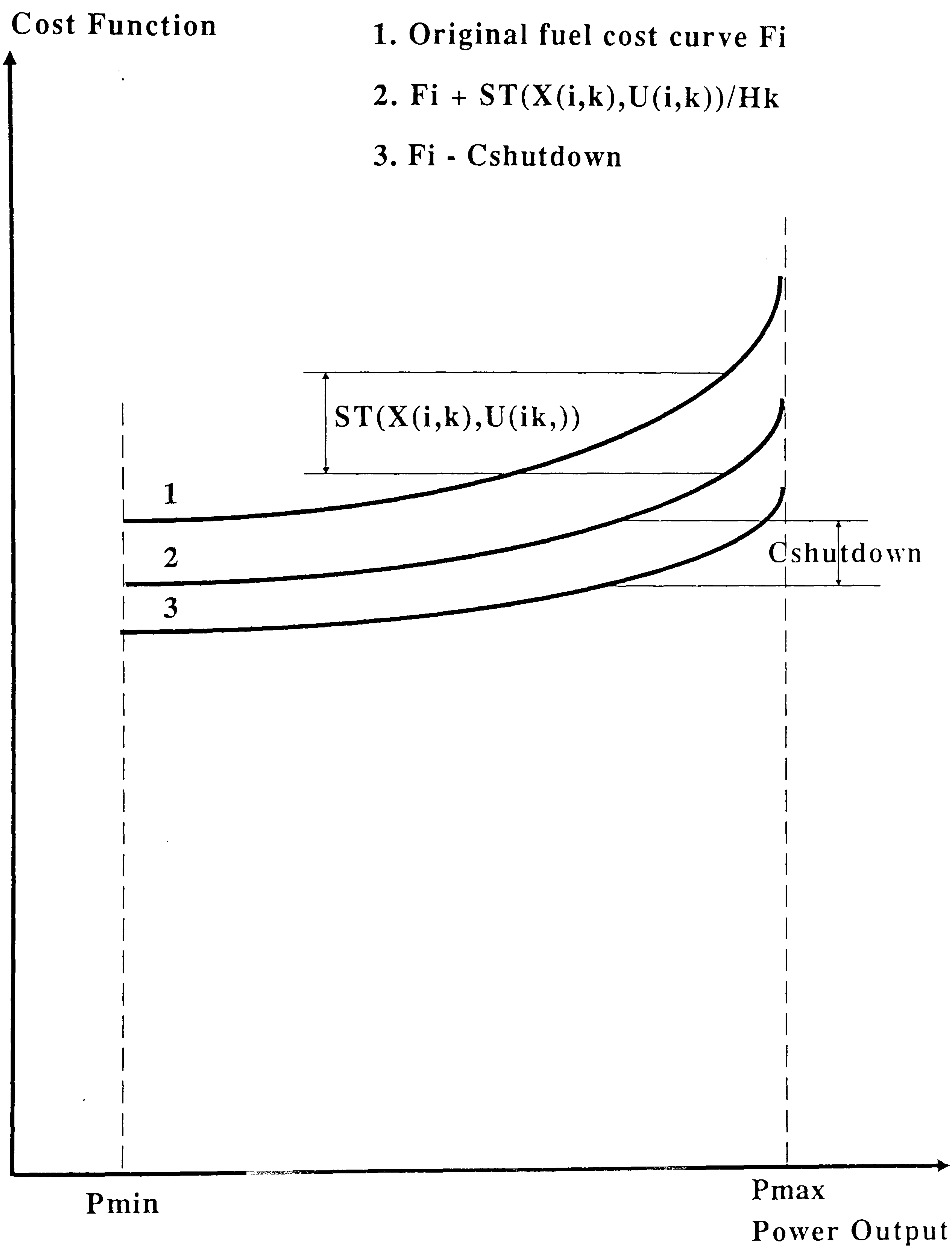


Diagram 5.2. Composite Cost Function Model

Given a power system with a total number of thermal units I including nuclear, coal-fired, oil-fired and gas-fired generating units, and each unit having a known quadratic fuel cost function, the generating capacity of each unit may be discretized to a multiple of G MW steps, and its fuel cost function may also be discretized according to these output levels. To find the optimal combination of I generating units at a certain load demand level L, in order to achieve a minimum production cost, according to the principle of optimality, the following equation must hold

$$COST(L) = \min \quad \{COST(L - J * G) + \Delta F_i(J, G)\} \quad J \in M, i \in I \qquad (5.17)$$

Where

$COST(L)$ is the optimal total production cost at load level L

$COST(L - J * G)$ is the optimal total fuel cost at load level $L - J * G$

$L = 0, 1 * G, 2 * G, ...T * G$ where $T * G$ equals to total generating capacity

$\Delta F_i(J, G)$ is the additional operating cost for unit i to generate further $J * G(MW)$ from its optimal loading at $COST(L - J * G)$

I is the number of generating units

M is the highest generating level of the largest unit

$COST(L) = 0.0$ for $L \leq 0.0$

Since $COST(L)$ is known at $L \leq 0.0$, the optimal operating cost and the corresponding optimal unit combination at load levels G, $2 * G$, $3 * G$, ... $T * G$ can be evaluated with the given unit cost function $F_i(P_(i, k))$, and the recursive cost function of dynamic programming can be easily implemented. It can be seen from the above equation that in finding $COST(L)$, M optimal unit combinations at optimal cost $COST(L-G)$, $COST(L-2*G)$, $COST(L-3*G)$.... $COST(L - M * G)$ should be evaluated. In this case, besides storing the cost function for each unit and other necessary data, the computer memory requirement for this algorithm is only $M * I$ (number of total generating levels * number of units) words. For a medium-sized system with approximately 100 thermal generating units with the largest equivalent unit as $4000MW$, and an accuracy up to $10MW$ step, there are totally $M = 4000/10 = 400$ steps and the computer storage requirement will be $400 * 100$, i.e. $40K$ words. Besides, further memory reduction can be achieved by breaking up the largest unit into several smaller units so that the maximum generating level M can be reduced. For example for the above system, if the largest unit is represented with two identical smaller units, each with capacity of $4000/2 = 2000MW$, then the memory requirement may be reduced to $200 * 101$, i.e. $20.2K$. Tests with the algorithm have proved that the number of generators in the system does not affect the computation time to obtain the optimal generator combination as much as the total generating capacity and the required accuracy. The computation time and memory is actually a function of total system capacity and required accuracy.

5.6.4 The Computational Scheduling Algorithm

The process of the composite cost dynamic programming approach to the solution of unit commitment problem has been implemented and can be summarized as follows:

1. Read in the load prediction data and thermal system data including generator parameters, must on/must off schedule and fixed generation requirements, etc.
2. Starting the main loop of dynamic programming with the first interval $k \Leftarrow 1$, consider the unit commitment problem interval by interval. Check unit availability and form the composite cost function model of each unit for this interval k.
3. Find the optimal unit combination for this time interval using dynamic programming with the derived composite cost function model.
4. Check whether the spinning reserve requirement is satisfied or not, if not, go back to step 3 to find another optimal combination, otherwise go back to step 2 with $k \Leftarrow k + 1$.
5. Check whether all the time intervals have been considered or not, if not, go back to step 2, otherwise calculate the total optimal production cost and output the commitment and loading schedule.

5.7 TEST SYSTEMS DATA AND COMPUTATIONAL EXPERIENCE

The test systems will include usually four types of thermal units: nuclear, coal-fired, oil-fired and gas-fired generating units. Results and comparisons of the application of the composite cost dynamic programming method, the merit-order heuristic scheme and Lagrangian relaxation dual decomposition technique to the solution of thermal unit commitment will be presented. The load prediction data for a 24 hour period with each hour demand data is shown as in Table 5.1, and a 12 thermal units test system data[49.] are presented in the following tables.

Table 5.2

Thermal generators data: number of units = **12**

Unit	P_{min}	P_{max}	A_i	B_i	C_i	$C_{coldstart}$	T_{start}	$C_{shutdown}$
1	0.5	2.0	29.0	190.0	100.0	113.0	2.0	13.5
2	0.5	1.5	29.0	200.0	150.0	113.0	1.5	11.0
3	0.2	0.7	25.0	210.0	170.0	101.0	1.0	10.0
4	0.1	0.5	15.0	210.0	170.0	85.0	0.5	8.5
5	0.1	0.5	15.0	210.0	170.0	85.0	0.5	8.5
6	0.1	0.5	15.0	210.0	170.0	85.0	0.5	8.5
7	0.5	2.0	29.0	190.0	100.0	113.0	2.0	13.5
8	0.5	1.5	29.0	200.0	150.0	113.0	1.5	11.0
9	0.2	0.7	25.0	210.0	170.0	101.0	1.0	10.0
10	0.1	0.5	15.0	210.0	170.0	85.0	0.5	8.5
11	0.1	0.5	15.0	210.0	170.0	85.0	0.5	8.5
12	0.1	0.5	15.0	210.0	170.0	85.0	0.5	8.5

Table 5.3

Thermal generators data: number of units = 12 (continued)

Unit	T_{minup}	$T_{mindown}$	Status	T_{change}	I_{ramp}	D_{ramp}	G_{init}
1	3.0	3.0	1	06/02/1985.23:00:00	0.040	0.040	0.5
2	3.0	3.0	0	06/02/1985.23:00:00	0.030	0.030	0.0
3	2.0	2.0	1	06/02/1985.23:00:00	0.014	0.014	0.2
4	1.0	1.0	1	06/02/1985.23:00:00	0.010	0.010	0.1
5	1.0	1.0	1	06/02/1985.23:00:00	0.010	0.010	0.1
6	1.0	1.0	1	06/02/1985.23:00:00	0.010	0.010	0.1
7	3.0	3.0	1	06/02/1985.23:00:00	0.040	0.040	0.5
8	3.0	3.0	1	06/02/1985.23:00:00	0.030	0.030	0.5
9	2.0	2.0	1	06/02/1985.23:00:00	0.014	0.014	0.2
10	1.0	1.0	1	06/02/1985.23:00:00	0.010	0.010	0.1
11	1.0	1.0	1	06/02/1985.23:00:00	0.010	0.010	0.1
12	1.0	1.0	0	06/02/1985.23:00:00	0.010	0.010	0.1

The thermal unit commitment scheduling program test starts at an absolute time of 07/02/1985.23 : 00 : 00. The per unit base of the test system is chosen to be 100.00(MW), the total thermal generation capacity is then 11.40($P.U.$), and it is assumed there are no units which must be "on" or must be "off". The merit-order lists of these thermal generating units will be considered from the following two aspects:

Table 5.1

Load prediction data

Created at: 07/02/1985.23:00:00

Interval no.	Absolute time	Load demand (P.U. value)
1	07/02/1985.23:00:00	2.7876
2	08/02/1985.00:00:00	2.4158
3	08/02/1985.01:00:00	3.3570
4	08/02/1985.02:00:00	2.3736
5	08/02/1985.03:00:00	2.4188
6	08/02/1985.04:00:00	2.4698
7	08/02/1985.05:00:00	2.5938
8	08/02/1985.06:00:00	3.1398
9	08/02/1985.07:00:00	4.0074
10	08/02/1985.08:00:00	4.3441
11	08/02/1985.09:00:00	4.3289
12	08/02/1985.10:00:00	4.2759
13	08/02/1985.11:00:00	4.3183
14	08/02/1985.12:00:00	4.1919
15	08/02/1985.13:00:00	4.2260
16	08/02/1985.14:00:00	4.2373
17	08/02/1985.15:00:00	4.2642
18	08/02/1985.16:00:00	4.5683
19	08/02/1985.17:00:00	4.5908
20	08/02/1985.18:00:00	4.4511
21	08/02/1985.19:00:00	4.3351
22	08/02/1985.20:00:00	4.2150
23	08/02/1985.21:00:00	4.0922
24	08/02/1985.22:00:00	3.8180

1. According to the average full load cost ($AFLC$), there is a merit order list to decide which unit must be committed or de-committed.

2. According to the average incremental cost ($AINC$), there is another merit order list to decide the committed unit generating level.

These two lists are evaluated as shown below:

Table 5.4

AFLC merit order list

Merit order no.	AFLC value ($/MW)	Unit no. i
1	325.00	12
2	325.00	11
3	325.00	10
4	325.00	4
5	325.00	5
6	325.00	6
7	364.71	3
8	364.71	9
9	404.50	1
10	404.50	7
11	444.33	8
12	444.33	2

Table 5.5

AINC merit order list

Merit order no.	AINC value ($/MW)	Unit no. i
1	312.00	12
2	312.00	11
3	312.00	10
4	312.00	4
5	312.00	5
6	312.00	6
7	363.00	3
8	363.00	9
9	440.00	1
10	440.00	7
11	500.00	8
12	500.00	2

The results and comparisons among the computational time of the three solution methodologies are shown in the following tables. For this particular test system, the Lagrangian relaxation decomposition approach starts with the initial Lagrangian multiplier estimate generated by a simple merit-order procedure as discussed previously. The dual function cost convergence criterion of the discrete problem is set to 1% and at each iteration, and the generating capacity is evaluated. If the load demand can be satisfied by this total committed generating capacity for all the time intervals, the discrete problem is solved without checking the individual infeasibility for each time interval. The continuous problem is solved with a tight convergence criterion such as 1.0×10^{-5} on both the continuous dual function cost and the line search used when maximizing the continuous dual function value. The CPU time for unit commitment versus the number of variables (without any reserve constraints) is as follows:

Table 5.6 Algorithms comparison: step = 5.0 MW (CCDP only)			
Algorithm	Number of variables	CPU time	Minimum cost ($)
CCDP	864	3.88 seconds	28195.44
MO	864	0.12 seconds	30063.36
LRD	864	0.46 seconds	28131.35

It can be seen that thermal unit commitment by Lagrangian relaxation achieve the best solution. The result of this optimization algorithm produces a $\epsilon-$ optimal solution with the cheapest total production cost among the three algorithms, nearly 0.23% cheaper than the total scheduling cost produced by $CCDP$ and 6% cheaper than the result of the merit-order scheme. The fuel costs, startup costs and shutdown costs resulting from these three algorithms are also shown in Table 5.7. It is obvious that the major cost savings actually come from avoiding unnecessary startups and shutdowns among the units.

Table 5.7 Algorithms comparison: step = 5.0 MW (CCDP only)				
Algorithm	Fuel ($)	Startup ($)	Shutdown ($)	Minimum cost ($)
CCDP	27978.50	185.94	31.00	28195.44
MO	29284.25	679.60	99.50	30063.36
LRD	28020.35	80.00	31.00	28131.35

The optimal generation schedules produced by the three algorithms are presented in the following tables (5.8-5.21). The balance (MW) represents the deviation between the power demand and the total scheduled generation, i.e. $P_D(k) - \sum_i^I P_T(i,k)$.

Table 5.8

Generation schedule for CCDP: step = 5.0 MW

Interval	Load (P.U.)	Capacity	Spinning reserve	Balance (MW)
1	2.7876	OK	OK	-0.1
2	2.4158	OK	OK	0.2
3	3.3570	OK	OK	0.1
4	2.3736	OK	OK	0.2
5	2.4188	OK	OK	0.2
6	2.4698	OK	OK	0.3
7	2.5938	OK	OK	0.1
8	3.1398	OK	OK	0.4
9	4.0074	OK	OK	-0.2
10	4.3441	OK	OK	0.4
11	4.3289	OK	OK	-0.3
12	4.2759	OK	OK	0.5
13	4.3183	OK	OK	0.2
14	4.1919	OK	OK	0.4
15	4.2260	OK	OK	0.3
16	4.2373	OK	OK	-0.2
17	4.2642	OK	OK	0.1
18	4.5683	OK	OK	0.3
19	4.5908	OK	OK	0.2
20	4.4511	OK	OK	0.1
21	4.3351	OK	OK	0.4
22	4.2150	OK	OK	0.1
23	4.0922	OK	OK	0.2
24	3.8180	OK	OK	0.1

Table 5.9 Generation schedule for CCDP: step = 5.0 MW												
(* 5.0 MW)	Unit number											
Load (steps)	1	2	3	4	5	6	7	8	9	10	11	12
56	14	0	0	6	6	6	12	0	0	6	6	0
48	12	0	0	6	5	5	10	0	0	5	5	0
67	16	0	0	7	7	7	16	0	0	7	7	0
47	12	0	0	5	5	5	10	0	0	5	5	0
48	12	0	0	6	5	5	10	0	0	5	5	0
49	12	0	0	5	5	5	12	0	0	5	5	0
52	12	0	0	6	5	5	12	0	0	6	6	0
63	14	0	0	7	7	7	14	0	0	7	7	0
80	16	0	0	8	8	8	16	0	0	8	8	8
87	18	0	0	8	8	9	18	0	0	8	9	9
87	16	0	0	7	8	7	16	10	0	7	8	8
86	16	0	0	7	8	7	16	10	0	7	7	8
86	16	0	0	7	8	7	16	10	0	7	7	8
84	16	0	0	7	7	7	16	10	0	7	7	7
85	16	0	0	7	8	7	16	10	0	7	7	7
85	16	0	0	7	8	7	16	10	0	7	7	7
85	16	0	0	7	8	7	16	10	0	7	7	7
91	16	0	0	8	8	9	16	10	0	8	8	8
92	18	0	0	8	8	8	16	10	0	8	8	8
89	16	0	0	8	8	7	16	10	0	8	8	8
87	16	0	0	7	8	7	16	10	0	7	8	8
84	16	0	0	7	7	7	16	10	0	7	7	7
82	16	0	0	7	7	7	14	10	0	7	7	7
76	14	0	0	6	6	7	14	10	0	6	6	7

Table 5.9 24 hours generation schedule

The system marginal cost at each time interval can be constructed according to the most expensive unit incremental cost as in the following table:

Table 5.10 Marginal cost schedule (* 5.0 MW)		
Interval no.	Load demand (steps)	Marginal cost ($/MW)
1	56	330.00
2	48	312.00
3	67	350.00
4	47	310.00
5	48	312.00
6	49	310.00
7	52	312.00
8	63	330.00
9	80	350.00
10	87	370.00
11	87	350.00
12	86	350.00
13	86	350.00
14	84	350.00
15	85	350.00
16	85	350.00
17	85	350.00
18	91	363.00
19	92	370.00
20	89	350.00
21	87	350.00
22	84	350.00
23	82	350.00
24	76	350.00

The cost summary of *CCDP* is as follows:

Table 5.11

Operating cost summary of each time interval for CCDP

Int.	Load	Fuel ($)	Startup ($)	Shutdown ($)	Sub-total ($)
1	56	856.50	0.00	31.00	887.50
2	48	733.80	0.00	0.00	733.80
3	67	1036.63	0.00	0.00	1036.63
4	47	718.63	0.00	0.00	718.63
5	48	733.80	0.00	0.00	733.80
6	49	748.63	0.00	0.00	748.63
7	52	794.15	0.00	0.00	794.15
8	63	968.63	0.00	0.00	968.63
9	80	1247.20	80.00	0.00	1327.20
10	87	1372.38	0.00	0.00	1372.38
11	87	1363.07	105.94	0.00	1469.01
12	86	1346.20	0.00	0.00	1346.20
13	86	1346.20	0.00	0.00	1346.20
14	84	1312.45	0.00	0.00	1312.45
15	85	1329.32	0.00	0.00	1329.32
16	85	1329.32	0.00	0.00	1329.32
17	85	1329.32	0.00	0.00	1329.32
18	91	1431.42	0.00	0.00	1431.42
19	92	1449.70	0.00	0.00	1449.70
20	89	1396.82	0.00	0.00	1396.82
21	87	1363.07	0.00	0.00	1363.07
22	84	1312.45	0.00	0.00	1312.45
23	82	1278.45	0.00	0.00	1278.45
24	76	1180.35	0.00	0.00	1180.35
Total cost		27978.50	185.94	31.00	28195.44

Table 5.12

Operating cost summary of each unit for CCDP

Unit	Fuel ($)	Startup ($)	Shutdown ($)	Sub-total ($)
1	5520.00	0.00	0.00	5520.00
2	0.00	0.00	0.00	0.00
3	0.00	0.00	10.00	10.00
4	2565.65	0.00	0.00	2565.65
5	2638.25	0.00	0.00	2638.25
6	2554.72	0.00	0.00	2554.72
7	5328.00	0.00	0.00	5328.00
8	2331.00	105.94	11.00	2447.94
9	0.00	0.00	10.00	10.00
10	2535.30	0.00	0.00	2535.30
11	2586.77	0.00	0.00	2586.77
12	1918.80	80.00	0.00	1998.80
Total cost	27978.50	185.94	31.00	28195.44

The schedules resulting from the merit-order scheme and the Lagrangian relaxation are also presented below for comparison. The individual unit schedules and the cost summary for each time interval and each generating unit will be compared together with the computational time and the total production cost.

Table 5.13

Generation schedule for MO: Step = 5.0 MW

Interval	Load (P.U.)	Capacity	Spinning reserve	Balance (MW)
1	2.7876	OK	OK	-0.2
2	2.4158	OK	OK	0.3
3	3.3570	OK	OK	0.1
4	2.3736	OK	OK	0.5
5	2.4188	OK	OK	0.4
6	2.4698	OK	OK	0.4
7	2.5938	OK	OK	-0.1
8	3.1398	OK	OK	-0.2
9	4.0074	OK	OK	0.1
10	4.3441	OK	OK	-0.1
11	4.3289	OK	OK	-0.4
12	4.2759	OK	OK	-0.5
13	4.3183	OK	OK	0.4
14	4.1919	OK	OK	-0.2
15	4.2260	OK	OK	-0.5
16	4.2373	OK	OK	-0.3
17	4.2642	OK	OK	0.3
18	4.5683	OK	OK	0.4
19	4.5908	OK	OK	0.2
20	4.4511	OK	OK	0.0
21	4.3351	OK	OK	-0.3
22	4.2150	OK	OK	0.3
23	4.0922	OK	OK	-0.2
24	3.8180	OK	OK	0.4

Table 5.14

Generation schedule for MO: Step = 5.0 MW

(* 5.0 MW)	Unit number											
Load (steps)	1	2	3	4	5	6	7	8	9	10	11	12
56	10	0	4	2	2	2	10	10	4	2	10	0
48	10	0	4	2	2	2	10	0	4	2	2	10
67	10	10	4	2	2	2	10	0	4	3	10	10
47	10	0	4	2	2	2	10	0	4	2	2	9
48	10	0	4	2	2	2	10	0	4	2	2	10
49	10	0	4	2	2	2	10	0	4	2	3	10
52	10	0	4	2	2	2	10	10	4	2	2	4
63	10	10	4	2	2	2	10	10	4	2	2	5
80	10	10	4	2	2	2	10	10	4	6	10	10
87	10	10	4	5	2	2	10	10	4	10	10	10
87	10	0	13	10	10	10	0	0	4	10	10	10
86	10	0	12	10	10	10	0	0	4	10	10	10
86	10	0	12	10	10	10	0	0	4	10	10	10
84	10	0	10	10	10	10	0	0	4	10	10	10
85	10	10	4	3	2	2	10	10	4	10	10	10
85	10	10	4	3	2	2	10	10	4	10	10	10
85	10	10	4	3	2	2	10	10	4	10	10	10
91	10	10	4	9	2	2	10	10	4	10	10	10
92	10	10	4	10	2	2	10	10	4	10	10	10
89	19	0	0	10	10	10	10	0	0	10	10	10
87	17	0	0	10	10	10	10	0	0	10	10	10
84	14	0	0	10	10	10	10	0	0	10	10	10
82	12	0	0	10	10	10	10	0	0	10	10	10
76	10	0	0	10	10	6	10	0	0	10	10	10

Table 5.15

Marginal cost schedule

(* 5.0 MW)

Interval no.	Load demand (steps))	Marginal cost ($/MW)
1	56	380.00
2	48	380.00
3	67	380.00
4	47	363.00
5	48	380.00
6	49	380.00
7	52	350.00
8	63	350.00
9	80	380.00
10	87	380.00
11	87	431.00
12	86	414.00
13	86	414.00
14	84	380.00
15	85	380.00
16	85	380.00
17	85	380.00
18	91	380.00
19	92	380.00
20	89	380.00
21	87	380.00
22	84	380.00
23	82	380.00
24	76	380.00

Table 5.16

Operating cost summary of each interval for MO

Int.	Load	Fuel ($)	Startup ($)	Shutdown ($)	Sub-total ($)
1	56	925.40	0.00	0.00	925.40
2	48	796.60	78.70	11.00	886.30
3	67	1100.53	110.18	0.00	1210.70
4	47	778.03	0.00	11.00	789.03
5	48	796.60	0.00	0.00	796.60
6	49	809.23	0.00	0.00	809.23
7	52	864.40	99.71	0.00	964.11
8	63	1045.23	96.86	0.00	1142.08
9	80	1310.00	0.00	0.00	1310.00
10	87	1419.63	0.00	0.00	1419.63
11	87	1431.13	0.00	35.50	1466.63
12	86	1410.00	0.00	0.00	1410.00
13	86	1410.00	0.00	0.00	1410.00
14	84	1370.30	0.00	0.00	1370.30
15	85	1391.83	294.16	0.00	1685.98
16	85	1391.83	0.00	0.00	1391.83
17	85	1391.83	0.00	0.00	1391.83
18	91	1485.43	0.00	0.00	1485.43
19	92	1504.00	0.00	0.00	1504.00
20	89	1423.75	0.00	42.00	1465.75
21	87	1386.75	0.00	0.00	1386.75
22	84	1335.00	0.00	0.00	1335.00
23	82	1303.00	0.00	0.00	1303.00
24	76	1203.80	0.00	0.00	1203.80
Total cost		29284.25	679.60	99.50	30063.35

Table 5.17 Operating cost summary of each unit for MO				
Unit	Fuel ($)	Startup ($)	Shutdown ($)	Sub-total ($)
1	3932.50	0.00	0.00	3932.50
2	1498.50	303.89	33.00	1835.39
3	1937.23	0.00	10.00	1947.23
4	2337.32	0.00	0.00	2337.32
5	2028.00	0.00	0.00	2028.00
6	1958.80	0.00	0.00	1958.80
7	2980.00	100.44	13.50	3093.94
8	1665.00	196.56	33.00	1894.56
9	1402.20	0.00	10.00	1412.20
10	2845.03	0.00	0.00	2845.03
11	3163.83	0.00	0.00	3163.83
12	3535.85	78.70	0.00	3614.55
Total cost	29284.25	679.60	99.50	30063.36

Table 5.18

Generation schedule of LRD

Interval	Load (P.U.)	Capacity	Reserve	Balance (MW)
1	2.7876	OK	OK	0.016
2	2.4158	OK	OK	0.307
3	3.3570	OK	OK	0.056
4	2.3736	OK	OK	0.007
5	2.4188	OK	OK	0.002
6	2.4698	OK	OK	0.004
7	2.5938	OK	OK	-0.002
8	3.1398	OK	OK	2.028
9	4.0074	OK	OK	1.170
10	4.3441	OK	OK	0.631
11	4.3289	OK	OK	0.676
12	4.2759	OK	OK	0.861
13	4.3183	OK	OK	0.732
14	4.1919	OK	OK	1.009
15	4.2260	OK	OK	0.962
16	4.2373	OK	OK	0.942
17	4.2642	OK	OK	0.901
18	4.5683	OK	OK	-0.151
19	4.5908	OK	OK	-0.228
20	4.4511	OK	OK	0.259
21	4.3351	OK	OK	0.661
22	4.2150	OK	OK	0.697
23	4.0922	OK	OK	0.903
24	3.8180	OK	OK	1.384

Table 5.19 Generation schedule of LRD												
(P.U.)	Unit number											
Load	1	2	3	4	5	6	7	8	9	10	11	12
2.78	0.62	0.0	0.0	0.31	0.31	0.31	0.62	0.0	0.0	0.31	0.31	0.0
2.41	0.55	0.0	0.0	0.26	0.26	0.26	0.55	0.0	0.0	0.26	0.26	0.00
3.35	0.74	0.0	0.0	0.38	0.38	0.38	0.74	0.0	0.0	0.38	0.38	0.00
2.37	0.54	0.0	0.0	0.26	0.26	0.26	0.54	0.0	0.0	0.26	0.26	0.00
2.41	0.55	0.0	0.0	0.26	0.26	0.26	0.55	0.0	0.0	0.26	0.26	0.00
2.46	0.56	0.0	0.0	0.27	0.27	0.27	0.56	0.0	0.0	0.27	0.27	0.00
2.59	0.58	0.0	0.0	0.28	0.28	0.28	0.58	0.0	0.0	0.28	0.28	0.00
3.14	0.69	0.0	0.0	0.35	0.35	0.35	0.69	0.0	0.0	0.35	0.35	0.00
4.01	0.79	0.0	0.0	0.40	0.40	0.40	0.79	0.0	0.0	0.40	0.40	0.40
4.34	0.85	0.0	0.0	0.44	0.44	0.44	0.85	0.0	0.0	0.44	0.44	0.44
4.33	0.85	0.0	0.0	0.44	0.44	0.44	0.85	0.0	0.0	0.44	0.44	0.44
4.28	0.84	0.0	0.0	0.43	0.43	0.43	0.84	0.0	0.0	0.43	0.43	0.43
4.32	0.84	0.0	0.0	0.44	0.44	0.44	0.84	0.0	0.0	0.44	0.44	0.44
4.19	0.82	0.0	0.0	0.42	0.42	0.42	0.82	0.0	0.0	0.42	0.42	0.42
4.23	0.83	0.0	0.0	0.43	0.43	0.43	0.83	0.0	0.0	0.43	0.43	0.43
4.24	0.83	0.0	0.0	0.43	0.43	0.43	0.83	0.0	0.0	0.43	0.43	0.43
4.26	0.83	0.0	0.0	0.43	0.43	0.43	0.83	0.0	0.0	0.43	0.43	0.43
4.57	0.89	0.0	0.0	0.46	0.46	0.46	0.89	0.0	0.0	0.46	0.46	0.46
4.59	0.89	0.0	0.0	0.47	0.47	0.47	0.89	0.0	0.0	0.47	0.47	0.47
4.45	0.87	0.0	0.0	0.45	0.45	0.45	0.87	0.0	0.0	0.45	0.45	0.45
4.34	0.85	0.0	0.0	0.44	0.44	0.44	0.85	0.0	0.0	0.44	0.44	0.44
4.22	0.82	0.0	0.0	0.43	0.43	0.43	0.82	0.0	0.0	0.43	0.43	0.43
4.09	0.80	0.0	0.0	0.41	0.41	0.41	0.80	0.0	0.0	0.41	0.41	0.41
3.82	0.75	0.0	0.0	0.38	0.38	0.38	0.75	0.0	0.0	0.38	0.38	0.38

Table 5.20

Operating cost summary of each interval for LRD

Int.	Load	Fuel ($)	Startup ($)	Shutdown ($)	Sub-total ($)
1	56	851.85	0.00	31.00	882.85
2	48	736.76	0.00	0.00	736.76
3	67	1037.49	0.00	0.00	1037.49
4	47	725.05	0.00	0.00	725.05
5	48	738.58	0.00	0.00	738.58
6	49	753.91	0.00	0.00	753.91
7	52	791.68	0.00	0.00	791.68
8	63	958.60	0.00	0.00	958.60
9	80	1245.65	80.00	0.00	1325.65
10	87	1366.58	0.00	0.00	1366.58
11	87	1360.95	0.00	0.00	1360.95
12	86	1341.31	0.00	0.00	1341.31
13	86	1356.98	0.00	0.00	1356.98
14	84	1310.91	0.00	0.00	1310.91
15	85	1323.18	0.00	0.00	1323.18
16	85	1327.26	0.00	0.00	1327.26
17	85	1336.99	0.00	0.00	1336.99
18	91	1451.00	0.00	0.00	1451.00
19	92	1459.57	0.00	0.00	1459.57
20	89	1406.62	0.00	0.00	1406.62
21	87	1363.23	0.00	0.00	1363.23
22	84	1320.21	0.00	0.00	1320.21
23	82	1276.17	0.00	0.00	1276.17
24	76	1179.80	0.00	0.00	1179.80
Total cost		28020.35	80.00	31.00	28131.35

Table 5.21

Operating cost summary of each unit for LRD

Unit no.	Fuel ($)	Startup ($)	Shutdown ($)	Sub-total ($)
1	5561.38	0.00	0.00	5561.38
2	0.00	0.00	0.00	0.00
3	0.00	0.00	10.00	10.00
4	2939.64	0.00	0.00	2939.64
5	2939.64	0.00	0.00	2939.64
6	2939.64	0.00	0.00	2939.64
7	5561.38	0.00	0.00	5561.38
8	0.00	0.00	11.00	11.00
9	0.00	0.00	10.00	10.00
10	2939.64	0.00	0.00	2939.64
11	2939.64	0.00	0.00	2939.64
12	2199.41	80.00	0.00	2279.41
Total cost	28020.35	80.00	31.00	28131.35

To investigate the practical applicability of the three algorithms for the solution of unit commitment problems in large scale thermal power systems, the commitment algorithms have been applied to EPRI Scenario system A. [49.] In this test thermal system, there are a total of 224 thermal generating units with a total capacity of $51,750MW$. The tests have shown that the CPU time of the $CCDP$ algorithm increases drastically despite the fact that it is more efficient than standard dynamic programming routines for medium-sized systems. The Lagrangian relaxation decomposition is much more efficient and the total production cost is actually about 0.1% cheaper than that of the CCDP algorithm, and certainly much cheaper than the merit-order scheme.

Table 5.22

Algorithms comparison: step = 25.0 MW

Algorithm	No. of Variables	CPU Time	Minimum Cost($)
CCDP	16128	10 minutes 37.64 seconds	1181167.00
M.O.	16128	02.21 seconds	1358989.25
LRD	16128	29.35 seconds	1180340.00

Table 5.23 Operating cost summary of each interval for CCDP					
Int.	Load	Fuel ($)	Startup ($)	Shutdown ($)	Sub-Total ($)
1	844	39656.81	0.00	2334.00	41990.81
2	803	37428.93	0.00	51.00	37479.93
3	776	35852.12	0.00	102.00	35954.12
4	775	35800.11	0.00	0.00	35800.11
5	774	35747.28	0.00	0.00	35747.28
6	787	36480.16	188.44	0.00	36668.60
7	884	41888.09	1154.05	0.00	43042.14
8	1026	49816.92	1716.09	0.00	51533.01
9	1126	55427.58	1307.42	16.00	56751.00
10	1165	57699.65	751.15	0.00	58450.80
11	1166	57753.71	0.00	0.00	57753.71
12	1169	58001.82	0.00	0.00	58001.82
13	1129	55622.87	0.00	80.00	55702.87
14	1147	56663.17	303.33	0.00	56966.51
15	1144	56467.28	0.00	20.00	56487.28
16	1114	54746.18	0.00	60.00	54806.18
17	1087	53218.44	0.00	60.00	53278.44
18	1050	51162.82	0.00	60.00	51222.82
19	1075	52601.43	424.57	0.00	53026.00
20	1086	53196.49	180.90	20.00	53397.39
21	1118	54994.08	361.80	0.00	55355.88
22	1072	52405.64	0.00	90.00	52495.64
23	983	47426.73	0.00	160.00	47586.73
24	876	41467.31	0.00	201.00	41668.31
Total cost		1171525.88	6387.74	3254.00	1181167.50

Table 5.24

Operating cost summary of each interval for MO

Int.	Load	Fuel ($)	Startup ($)	Shutdown ($)	Sub-Total ($)
1	844	39932.09	0.00	2232.00	42164.09
2	803	37612.80	0.00	102.00	37714.80
3	776	36116.46	0.00	51.00	36167.46
4	775	36064.58	0.00	0.00	36064.58
5	774	36012.86	0.00	0.00	36012.86
6	787	52563.97	16085.41	0.00	68649.22
7	884	55985.29	0.00	0.00	55985.29
8	1026	63026.41	0.00	0.00	63026.41
9	1126	68173.16	0.00	0.00	68173.16
10	1165	70183.60	0.00	0.00	70183.60
11	1166	70236.28	0.00	0.00	70236.28
12	1169	70389.54	0.00	0.00	70389.54
13	1129	60642.10	0.00	0.00	60642.10
14	1147	69256.91	0.00	0.00	69256.91
15	1144	56862.68	0.00	1572.00	58434.68
16	1114	55129.01	0.00	80.00	55209.01
17	1087	53555.80	0.00	80.00	53635.80
18	1050	51609.16	0.00	0.00	51609.16
19	1075	65550.16	11786.48	0.00	77336.39
20	1086	66112.58	0.00	0.00	66112.58
21	1118	67761.30	0.00	0.00	67761.30
22	1072	52693.05	0.00	1772.00	54465.05
23	983	47648.39	0.00	190.00	47838.39
24	876	41704.32	0.00	210.00	41914.32
Total cost		1324822.63	27871.89	6289.00	1358983.13

Table 5.25 Operating cost summary of each interval for LRD					
Int.	Load	Fuel ($)	Startup ($)	Shutdown ($)	Sub-Total ($)
1	844	40084.80	0.00	2082.00	42166.80
2	803	37524.36	0.00	252.00	37776.36
3	776	36127.98	0.00	0.00	36127.98
4	775	36032.16	0.00	0.00	36032.16
5	774	35993.83	0.00	0.00	35993.83
6	787	36697.21	0.00	0.00	36697.21
7	884	41840.02	0.00	0.00	41840.02
8	1026	50207.98	3261.46	0.00	53469.43
9	1126	55384.36	0.00	0.00	55384.36
10	1165	57676.00	0.00	0.00	57676.00
11	1166	57676.00	0.00	0.00	57676.00
12	1169	57676.00	0.00	0.00	57676.00
13	1129	55537.48	0.00	0.00	55537.48
14	1147	56508.85	0.00	0.00	56508.85
15	1144	56377.14	0.00	0.00	56377.14
16	1114	54768.57	0.00	0.00	54768.57
17	1087	53310.79	0.00	0.00	53310.79
18	1050	51779.63	0.00	0.00	51779.63
19	1075	52765.70	0.00	0.00	52765.70
20	1086	53258.59	0.00	0.00	53258.59
21	1118	54853.73	0.00	0.00	54853.73
22	1072	52547.52	0.00	0.00	52547.52
23	983	48175.68	0.00	0.00	48175.68
24	876	41520.63	0.00	420.00	41940.63
Total cost		1174325.13	3261.46	2754.00	1180340.50

Note that the unit of the load demand in the above three tables is in *STEPS* The actual demand is discretized into many steps with each step representing some amount of (MW)s. The total fuel cost, start up and shut down cost resulting from these three algorithms are also shown as follows.

Table 5.26

Algorithms comparisons: step = 25.0 MW
(CCDP only)

Algorithm	Fuel ($)	Startup ($)	Shutdown ($)	Total Cost ($)
CCDP	1171525.88	6387.74	3254.00	1181167.50
M.O.	1324828.00	27871.89	6289.00	1358989.25
LRD	1174324.75	3261.46	2754.00	1180340.00

A set of test results obtained with the EPRI 224 unit system using Lagrangian relaxation technique is presented. The following figures will show how the dual and the primal cost functions change together with the Lagrangian multipliers. Figure 5.1 shows the cost change at each iteration. Since the sub-gradient optimization algorithm is used to maximize the dual function, no line search is performed in the sub-gradient direction, hence the change of the dual cost function value is not monotone. The convergence criterion for the discrete problem is to ensure feasibility, i.e. the total generating capacity should be sufficient to cover the load demand at each time interval. Thus, even though the primal cost value is very near to the dual cost value at some iterations, the discrete problem is not terminated. The increases and decreases in the dual and the primal function values actually show the computational process of seeking a potential feasible unit commitment schedule.

The profile of Lagrangian multipliers in the discrete problem is shown in Figure 5.2 and 5.3. For this example, the discrete problem converges at the 7th iteration. Figure 5.4 demonstrates the primal and dual cost changes for the continuous problem. This shows that for the continuous problem,

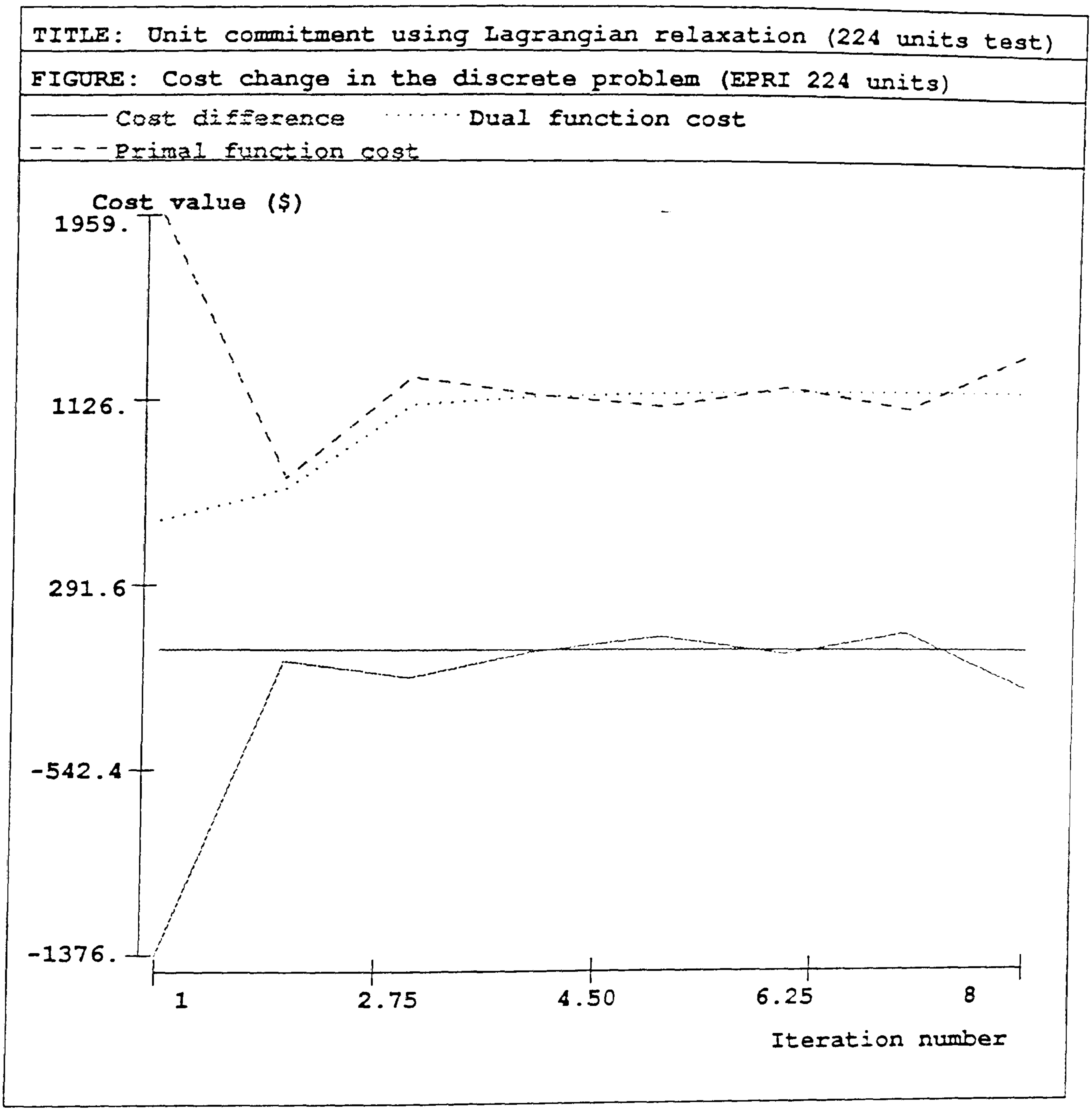

TITLE: Unit commitment using Lagrangian relaxation (224 units test)
FIGURE: Cost change in the discrete problem (EPRI 224 units)
Cost difference
Dual function cost
Primal function cost
Cost value ($)
1959.
1126.
291.6
-542.4
-1376.
1
2.75
4.50
6.25
8
Iteration number

Figure 5.1

without integer variables, the duality gap between the primal problem and the dual problem disappears, since the continuous problem of power dispatch is actually a convex optimization problem. In this case, it is assumed that the gradient direction is very accurate, hence, a line search is performed in order to maximize the dual function value along the chosen ascending direction. The conventional sub-gradient optimization algorithm without line search has also been tested. Results show that in the continuous problem, this algorithm will cause instability or oscillation near the optimal point. Also, depending on the optimal dual cost estimation, the convergence will be slow. To ensure the stability and optimality of the solution, a line search should be performed. Since economic dispatch among the committed units at each iteration with specified Lagrangian multipliers takes little computational time, the overall continuous problem converges efficiently despite the line search evaluation.

Two line search approaches have been proposed, namely, the golden section search and quadratic interpolation search. Details about these two line search techniques are presented in Appendix 1 and Appendix 2. A quadratic interpolation line search has been proved to perform much better than the golden section line search. Normally 3 to 4 interpolations at each iteration will be sufficient to maximize the dual function along the chosen ascending direction. The golden section search is well-known for its robustness, but its linear convergence rate tends to be unsatisfactory, and this line search is very time consuming especially for the Lagrangian min-max dual problem since for each set of specified Lagrangian multipliers, an inner optimization must be carried out to find the minimum of the primal function value. To achieve an accurate line search result may require the golden section search to evaluate the dual function value along the chosen sub-gradient direction many times. The quadratic interpolation search has a higher efficiency since the dual objective function is usually a concave function and can be closely approximated using a quadratic function near the optimal point.

For this particular example, the continuous problem converges at the 26th iteration. The Lagrangian multiplier profile for each iteration is shown in Figure 5.5-9. It can be seen that Lagrangian dual variables in the continuous problem

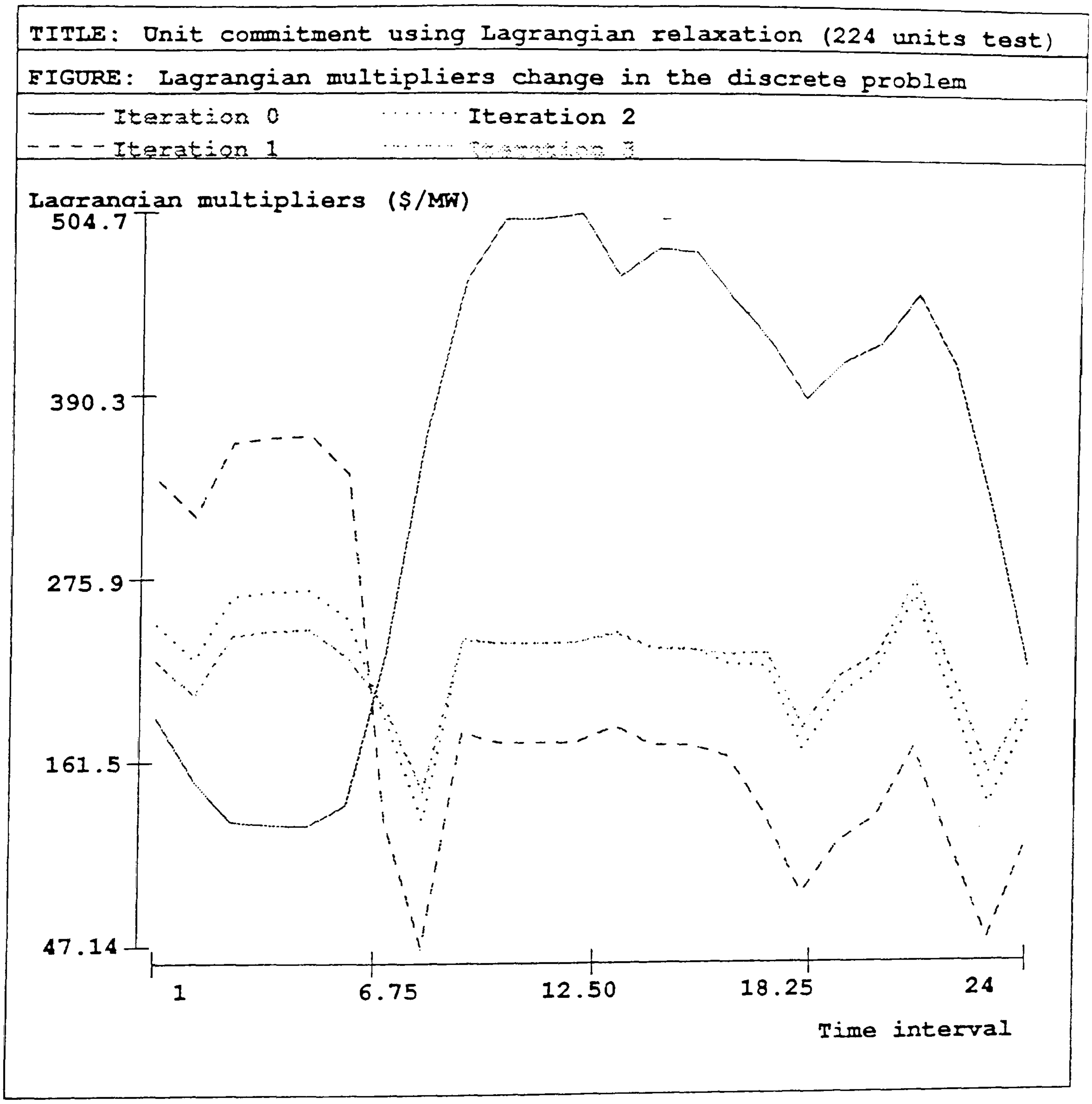
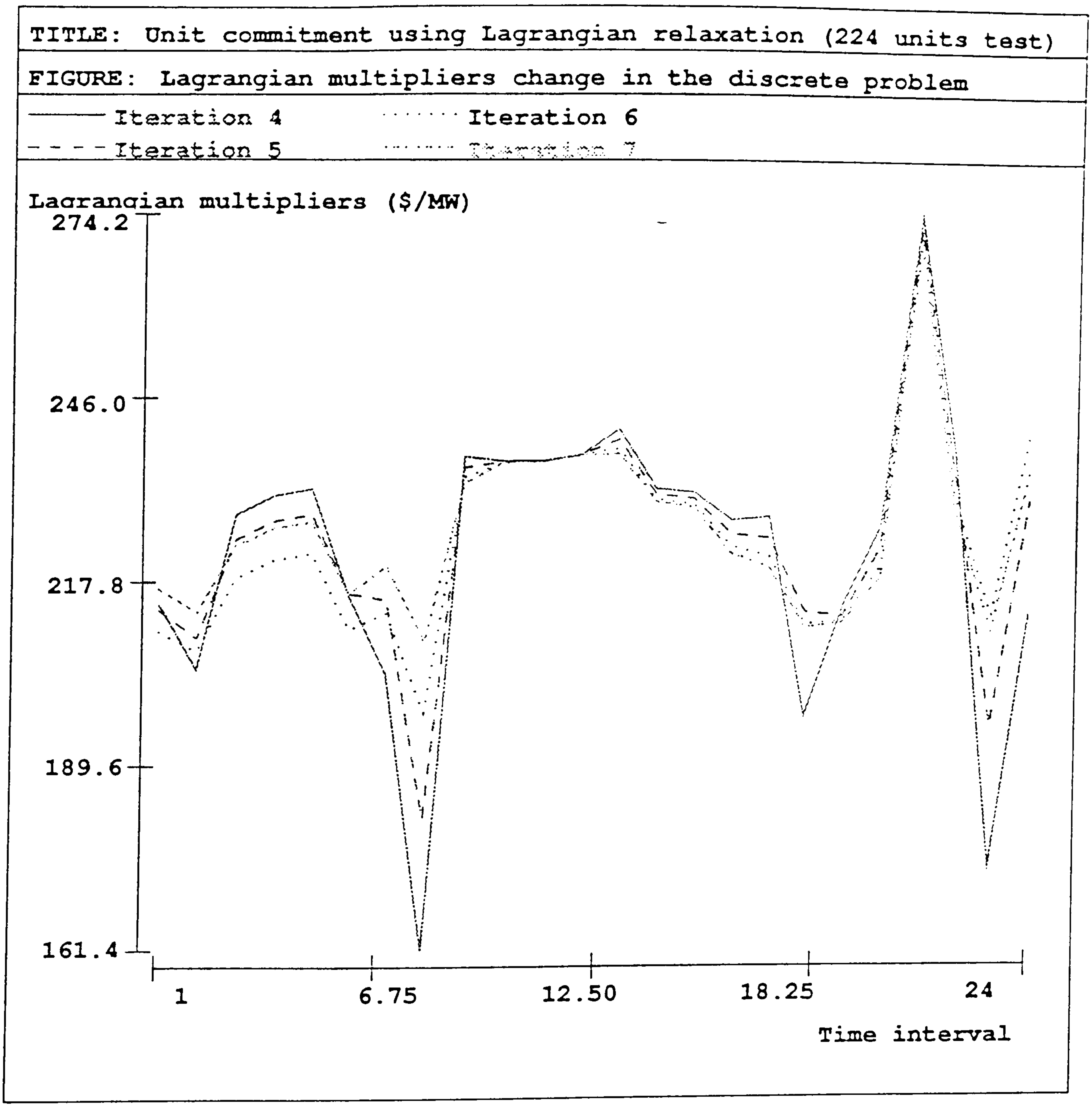

TITLE: Unit commitment using Lagrangian relaxation (224 units test)
FIGURE: Lagrangian multipliers change in the discrete problem
Iteration 0
Iteration 2
Iteration 1
Lagrangian multipliers ($/MW)
504.7
390.3
275.9
161.5
47.14
1
6.75
12.50
18.25
24
Time interval

Figure 5.2



TITLE: Unit commitment using Lagrangian relaxation (224 units test)
FIGURE: Lagrangian multipliers change in the discrete problem
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Lagrangian multipliers ($/MW)
274.2
246.0
217.8
189.6
161.4
1
6.75
12.50
18.25
24
Time interval

Figure 5.3

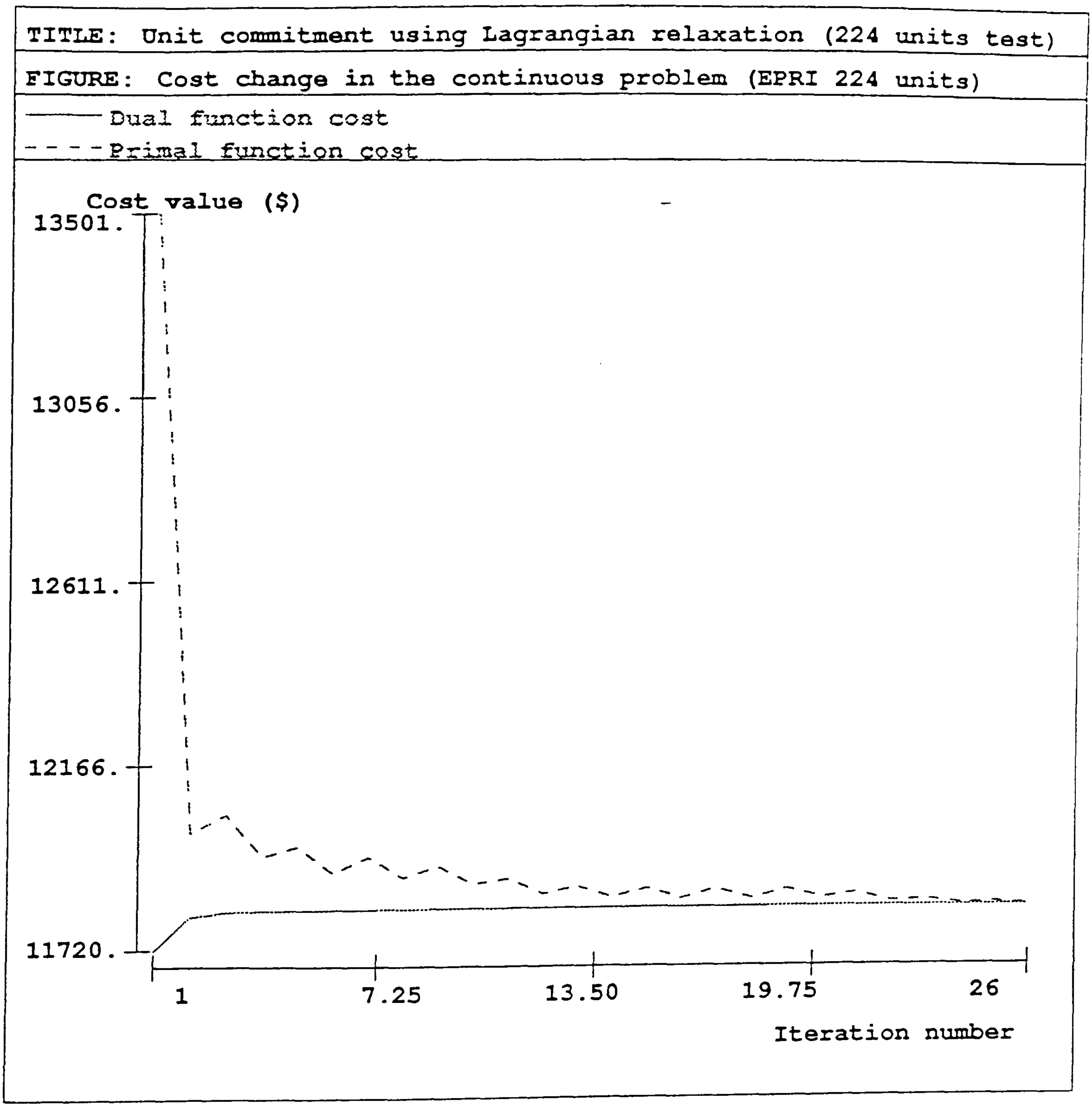

TITLE: Unit commitment using Lagrangian relaxation (224 units test)
FIGURE: Cost change in the continuous problem (EPRI 224 units)
Dual function cost
Primal function cost
Cost value ($)
13501.
13056.
12611.
12166.
11720.
1
7.25
13.50
19.75
26
Iteration number

Figure 5.4

always change in very small steps. The change of Lagrangian multipliers is very sensitive. However, the figures also show that apart from a few time intervals, the Lagrangian multipliers in other time intervals hardly change.

Tests also show that Lagrangian relaxation is very sensitive to the initial estimate of Lagrangian multipliers. Using the feasible generating approach which has been developed can always ensure fast convergence of the discrete problem as well as ensuring a feasible power dispatch schedule among the committed units. The fixed unit commitment may not guarantee an exact optimal solution, however, tests have shown that a near optimal solution is certainly possible to achieve. Compared with the result of the unit commitment using the CCDP algorithm, the algorithm based on Lagrangian relaxation usually has a cheaper total production cost. In a few cases the result may be more or less the same as that of CCDP, nevertheless, the more important advantage of Lagrangian relaxation over the CCDP algorithm is that large scale unit commitment problems can be solved much more efficiently. For this 224 unit system, a minute or so is sufficient for the Lagrangian relaxation algorithm to converge while CCDP needs nearly 11 minutes.

To conclude, tests have shown that the Lagrangian relaxation technique is so far the most efficient method for the solution of unit commitment problems in large scale thermal power systems with hundreds of units over 24 hourly intervals. The difficulty associated with the existence of the duality gap, which is created by the non-convexity of the primal problem due to integer variables, has been overcome using a method which generates a feasible solution. The results obtained are very satisfactory compared with the other two approaches of a merit-order scheme and a CCDP algorithm. Furthermore, tests have been carried out for two sets of system data with 12 units and 224 units respectively, and the results have proved that the computation time with Lagrangian relaxation algorithm increases only linearly with the number of units in the system, and unlike the CCDP algorithm, is not affected by the system capacity and the required accuracy of the schedule.

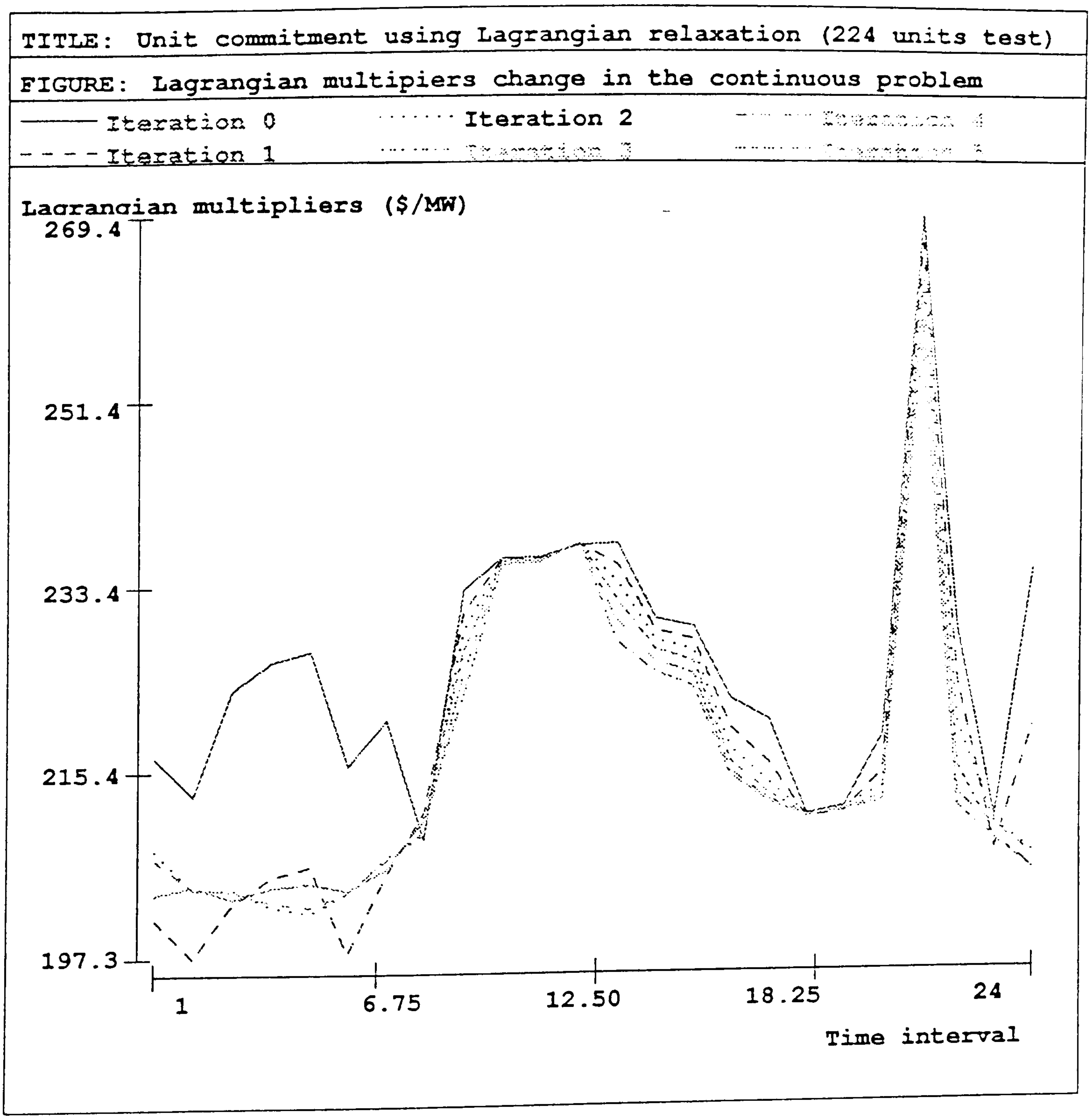
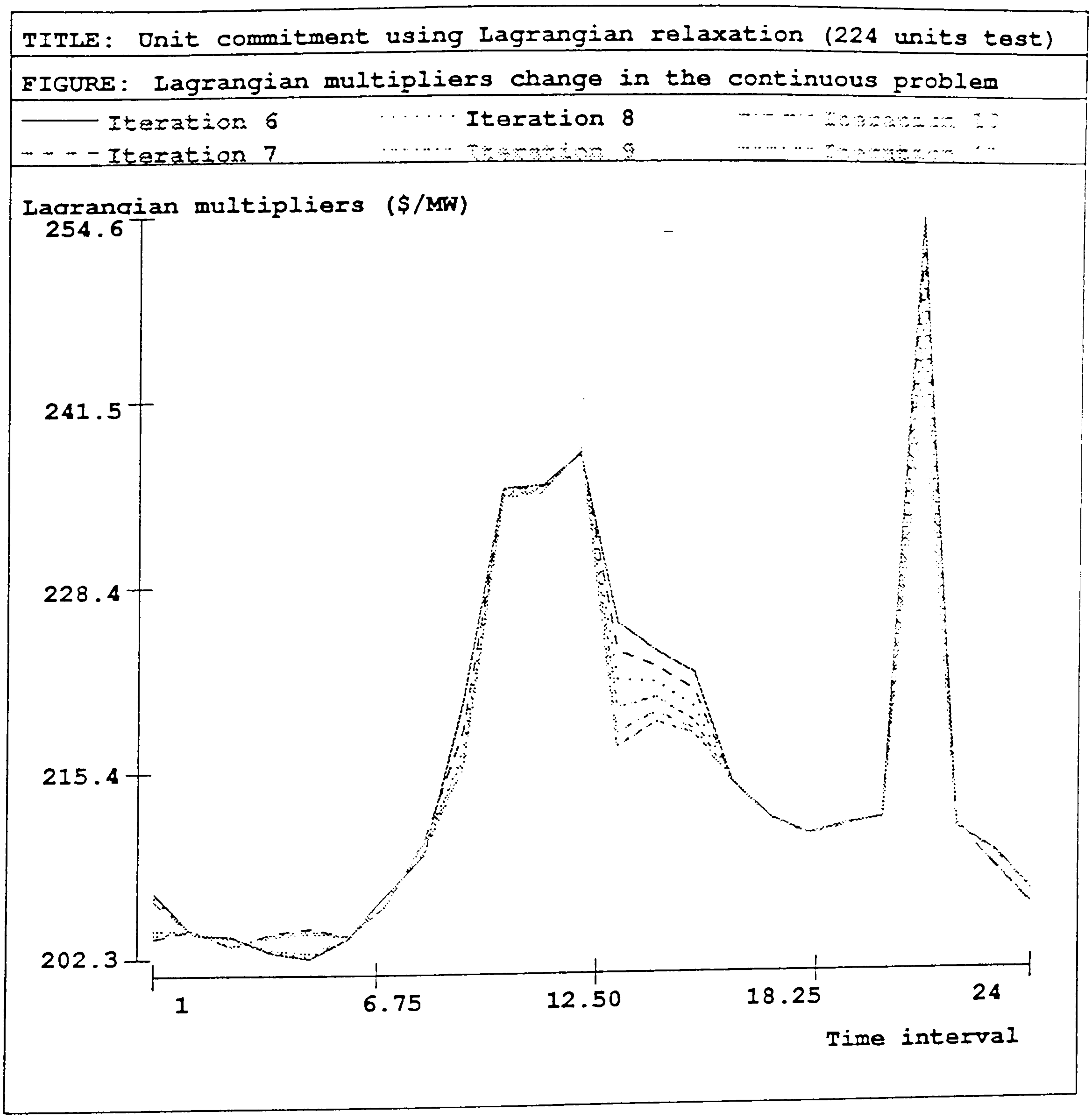

TITLE: Unit commitment using Lagrangian relaxation (224 units test)
FIGURE: Lagrangian multipiers change in the continuous problem
Iteration 0
Iteration 1
Iteration 2
Lagrangian multipliers ($/MW)
269.4
251.4
233.4
215.4
197.3
1
6.75
12.50
18.25
24
Time interval

Figure 5.5



TITLE: Unit commitment using Lagrangian relaxation (224 units test)
FIGURE: Lagrangian multipliers change in the continuous problem
Iteration 6
Iteration 8
Iteration 7
Lagrangian multipliers ($/MW)
254.6
241.5
228.4
215.4
202.3
1
6.75
12.50
18.25
24
Time interval

Figure 5.6

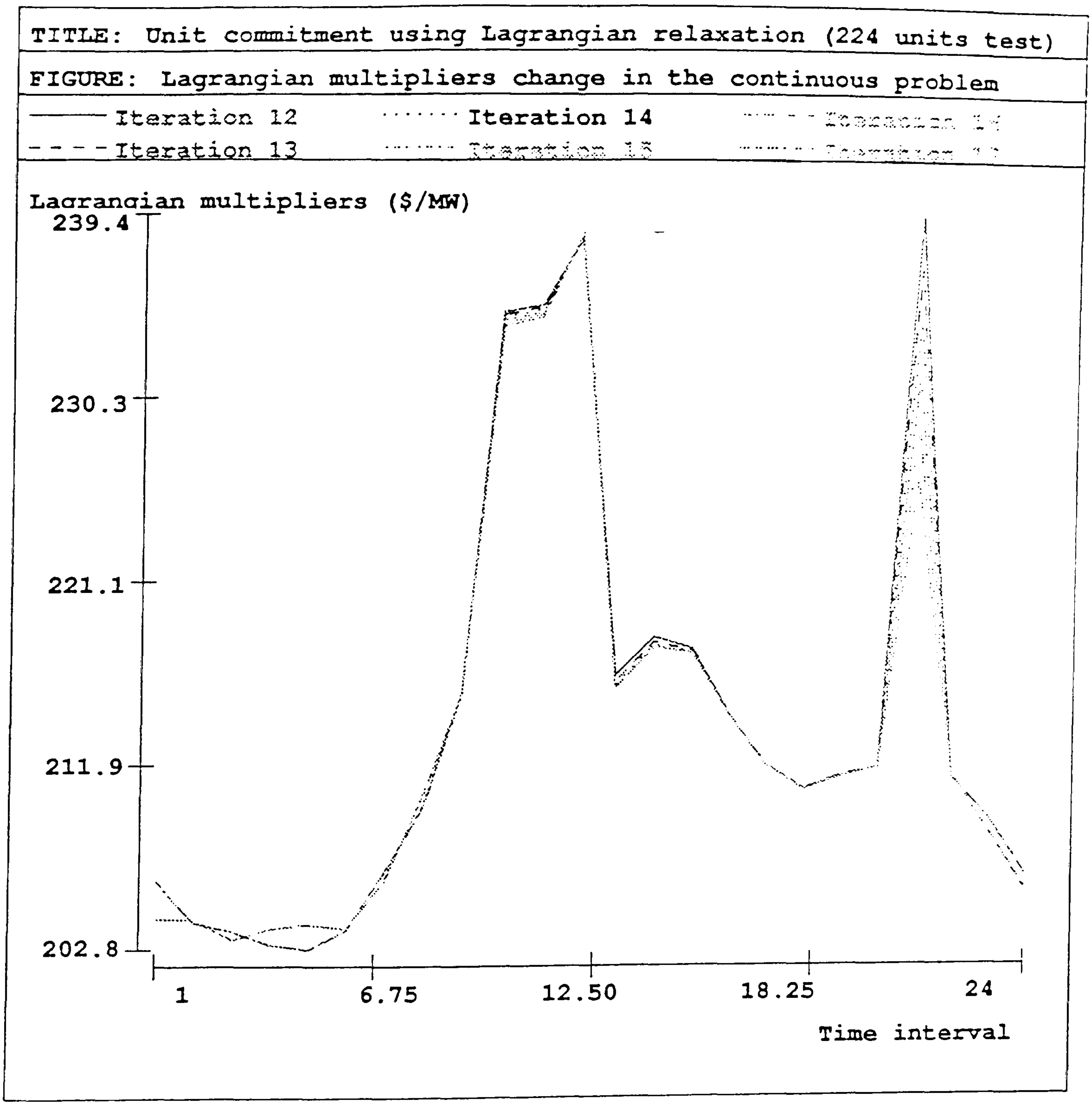

TITLE: Unit commitment using Lagrangian relaxation (224 units test)
FIGURE: Lagrangian multipliers change in the continuous problem
Iteration 12
Iteration 14
Iteration 13
Lagrangian multipliers ($/MW)
239.4
230.3
221.1
211.9
202.8
1
6.75
12.50
18.25
24
Time interval

Figure 5.7

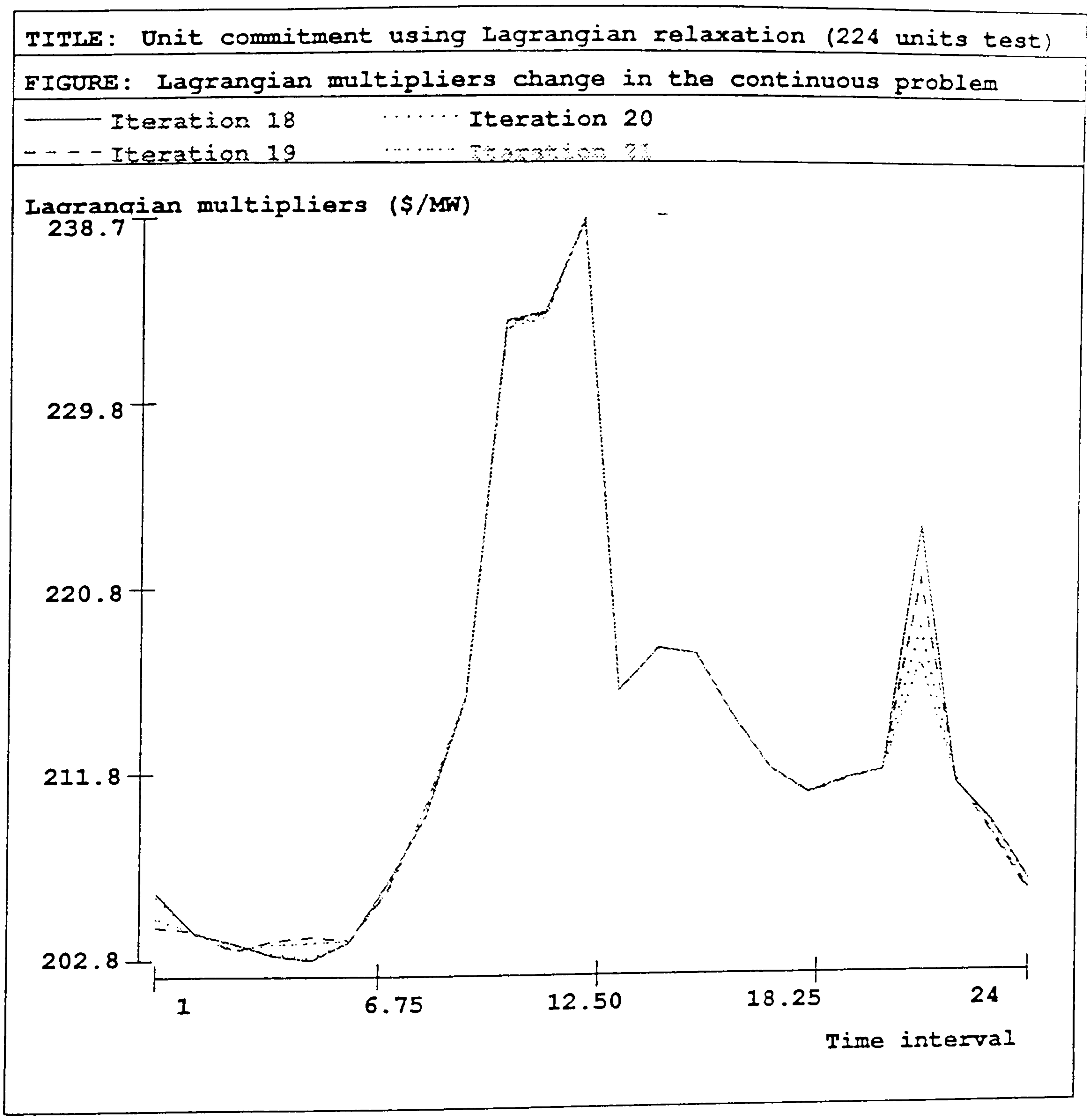
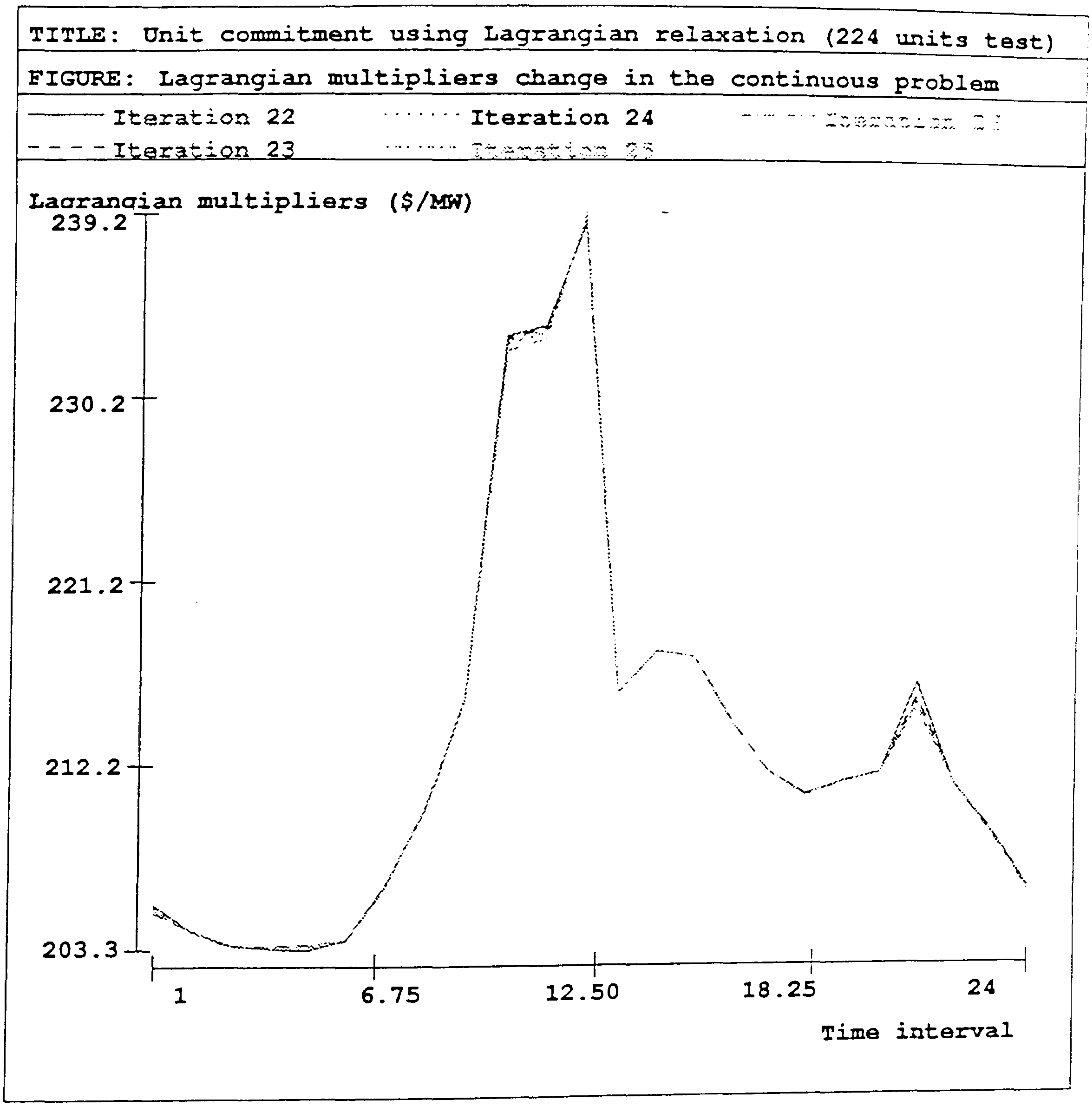

TITLE: Unit commitment using Lagrangian relaxation (224 units test)
FIGURE: Lagrangian multipliers change in the continuous problem
Iteration 18
Iteration 20
Iteration 19
Lagrangian multipliers ($/MW)
238.7
229.8
220.8
211.8
202.8
1
6.75
12.50
18.25
24
Time interval

Figure 5.8



TITLE: Unit commitment using Lagrangian relaxation (224 units test)
FIGURE: Lagrangian multipliers change in the continuous problem
Iteration 22
Iteration 24
Iteration 23
Lagrangian multipliers ($/MW)
239.2
230.2
221.2
212.2
203.3
1
6.75
12.50
18.25
24
Time interval

Figure 5.9

CHAPTER 6

SOLUTION TECHNIQUES FOR HYDRO SCHEDULING

6.1 INTRODUCTION

This chapter is aimed at developing efficient techniques that are applicable to the optimal solution of the generation scheduling problem in large scale realistic sized, purely hydroelectric generation systems. The application of these mathematical programming techniques has mainly two objectives. Firstly, these methods are proposed for the solution of short-term generation scheduling of large scale hydroelectric systems, hence, the development of these techniques was independent of hydrothermal scheduling coordination. Secondly, since the short-term hydrothermal generation scheduling problem can be decomposed into hydro and thermal subproblems through mathematical decomposition techniques, the special features can be exploited of the two different subsystems respectively, these solution techniques can also be applied to the hydro subproblem in the short-term generation scheduling of large scale hydrothermal systems. By modification of the computer program and taking into account some additional constraints from hydrothermal coordination, the hydro subproblem scheduling program can be combined with the thermal subproblem scheduling program into hydrothermal scheduling through a coordination procedure. Details about hydrothermal coordination will be presented later in Chapter 7.

Several algorithmic approaches for determining the optimal short-term scheduling of multi-reservoir hydroelectric power systems are considered. The aim of this chapter is to introduce in great detail the development of efficient solution techniques that are of practical use in the hydro generation scheduling problem as well as the hydro subsystem scheduling in a hydrothermal scheduling problem. The hydro subproblem scheduling is actually the maximization of hydro power production benefit with respect to the marginal prices of the whole system. This problem is normally a large scale nonlinear programming problem with an embedded network structure.

Numerous optimization techniques have been applied to this problem including linear programming, nonlinear programming and dynamic programming techniques. During the progress of the project, this problem is firstly formulated as a linear network flow model by linear approximation of the original nonlinear problem, different algorithms are presented for the solution of this linear programming problem, such as the out-of-kilter network flow algorithm, a sparse dual revised simplex linear programming method (SDRSLP) and the simplex method on a graph algorithm for a minimal cost network flow problem (NFLP), i.e. a network flow linear programming method. The results have shown the efficiency of the minimal network flow algorithm. A feasible direction approach based on the Frank-Wolfe method is then applied to take into the consideration the nonlinearity of the formulated objective function. Finally, the Lagrangian relaxation technique is used to take into account the power balance constraints for short-term purely hydroelectric generation scheduling.

The sections of this chapter are organised as follows. Firstly, the mathematical modelling of a hydroelectric power system for the short-term optimal operational scheduling problem is considered in Section 6.2, including a brief description of the overall formulation of the optimization model for the problem. The linear approximation of the model enables the problem to be formulated as a large scale linear programming problem with embedded network structure. In Section 6.3, a linear programming approach using a sparse dual revised simplex method is applied to solve the problem, which can be used directly but it does not exploit the network structure of the problem. A brief discussion on how the special network structure of the hydro subproblem can be exploited to derive a linear network flow problem formulation is presented in Section 6.4. The later sections then concentrate on presenting the details about the algorithms which are employed for the solution of this hydro scheduling network flow problem. The implementation considerations for each solution method are also outlined.

The out-of-kilter minimal cost network flow algorithm is considered in Section 6.5. Another linear network flow approach based on the *Simplex method on a graph* introduced by Kennington[116.] is described in Section 6.6. This method, namely the network flow simplex method, is shown to be an very efficient approach to exploit the network structure of the hydro subproblem as shown through numerous tests. However, the nonlinearity of hydroelectric power production function was not considered and the head variations were neglected in this linear network flow minimal cost model. This may result in a fairly approximate optimal solution (approximately with 5% of its real optimum) within a very low CPU time and memory requirement. In Section 6.7, a Frank-Wolfe feasible direction algorithm is applied to consider the nonlinearity of the objective function which stems from the head variations of a hydro power station. A more realistic model is thus derived and the problem is solved by this feasible direction successive approximation method to obtain a more accurate optimal solution.

Some system security requirements may be considered in the constraint set of the hydro scheduling. These constraints are termed system security constraints. Usually, they include the limitations on transmission lines, the consideration of transmission losses and the requirement for spinning reserves. The system security requires some predetermined spinning reserves, due to the rapid response capabilities of hydroelectric units, these spinning reserves in a hydrothermal power system are often imposed on the hydro units. For a large scale hydrothermal power system with a high proportion of hydro generation capacity, it is assumed that the spinning reserves will be easily satisfied by the available hydro units, hence these constraints are not considered explicitly in the hydro scheduling model. The Dantzig-Wolfe decomposition technique and partitioning techniques have been applied to incorporate the transmission limitations and transmission constraints into the problem.[40.] Since all these security constraints are non-network constraints, they destroy the direct application of network flow algorithms. As the hydro subproblem scheduling needs to be coordinated with the thermal subproblem in the hydrothermal scheduling problem, all these additional constraints will be considered in hydrothermal coordination. For generation scheduling problem in a purely hydroelectric system, the power demand requirement constraints must be considered. A Lagrangian relaxation dual decomposition methodology is proposed in Section 6.8, which allows the application of network flow algorithms directly. Other security constraints such as reserve constraints and transmission limitations could be included in the same way.

A brief comparison and discussion of all the solution algorithms is presented in Section 6.9, together with some test results to show the efficiency of employing the network structure of hydroelectric generation scheduling problem.

6.2 PROBLEM FORMULATION

The objective of short-term hydroelectric scheduling is to optimize the total production benefit with respect to the marginal prices of the system over a scheduling period of typically 24 hours to one week. Solution algorithms of this problem should determine the hourly schedule of each hydroelectric plant within the system and at the same time satisfy all the hydro system operating constraints.

This scheduling problem is conventionally formulated as a large nonlinear programming problem. The problem is usually very complex for realistically sized hydroelectric power systems and in order to achieve the solution of the problem efficiently, the special features involved in the problem must be exploited. Fortunately, the reservoir dynamics constraint has a special embedded network structure and some solution techniques based on the network flow formulation and fast network flow algorithms can be employed to solve the problem efficiently.

The following simplifications are introduced for the sake of clarity in the explanation, however, in the test results that follow in the later sections, these simplifying assumptions are not used.

- The distance between an upstream reservoir and its downstream reservoir is assumed to be insignificant, so the transport time delay of water travelling from an upstream reservoir to its downstream reservoir is assumed to be much less than a hour, which is the time of an discretized interval of the mathematical model. Hence, the water transport time delay can be neglected.

- No reservoir has more than one downstream reservoir.
- Spillage is not considered.

6.2.1 Reservoir Operating Constraints

As stated in Chapter 4, hydro reservoir dynamics and hydraulic network modelling form the major part of the constraints in hydroelectric generation scheduling problem. These hydro constraints can be summarized by the following equations.

- The dynamics of cascaded hydro reservoirs can be represented by the following difference equation:

$$V(j,k+1) - V(j,k) + Q(j,k) - \sum_{m}^{M} Q(n,k) = INF(j,k) \tag{6.1}$$

- The upper and lower limits on reservoir contents can be time-dependent if necessary and they are described by:

$$V_{jmin} \leq V(j,k) \leq V_{jmax} \tag{6.2}$$

- The upper and lower limits on reservoir discharge rates can also be time-dependent if necessary and they are represented by:

$$Q_{jmin} \leq Q(j,k) \leq Q_{jmax} \tag{6.3}$$

- The specification of the initial condition on reservoir volumes, i.e. the initial reservoir volume is fixed:

$$V(j,1) = V(j,0) \tag{6.4}$$

- The specification of the final stage constraints on reservoir volumes, i.e. the final reservoir volume is fixed:

$$V(j,K+1) = V(j,K) \tag{6.5}$$

6.2.2 Hydro Scheduling Objective Function

A common objective function of hydroelectric power scheduling is chosen to be the maximization of the benefit of the amount of electrical energy (MWh) production over a specified scheduling period, typically on a daily or weekly basis. The hydropower benefit function is dependent on the hydro power production function and the system marginal price at each time interval. The power balance requirement is considered to be an equality constraint rather than an objective in the generation scheduling problem of a purely hydroelectric system since hydroelectric generation should always satisfy the hourly load demand for a purely hydro generation system. However, for the hydro subproblem scheduling as a part of the hydrothermal generation scheduling, these constraints are assumed to be satisfied through hydrothermal coordination. They are therefore not considered in the formulation of hydro subproblem scheduling.

The electrical energy production from a hydroelectric power station j during a time interval k can be expressed as:

$$E(j,k) = T * P_H(j,k) \tag{6.6}$$

Where T is the amount of time at each time interval. Since T in this model is taken as one hour unit,

$$E(j,k) = P_H(j,k) \tag{6.7}$$

The total energy production during time interval k will be the sum of the energy production from each hydro power station, thus

$$E(k) = \sum_{j}^{J} P_H(j,k) \tag{6.8}$$

Therefore, the objective of hydro generation scheduling can be represented as:

$$max \quad \sum_{k}^{K} \lambda(k) \sum_{j}^{J} P_H(j,k) \tag{6.9}$$

Where $\lambda(k)$ is the marginal price during time interval k.

The objective of the maximum of electrical energy can be transformed into an objective of the minimum of a cost by considering electrical energy output as a negative cost or a profit. The optimization algorithms applied for the solution of this problem should make this "cost" as large a negative number as possible. So the above optimization problem becomes

$$min \quad -\sum_{k}^{K} \lambda(k) \sum_{j}^{J} P_H(j,k) \tag{6.10}$$

The hydro power production function $P_H(j,k)$ is a nonlinear function of station discharge rate and station net head as shown in Chapter 4. For short-term daily hydro scheduling with large reservoirs, the station net head will not change drastically, therefore the head variations may be ignored. With further piecewise linear approximation, the hydro power production function for hydro power station j at a given head during time interval k can be written as:

$$P_H(j,k) = \sum_{n}^{N} C_{nj} * Q_n(j,k) \tag{6.11}$$

and the station discharge rate is

$$Q(j,k) = \sum_{n}^{N} Q_n(j,k) \tag{6.12}$$

6.2.3 System Operating Constraints

Apart from the constraints imposed on individual hydroelectric power stations, other operating constraints imposed on the system must be taken into account in the problem formulation as well. These system requirements are mainly for the purpose of power system security. One of the security constraints is the reserve requirement, i.e. the total possible maximum spare power generation must be larger than a specified threshold reserve requirement in order to cover the sudden loss of the generation:

$$\sum_{j}^{J} P_{Hjmax} - \sum_{j}^{J} P_H(j,k) \geq R(k), \qquad k \in K \tag{6.13}$$

Where P_{Hjmax} represents the possible maximum power generation of plant j during time interval k and $R(k)$ represents the required system reserve during time interval k.

For hydroelectric generation scheduling in a purely hydroelectric system, only hydroelectric power stations are involved without other power utilities. Therefore, the total hydroelectric power generation must satisfy the predicted load demand plus the transmission losses. This power balance requirement can be expressed as:

$$\sum_{j}^{J} P_H(j,k) \geq D(k) + L(k), \qquad k \in K \tag{6.14}$$

Where $D(k)$ represents the forecasted load demand during time interval k and $L(k)$ represents the estimated transmission losses during time interval k. For hydro subproblem scheduling problem, these security constraints will be considered as the coupled constraints (reserve requirement, power balance requirement) between thermal subsystem problem and hydro subsystem problem, hence they are not considered in the model here.

6.2.4 The Overall Optimization Model

To summarize, the overall optimization model for hydro subproblem scheduling can be derived as:

$$min \quad F = -\sum_{k}^{K} \lambda(k) * \sum_{j}^{J} \sum_{n}^{N} C_{nj} * Q_n(j,k)$$

Subject to the following constraints:

$$V(j,k+1) - V(j,k) + Q(j,k) - \sum_{m}^{M} Q(n,k) = INF(j,k)$$

$$V_{jmin} \leq V(j,k) \leq V_{jmax}$$

$$Q_{jmin} \leq Q(j,k) \leq Q_{jmax}$$

$$V(j,1) = V(j,0)$$

$$V(j,K+1) = V(j,K)$$

This hydro subproblem scheduling model is a large scale linear programming problem with a pure network structure from reservoir dynamics, various linear programming methods and network flow algorithms can be applied to solve the problem. Since the total number of variables and equality and inequality constraints from reservoir dynamics rises very rapidly as the system size increases, the problem will become very complicated for a large scale system. For example, for a medium-sized system consisting of 30 hydro plants and a scheduling period of one day with the scheduling interval of one hour, the number of equality constraints will be $24 * 30 = 720$ from reservoir dynamics, the number of variables will be $24 * 30 + 23 * 30 = 1410$ even if the plant power output is modelled as a linear function of the plant discharge rate and at a constant head. Moreover, the hydraulic coupling between the reservoirs at the same river valley will make this problem highly constrained and very complex. The general linear programming methods, despite being robust, fast and simple in implementation, may find difficulty in coping with the dimensionality of this problem. More efficient solution techniques may be applied to exploit the special features involved in this hydro subsystem scheduling problem as much as possible.

6.3 A SPARSE DUAL REVISED SIMPLEX METHOD

6.3.1 Linear Programming Methods

The simplex method of linear programming is a well-known solution technique for linear optimization problems. The hydro subproblem scheduling, as described above, is a large scale linear optimization program, thus, standard linear programming algorithms can be easily applied for the solution of this problem. Specialisations and modifications may be considered in the implementation of the hydro subsystem scheduling program in order to achieve a time saving and a reduction in memory requirements. The sparse dual revised simplex linear programming (SDRSLP) program was initially developed in the OCEPS project for dynamic and static active power dispatch problems. This efficient FORTRAN code is available and has been revised to be applied to solve the hydroelectric scheduling problem. Some implementation considerations will be used in the program for hydro subproblem scheduling.

6.3.2 The SDRSLP Method

The SDRSLP programme is initialised with an optimal solution of a subset of the problem constraints and it will proceed towards an optimal feasible solution of the overall problem through a successive introduction of overloaded constraints. One of the most important features of this linear programming routine is that owing to the dual approach, very large numbers of constraints may be handled without any increase in the dimensionality of the basis matrix and constraints which have both upper and lower limits may be handled efficiently.

The dual simplex method is often used under the circumstance when there is an initial basic solution readily or easily available for the linear programming problem being considered. This solution may be infeasible but it is priced out optimally. This is equivalent to the fact that its dual problem is feasible with the initial simplex multipliers. In the simplex tableau, this situation corresponds to having no negative elements in the bottom row (pricing) but having an infeasible solution. This situation often happens when an optimal feasible solution was once available, but with the re-arrangement of the constraint matrix B, this solution may become an optimally-priced but infeasible solution for the reformatted problem. Under this condition, a basic feasible solution of its dual problem is readily available. Hence it is convenient to optimize the dual problem to achieve a feasible and optimal solution for the primal problem using the concept of the dual simplex method. The dual simplex method operates by maintaining the optimality condition of the last row while working towards the feasibility of the primal problem. This corresponds to operating on the dual problem to maintain its feasibility while working towards its optimality.

Given a general form of linear programming problem as:

$$Min \qquad [C]^T * [X]$$

Subject to the constraints

$$[A] * [X] = [b]$$

and

$$[X] \geq [0]$$

According to the dual programming theorem, the corresponding dual problem of this linear primal problem becomes:

$$Max \qquad [\lambda]^T * [b]$$

Subject to the constraints

$$[A]^T * [\lambda] \leq [C]$$

and $[\lambda]$ is unbounded.

Suppose a basis B for the primal problem is known, the dual variable λ will be $\lambda = [C]_B * [B]^{-1}$ such that this solution λ is feasible for the dual problem as $[C]_B * [B]^{-1} * [N] \leq [C]_N$ is to be hold for the basic solution of the primal, thus $\lambda * [N] \leq [C]_N$ and this leads to $\lambda * [A] \leq [C]$. The corresponding basic solution for the primal problem $[X]_B = [B]^{-1} * [b]$ is called *dual feasible* . If at the same time $[X]_B \geq [0]$ then this solution is also primal feasible and hence the optimal solution.

The dual simplex method will start with a given dual feasible solution $[X]_B$ of a linear programming problem. If all the $[X]_B \geq [0]$, the program will terminate with the current solution as an optimal and feasible solution. If $[X]_B$ is not non-negative, select an index i such that the ith variable component of the $[X]_B$, i.e. $[X]_{Bi} < 0$. If all the components of $[B]^{-1} * [N]$ say y_{ij} hold as $y_{ij} \geq 0, j = 1, 2, 3, ...n$, then this dual problem has no maximum as the $[X]_{Bi} < 0$ can not be improved, this is equivalent to no feasible solutions for the primal problem. If some $y_{ij} < 0$ for some j, let the candidate to enter the basis be k with $\varepsilon_0 = \frac{z_k - c_k}{y_{ik}} = min(\frac{z_j - c_j}{y_{ij}} : y_{ij} < 0)$. Form the new basis B' by replacing i th row with k th row. Use this new basis to determine the new basic dual feasible solution $[X]_{B'}$ and go back to select the new candidate until termination occurs.

In order to take full advantage of the sparsity in linear programs, sparsity techniques may be applied. An algorithm is used to minimize the "fill in" non-zero elements in the basis matrix factors when they are modified. It is now well-known that the elimination form of a matrix inverse for basis factorisation is better than the product form of it. By introducing further interchanges between rows and columns, Reid[166.] has developed an algorithm which avoids the pivotal operations whenever possible. This approach has been implemented in SDRSLP program.

6.3.3 The SDRSLP Process for Hydro Subproblem Scheduling

As a summary, the dual revised simplex algorithm for hydro subproblem scheduling may be processed as follows:

1. Initialization. The process begins at an optimally-priced or dual feasible solution $[X]_B$ based on the consideration of the objective function of hydro subproblem scheduling, unit discharge rates and reservoir volumes lower and upper limits only (but the solution is not necessarily feasible with respect to network conservation constraints). For hydro subproblem scheduling, an easy and straightforward initial solution is obtained by assigning the discharge rates variables to their upper bound values and the rest of variables such as the reservoir volumes to their lower bound values.

2. Assemble the appropriate constraint coefficient rows into basis matrix B and the currently active constraint limits into vector L. Factorise the basis matrix B.

3. Select the most overloaded constraint based on the current state of the variables X. This constraint will enter the basis. If no such candidate is available, that is to say, no overloaded constraint exists, all $[X]_B \geq [0]$, terminate the program with the current solution as the obtained optimal and feasible solution, otherwise continue.

4. Compute the sensitivity vector S with $S = B^{-T} * e^T$, where e is the coefficient row of the entering constraint, and compute the incremental cost vector or the dual variable λ with $\lambda = B^{-T} * C^T$, where C^T is the vector of cost coefficients. The vectors λ and S should be computed by repeat solutions using the current basis factors from the matrix B.

5. Select a constraint to leave the basis. A constraint k is eligible if either it and entering constraint are both at upper limits or lower limits and S_k is positive, or it and entering constraint are on opposite limits and S_k is negative. If there is no such constraint eligible, there will be no feasible

solution, terminate the program with the indication of infeasibility or choose to relax the constraint to some degree. Otherwise the constraint to leave the basis is selected as the eligible constraint for which $|\lambda_k/S_k|$ is a minimum.

6. Update the factors of B and the vector L to allow for the replacement of the leaving constraint by the entering constraint. Form the new basis B.

7. Using the new basis B to determine the corresponding basic dual feasible solution, compute the new current state X as $X = B^{-1} * L$.

8. Repeat from Step 3.

The SDRSLP approach employed here with sparsity techniques is an extension of the ordinary dual simplex method. It will allow for hierarchical constraint relaxation and removal in cases where an infeasible problem has been specified inadvertently, hence the program is very robust. The other advantage is that the method, compared with standard linear programming routines, has a very low memory and computation time requirement. The availability of such constraint relaxation strategy may also permit the application of approximate methods for the inclusion of power generation constraints, reserve constraints and other security constraints within the hydroelectric generation scheduling model.

6.4 NETWORK PROGRAMMING TECHNIQUES

Some linear or nonlinear programming problems, termed network programming problems generally, have very interesting and fascinating special features that may be mathematically exploited so that the solution of these problems can be achieved far more efficiently through the application of specially developed algorithms.

Numerous applications of network flow algorithms and models have been extensively studied. Kennington's book[116.] serves as the first literature that contains an extensive summary of all the network programming models and applications. The purposes of network programming methods are to exploit the network structure of certain optimization problems and develop special algorithms for the solution of these types of problems. Among all the models, the minimal cost network flow problem, which aims to find a feasible flow solution within a network in order to minimize the total production cost in this network, has received much attention. Many applications have been reported in various fields and numerous solution algorithms have been employed.

A brief review on network flow problems and results from graph theories will be presented in Appendix 3. Details about an out-of-kilter minimal cost network flow algorithm and a simplex method employed to solve the network flow problems are discussed in the following two sections.

6.5 AN OUT-OF-KILTER LINEAR NETWORK FLOW ALGORITHM

6.5.1 Introduction

An out-of-kilter algorithm for minimal cost flow problems was developed by Fulkerson in 1961.[75.] Unlike general linear programming methods or the simplex on a graph algorithm for network flow problems, the out-of-kilter algorithm is not a specialization of the general simplex methods where the basis concept is used, instead, the concept of kilter numbers and arc flows *in kilter* or *out of kilter* were developed to describe the problem. This algorithm is specially designed for network programming problems and is unique in the mathematical programming literature.[116.]

The out-of-kilter algorithm employed in this project to solve the hydro-electric scheduling problem is applied to determine the least cost flow over an upper and lower bounded capacitated flow network. This algorithm is developed for a special class of network flow problems defined on a network G called *circulation problems*. The algorithm accepts a network model defined by the parameters associated with all arcs. The program starts with an arbitrary flow value on each arc, either feasible or not, together with an arbitrary price vector number for each vertex. A special labelling procedure is used to adjust the flow in an arc which fails to satisfy the appropriate optimality properties, so that by adjusting continuously the flow in arcs and at the same time satisfying the flow conservation law, the overall cost due to these flows will be minimized. i.e. a minimal cost flow circulation in this network with respect to these parameters will be achieved.

All arcs in the formulation are directed, i.e. the flow direction along an arc is from a sending vertex to a receiving vertex. The initial flows along the arcs of the network need not satisfy all the upper and lower flow capacities of the arcs, but they must be flow-conservative. In electrical terms, Kirchhoff's first law must be obeyed at all vertexes. e.g. a zero flow circulation is an acceptable initial state. However, a good initial estimate of the arc flows and the price vectors does reduce the computation time.

The out-of-kilter algorithm works on a sinkless and sourceless network only, i.e. the flows are in a circulation. This implies that in order to solve an ordinary transportation problem having sinks and sources, additional arcs are required to link all sinks to a supersink, and all sources to a supersource. These supervertices have to be linked by another arc. All these linking arcs do increase considerably the total number of arcs in the network for the short-term hydroelectric scheduling problem and result in a much larger network flow problem and inefficiency. This is the main drawback of this technique applied to hydroelectric scheduling problem as shown later in the test results.

6.5.2 The Out-of-kilter Algorithm

A general linear programming problem can be mathematically expressed in the following form:

$$\min_{[X]} \quad \sum_{j}^{N} C_j * X_j$$

Subject to the constraints

$$\sum_{j}^{N} A_{ij} * X_j = B_i, \qquad i \in M$$

and

$$L_j \le X_j \le U_j, \qquad j \in N$$

According to linear programming dual theory, suppose that $[X] = (X_1, X_2, ..., X_N)$ is a vector which satisfies the constraints set, this $[X]$ is called a feasible solution. If there is a dual (or pricing) vector $[\Pi] = (\Pi_1, \Pi_2, ..., \Pi_M)$ such that the implication holds for all the j^s as follows:

$$C_j + \sum_i^M \Pi_i * A_{ij} \begin{cases} > 0, & \text{when } X_j = L_j \\ < 0, & \text{when } X_j = U_j \end{cases}$$

then this $[X]$ is called the minimum solution of this linear programming problem, and the above equation is termed the optimal property for a linear program.

For a linear programming problem, given a solution $[X]$ satisfying the equality constraints and for any corresponding dual variables $[\Pi]$, the following cases of classification of all the j th components of the program are *exclusive* and *exhaustive* as illustrated in Table 6.1 and the corresponding equations below.

Table 6.1

Kilter states for a component j

$C_j + \sum_i^M \Pi_i * A_{ij}$	> 0	$= 0$	< 0
$X_j = U_j$	out of kilter	in kilter	in kilter
$L_j \le X_j \le U_j$	out of kilter	in kilter	out of kilter
$X_j = L_j$	in kilter	in kilter	out of kilter

(α) $\quad C_j + \sum_i^M \Pi_i * A_{ij} > 0, \quad X_j = L_j$

(β) $\quad C_j + \sum_i^M \Pi_i * A_{ij} = 0, \quad L_j \le X_j \le U_j$

(γ) $\quad C_j + \sum_i^M \Pi_i * A_{ij} < 0, \quad X_j = U_j$

(α_1) $\quad C_j + \sum_i^M \Pi_i * A_{ij} > 0, \quad X_j < L_j$

(β_1) $\quad C_j + \sum_i^M \Pi_i * A_{ij} = 0, \quad X_j < L_j$

(γ_1) $\quad C_j + \sum_i^M \Pi_i * A_{ij} < 0, \quad X_j < U_j$

(α_2) $\quad C_j + \sum_i^M \Pi_i * A_{ij} > 0, \quad X_j > L_j$

(β_2) $\quad C_j + \sum_i^M \Pi_i * A_{ij} = 0, \quad X_j > U_j$

$(\gamma_2) \quad C_j + \sum_i^M \Pi_i * A_{ij} < 0, \quad X_j > U_j$

If all the components are in the one of the states α, β or γ, then $[X]$ is a feasible and optimal solution, these three states are termed "in-kilter" states, the others "out-of-kilter" states. The out-of-kilter algorithm for network problems will select a particular component in an out-of-kilter state and gradually put it into an in-kilter state. The program proceeds until all in-kilter components stay in kilter, and all other out-of-kilter states either being improved or stay the same.

If a linear programming problem has a network structure as defined, this linear programming problem becomes a linear network flow problem. Given a special class of linear program with a network structure consisting of M nodes, a directed network arc a is defined by a sending node i and a receiving node j, each arc ij is associated with its lower bound L_{ij}, upper bound U_{ij} and unit cost C_{ij}. The constraint set has a network structure means that the constraint coefficient matrix $[A_{ij}]$ has a special feature, that is, all the coefficients in this matrix either equal to 1, -1 or 0.

In order to apply the network flow linear programming algorithms for the solution of the hydro subsystem scheduling problem, the hydro subsystem problem formulation must be described in the form of a network with nodes and branches, see Diagram 4.11. The reservoir dynamics constraints in the linear programming model can be written in form of $\mathbf{A} \cdot \mathbf{X} = \mathbf{b}$ with

$$\mathbf{A} \cdot \begin{vmatrix} Q(j,k) \\ \vdots \\ V(j,k) \\ \vdots \end{vmatrix} = \begin{vmatrix} \vdots \\ INF(j,k) \\ \vdots \\ \vdots \end{vmatrix} \qquad (6.15)$$

Through analyzing these constraints, It is obvious that the coefficient matrix $\mathbf{A}$ has the special network structure since all the coefficients either equal to 1, -1 or 0. Given an example of a hydro scheduling subproblem with two reservoirs in cascade on the same river and over a two time intervals, this network structure can be illustrated in the following matrix form and in Diagram 6.1.

$$\mathbf{A} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{array}{c} \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} \\ \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 \end{pmatrix} \end{array} \tag{6.16}$$

with 7 nodes and 10 arcs in total. The nodes in this network represent reservoirs and the junctions of rivers and streams, whereas network branches (arcs) represent the river channels, pipelines and reservoir contents. The branch flows represent discharge rates for each time interval and reservoir volumes at the beginning of each time interval. The constraints coefficients matrix **A** is thus called the node-arc incidence matrix with the number of rows equals to the node numbers, and the number of columns equals to the arc numbers. The node-arc elements of this matrix are defined as,

$$A_{ij} = \begin{cases} +1, & \text{if arc } j \text{ is directed away from node } i, \\ -1, & \text{if arc } j \text{ is directed towards node } i, \\ 0, & \text{if arc } j \text{ is not incident on node } i, \end{cases} \tag{6.17}$$

In this way, the hydro subproblem scheduling has been formulated as a capacitated transportation network problem in order to use the out-of-kilter algorithm and the simplex method on a graph technique.

Thus, the variable cost for j th component, as discussed before, can be rewritten as $C_{ij} + \Pi_i - \Pi_j$ and all the in-kilter or out-of-kilter conditions can be rewritten as follows:

$$\begin{array}{lll} (\alpha) & C_{ij} + \Pi_i - \Pi_j > 0, & X_{ij} = L_{ij} \\ (\beta) & C_{ij} + \Pi_i - \Pi_j = 0, & L_{ij} \leq X_{ij} \leq U_{ij} \\ (\gamma) & C_{ij} + \Pi_i - \Pi_j < 0, & X_{ij} = U_{ij} \end{array}$$

A two stations on one river,

The scheduling period = 3 hours

With one hour time step.

Constraints Set:

Reservoir Dynamics : 2 x 3 = 6

Lower and Upper Limits:

2 x 3 = 6 (Discharge)

2 x (3-1) = 4 (Volume)

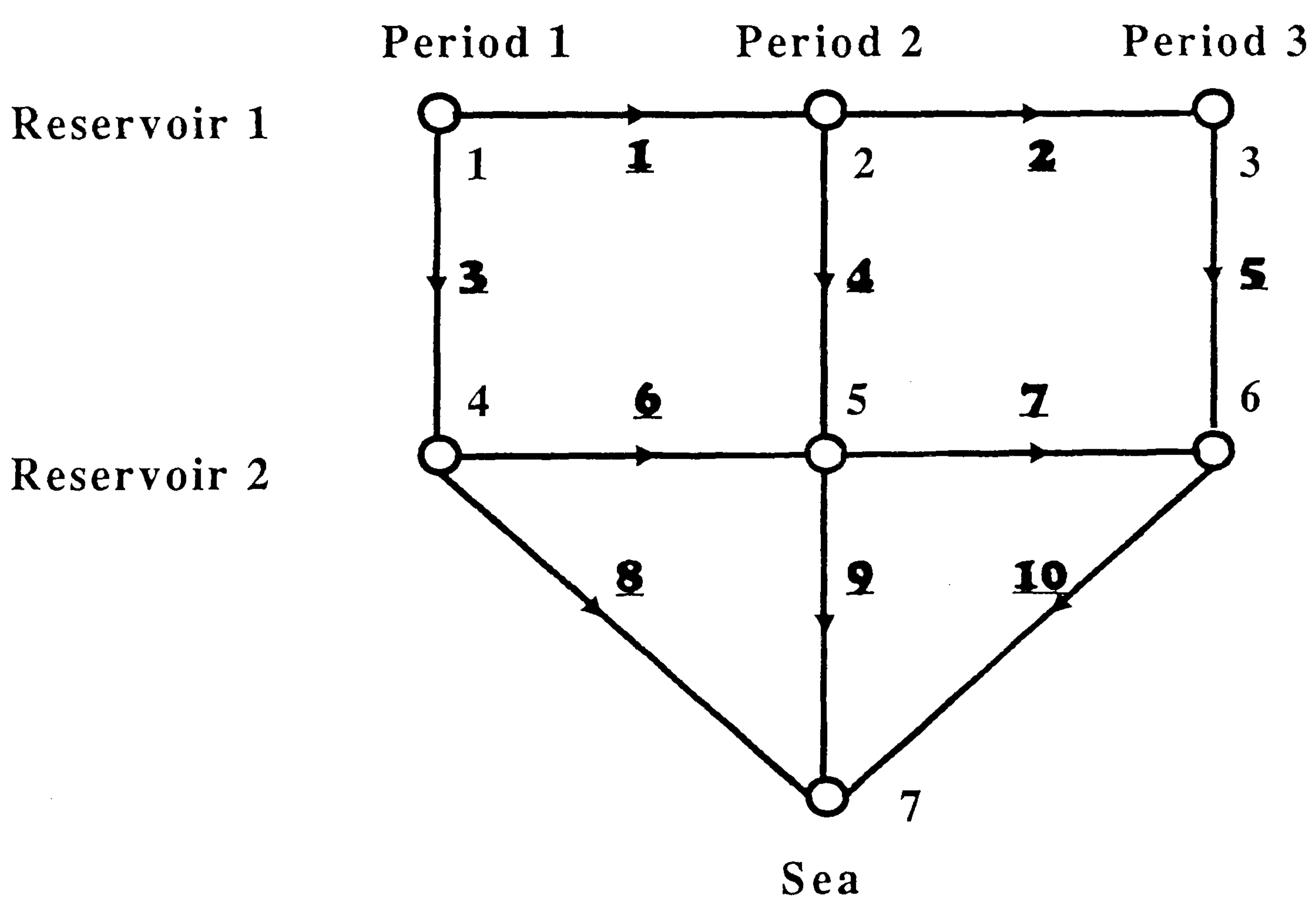


Diagram 6.1 Network Structure of Reservoir Dynamics

$(\alpha_1)\quad C_{ij} + \Pi_i - \Pi_j > 0, \quad X_{ij} < L_{ij}$

$(\beta_1)\quad C_{ij} + \Pi_i - \Pi_j = 0, \quad X_{ij} < L_{ij}$

$(\gamma_1)\quad C_{ij} + \Pi_i - \Pi_j < 0, \quad X_{ij} < U_{ij}$

$(\alpha_2)\quad C_{ij} + \Pi_i - \Pi_j > 0, \quad X_{ij} > L_{ij}$

$(\beta_2)\quad C_{ij} + \Pi_i - \Pi_j = 0, \quad X_{ij} > U_{ij}$

$(\gamma_2)\quad C_{ij} + \Pi_i - \Pi_j < 0, \quad X_{ij} > U_{ij}$

The out-of-kilter algorithm uses a labelling rule to search for a path that has a certain desired property from the chosen node, which is in one of the out-of-kilter states, to another node. The rule will label from a given node termed the origin and attempt to reach some other node termed the terminal. There are only two ways of terminating the labelling: **non-breakthrough** when the terminal can not receive a label and **breakthrough** when the terminal receives a label. If breakthrough occurs, the backtracking process is performed to change the arc flows in the path; if non-breakthrough occurs, the node price change must be found. This labelling process is repeated until either the arc is put in kilter or until a non-breakthrough occurs for which a node price change becomes infinite. In the latter case, the program will terminate with an indication of no feasible solution; in the former case, another out-of-kilter arc is located and the program continues. The computational process of the out-of-kilter algorithm can be summarized as follows:

1. Initialization with a flow conservation circulation $[X]$ and the pricing vector $[\Pi]$.

2. The main loop begins with the first arc KA. Calculate the variable cost $CV_{ij} = C_{ij} + \Pi_i - \Pi_j$ for the chosen arc and determine the out-of-kilter status (IKS) of the current arc KA. If $IKS = 0$, this arc KA is in kilter, go to select the next arc; if $IKS \neq 0$, this arc KA is out of kilter, clear all the labels but the source label, start the new labelling rule process. If $CV_{ij} > 0$ and $X_{ij} < L_{ij}$ or $CV_{ij} = 0$ and $X_{ij} < L_{ij}$ or $CV_{ij} < 0$ and $X_{ij} < U_{ij}$, the origin for labelling is j and the terminal is i. If $CV_{ij} > 0$ and $X_{ij} > L_{ij}$ or $CV_{ij} = 0$ and $X_{ij} > U_{ij}$ or $CV_{ij} < 0$ and $X_{ij} > U_{ij}$, the origin for labelling is i and the terminal is j.

3. Labelling process begins. If all the nodes are labelled, test for breakthrough, otherwise repeat this step with another node.

4. Non-breakthrough occurs. Determine the change of the vertex price δ. Suppose that L denotes the set of labelled nodes and L^n denotes the set of unlabelled nodes, calculate δ_1 from the arc whose sending node is in the set of labelled nodes and δ_2 from the arc whose sending node is in the set of unlabelled nodes with:

$$\delta_1 = \{Min \quad [CV_{ij}] : CV_{ij} > 0, X_{ij} \leq U_{ij}, i \in L, j \in L^n\}$$

and

$$\delta_2 = \{Min \quad [-CV_{ij}] : CV_{ij} < 0, X_{ij} \geq L_{ij}, i \in L^n, j \in L\}$$

If no such arc candidate in the above two subsets is available, i.e. two sets are empty, δ_1 and δ_2 will be assigned with infinite numbers: $\delta_1 \Leftarrow \infty$ and $\delta_2 \Leftarrow \infty$.

5. Set $\delta = \{Min \quad [\delta_1, \delta_2]\}$ and test for feasibility. If the node price change δ is infinite, there will be no feasible solution, terminate the program with such an indication; if δ is a positive real number other than infinite, change the price vector Π by computed value δ for all the sending nodes in the unlabelled node set. Go to step 2.

6. If breakthrough occurs, change the arc flow along the chain or the path of arcs by backtracking using the labels from the predecessor vertex vectors. Compute the incremental flow ϵ by

$$\epsilon = \begin{cases} Min \quad \{\epsilon_i, L_{ij} - X_{ij}\}, & (\alpha_1) \\ Min \quad \{\epsilon_i, U_{ij} - X_{ij}\}, & (\beta_1, \gamma_1) \\ Min \quad \{\epsilon_j, X_{ij} - L_{ij}\}, & (\alpha_2, \beta_2) \\ Min \quad \{\epsilon_j, X_{ij} - U_{ij}\}, & (\gamma_2). \end{cases}$$

Add this incremental flow ϵ to the flow in all forward arcs in the path from its origin to the terminal and subtract ϵ from the flow in all reverse arcs, also add ϵ to X_{ij} in arc ij in case of α_1, β_1 or γ_1, or subtract ϵ from X_{ij} in arc ij in case of α_2, β_2 or γ_2. Go to step 2.

The labelling rules for step 3 are summarized as follows: Initialize the labelling arrays. Labelling each arc according to the status (IKS) of the current arc KA. If $IKS \leq 3$, the arc flow in KA must increase; if $IKS = 4$, 5 or 6, the arc flow in KA must decrease. Skip the current out-of-kilter arc KA which has been labelled, and skip the arc with both of its vertices unlabelled, also skip the arc with both of its vertices labelled. Consider only those arcs each has one labelled vertex and one unlabelled vertex. The labelling rules are as follows:

A. If a sending node i is labelled with a predecessor vertex k and an incremental flow ϵ_i denoted by $[k^{\pm}, \epsilon_i]$ and the receiving node j of arc ij is not labelled, if arc ij is in one of the states of α_1, β_1 or γ_1, then node j will receive the label $[i^{+}, \epsilon_j]$ where

$$\epsilon_j = \begin{cases} Min \quad \{\epsilon_i, L_{ij} - X_{ij}\}, & (\alpha_1) \\ Min \quad \{\epsilon_i, U_{ij} - X_{ij}\}, & (\beta_1, \gamma_1) \end{cases}$$

B. If a receiving node i is labelled with a predecessor vertex k and an incremental flow ϵ_i denoted by $[k^{\pm}, \epsilon_i]$ and the sending node j of the arc ij is not labelled, if arc ij is in one of the states of α_2, β_2 or γ_2, then node j receives the label $[i^{-}, \epsilon_j]$ where

$$\epsilon_j = \begin{cases} Min \quad \{\epsilon_i, X_{ji} - L_{ji}\}, & (\alpha_2, \beta_2) \\ Min \quad \{\epsilon_i, X_{ji} - U_{ji}\}, & (\gamma_2) \end{cases}$$

It can be proved that all arc kilter numbers are monotonic non-increasing throughout the computation of the out-of-kilter algorithm. Suppose that arc ij is out of kilter, say in state α_2, that means the flow in this arc should decrease. In the labelling process, the origin for labelling will be i with the terminal j, the arc ij can not be labelled to the terminal j directly. If breakthrough occurs, the resulting path from i to j plus the arc ij will be a cycle. The flow changes in the arcs of this cycle will be a circulation to ensure feasibility. The labelling rules will ensure that the kilter numbers for arcs of this cycle will not increase, and at least one, namely the kilter number of arc ij will decrease. Obviously, the kilter numbers of the arcs not in the cycle will not change.

It has also been proved that after finitely many non-breakthroughs with $\delta < \infty$, either there will be a non-breakthrough with $\delta = \infty$ or the arc will be finally in kilter and obtain a breakthrough. In the former case, there is no feasible solution to the problem, the program will terminate without an optimal and feasible solution and with error message for problem model reformulation; in the latter case, the program will continue until a feasible and optimal solution is found.

6.6 THE SIMPLEX METHOD ON A GRAPH

Recently, several network flow optimization algorithms based on the simplex method have been proposed for hydro subproblem scheduling, such as the primal simplex method, the dual simplex method and the primal-dual simplex method. The primal simplex method is adopted here due to its simplicity and efficient computational speed. It is fairly easy to find a feasible initial solution in the network through examining the flow conservation constraints, also the huge number of variables suggest that the primal simplex method may be most appropriate.

The linear network flow approach based on the primal simplex method on a graph is introduced by Kennington and Helgason.[116.] The algorithm optimises the flow in a transportation network using a graph theory based primal simplex method. This method is actually a specialization of Dantzig's original primal simplex method applied to network programming problems. The advantages of this algorithm are enormous since, by introducing in the concepts from graph theory and the network flow formulation, the algorithm completely eliminates the necessity of carrying and updating the basis inverse as in general linear programming algorithms and results in efficient solutions.

The program applied is a modification of the FORTRAN code from the NETFLO routine introduced by Kennington and Helgason.[116.] The purpose of this routine is to solve a minimal cost network flow problem with lower and upper bounds on arcs such as

$$\min_{[Y]} \quad [C]^{T} * [Y]$$

Subject to

$$[A] * [Y] = [b]$$

and

$$[L] \leq [Y] \leq [U]$$

Assuming the number of nodes is I, the number of arcs in the network is J and each network constraint corresponds to a nodal flow conservation equation. $[A]$ is an $I * J$ node-arc incidence matrix, $[C]$ is a $1 * J$ vector of the unit costs, $[b]$ is a $I * 1$ vector of the node requirements, $[L]$ and $[U]$ are both $J * 1$ vectors, representing the lower bound and upper bound on arc flows respectively. If $b_i > 0$, node i is termed a supply node (point) with a supply equal to b_i; a node i with $b_i < 0$ is called a demand node (point) with a demand equal to b_i; a node i with $b_i = 0$ is a transshipment node.

To simplify the initial solution and the optimization process of the network flow problem, the above formulation is transformed so that the lower bounds are equal to zero by substituting the variable $[Y]$ with $[X] = [Y] - [L]$ such that the above problem formulation becomes:

$$\min_{[X]} \quad [C]^T * [X] + [C]^T * [L] = [C]^T * [X] + C_0$$

Subject to

$$[A] * [X] = [b] - [A] * [L] = [b']$$

and

$$[0] \leq [X] \leq [U] - [L] = [U']$$

To solve this problem, firstly, the idea of an artificial variables start procedure similar to the start procedure in an ordinary linear programming method is applied. This implies the network is enlarged by adding artificial arcs from an original node i with an induced node supply to the root node l with an infinitely large unit cost. All these artificial arcs then form part of a spanning tree with their initial flow equal to the induced supplies at the nodes that are connected to the root node l, that is $X_{il} = b_i$. The flow X_{il} will be decreased if a set of arcs is found to allow for the distribution of b_i of the artificial arcs to one or more demand nodes.

A heuristic procedure is followed to obtain an initial basic feasible solution of the network flow problem. The main idea of this procedure is to quickly find the low-cost paths through the network that will transport a large amount of commodity to the demand nodes. The procedure starts by forming a list L of the nodes with induced demand, ordered by the magnitude of the demand. The node with the largest demand will appear first in the list. For each node i in the list L, a quantity Q_i is defined to be the unsatisfied demand. Initially for a demand node i put in the list L, the unsatisfied quantity will be $Q_i = -b_i$. An attempt is then made to build backward chains, i.e. a directed path, beginning at each demand node and terminating at some supply node. Each chain initially consists of a single node and may be extended by the addition of new nodes and connecting arcs. The node most recently added to the chain will be referred to as the *lowest* node. Eventually each chain is connected to the spanning tree either by an artificial arc from the root to the lowest node in the chain or by an arc ij where i is an induced supply node and j is the lowest node in the chain.

This procedure contains two phases: In phase one, part of the spanning tree is formed so as to satisfy the induced demand through the chains from sources to demand nodes and through artificial arcs. In phase two, arcs are added with a flow of zero so as to complete the spanning tree. After these two phases have been completed, the spanning tree contains all nodes in the network and all basic and nonbasic variables have feasible values. The process above for finding an initial feasible basis of the primal simplex method on a graph can be summarized in the following steps:

1. Select the first node in the demand node list L, i.e. the demand node with the highest demand. If the list is empty, this implies all the demand nodes have been linked with supply nodes and the demand is satisfied, go to step 5, otherwise let a node i be the first node in list L with the highest demand, check its unsatisfied quantity Q_i, if $Q_i = 0$, go to step 4.
2. Find supply nodes to satisfy the demand, firstly find arcs that may be set to their upper bound if $U_{ji} \leq Min \quad [Q_i, X_{jl}]$, where ji belongs to the set

$D_i{}^1 = \{ji : ji \in B_i, b_j > 0, U_{ji} \le Min \quad [Q_i, X_{jl}]\}$ and B_i is called the "before set" of arc ji and is simply the set of all arcs whose "to" nodes is i. Also find arcs that may become basic if $U_{ji} > Q_i$ where ji belongs to the set $D_i{}^2 = \{ji : ji \in B_i, b_i > 0, Q_i < U_{ji}, Q_i \le X_{jl}\}$. Select a candidate with the least cost. If no such candidates are available, go to step 3, otherwise find a node k where $C_{ki} = Min \quad \{C_{ji} : ji \in D_i{}^1 \cup D_i{}^2\}$, either set arc ki to its upper bound or make it basic. If $ki \in D_i{}^1$, put $X_{ki} = U_{ki}$, Q_i changes to $Q_i - U_{ki}$ and X_{kl} changes to $X_{kl} - U_{ki}$ and go to step 1, otherwise if $ki \in D_i{}^2$, then put $X_{ki} = Q_i$, X_{kl} changes to $X_{kl} - Q_i$ and Q_i changes to zero, and connect the chain with i as the lowest node to the tree through arc ki, remove i from the list L and go to step 1.

3. Find the transshipment nodes to transfer the demand. Start by determining the candidates that have $D_i{}^3 = \{ji : ji \in B_i, b_j = 0\}$, and j is not part of the chain. Select from these candidates an arc with the least cost. If $D_i{}^3 = \phi$, go to step 4, otherwise find $C_{ki} = Min \quad \{C_{ji} : ji \in D_i{}^3\}$, either set ki to its upper bound or make it basic, similar to the process of supply nodes. If $U_{ki} < Q_i$, then $X_{ki} = U_{ki}$, $Q_i = Q_i - U_{ki}$, place k in the first position of L with $Q_k = U_{ki}$, begin a new chain with k and go to step 1, otherwise put $X_{ki} = Q_i$, remove i from L, place k in the first position of L, and with $Q_k = Q_i$, $Q_i = 0$, extend the chain with i as the lowest node to k via ki and go to step 1.
4. Connect i to the spanning tree with an artificial arc, remove i from the list L, create an artificial arc li with $C_{li} = U_{li} = \infty$ and $X_{lk} = Q_i$, connect the chain with i as the lowest node to the tree through lk and go back to step 1.
5. Initialize the node counter with $i \Leftarrow 1$.
6. Test for termination, if all nodes in the network are connected to the tree, terminate.
7. Save the starting node for current search with $i' \Leftarrow i$.
8. Find the low-cost connectable arc to add to the tree, the candidates hold $D_i{}^4 = \{ji : ji \in B_i\}$ and only one of j and i is not connected to the tree, select one of the candidates with the least cost. If $D_i{}^4 = \phi$ go to step 9, otherwise find k with $C_{ki} = Min \quad \{C_{ji} : ji \in D_i{}^4\}$, and make ki

basic by putting $X_{ki} = 0$ and connect the arc ki to the tree. Increase the node counter $i \Leftarrow i + 1$. If $i > I$ then $i \Leftarrow 1$ and go to step 6.

9. Increase the node counter by $i \Leftarrow i + 1$. If $i > I$ then $i \Leftarrow 1$, if $i \neq i'$ go to step 8.
10. Determine the isolated nodes, for each isolated node, create an artificial arc with an infinitely large cost, an infinite upper bound and a zero flow and connect this arc to the tree.

Steps 1-4 can be viewed as the process of phase one, the process for phase two is formed by Steps 5-10.

This NETFLO routine uses 6 node-length arrays and 5 arc length arrays for data storage. Node-length arrays include thread array $NEXT_i$, distance array $LEVEL_i$, predecessor array $DOWN_i$, a pointer array $ARCID_i$, an arc flow array $FLOW_i$ which contains the arc flow of the arc connecting i and $DOWN_i$ and a dual variable array $DUAL_i$ which contains the value of the dual variable of node i. The magnitude of $ARCID_i$ is a pointer to the spanning tree arc that connect i with $DOWN_i$ and the sign of $ARCID_i$ will be negative if the connecting arc is $(i, DOWN_i)$ and is positive if the connecting arc is $(DOWN_i, i)$. Arc-length arrays include arc unit cost array $COST_i$, capacity array $CAPAC_i$, lower bound array $FLOOR_i$, arc name array $NAME_i$ for users convenience and predecessor array of an arc $PRED_i$. A summary of the network flow algorithm using the primal simplex method is briefly as follows:

1. *Initialization* with $[X] = [X_B|X_N]$ as an initial basic feasible solution, find the spanning basis tree.
2. *Calculation* of the dual variables $[\Pi]$.
3. *Pricing* If no candidate, terminate the program either with no feasible solution or an optimal feasible solution.
4. *Updating* the flow in the cycle. If the blocking variable is a basic arc, go to the next step, otherwise if it is a nonbasic arc, go back to step 3.
5. *Updating* the spanning basic tree and dual variables, replacing the basic arc candidate to leave the basis with a nonbasic candidate to enter the basis. Go to step 3.

All the nodes in the network model are either supply nodes or demand nodes, and all the arcs are directed from supply nodes to demand nodes, furthermore, all the arcs are capacitated.

6.7 FRANK-WOLFE FEASIBLE DIRECTION METHOD

As discussed before, the solution of the hydroelectric scheduling subproblem is obtained from a linear optimization problem with a piecewise linear approximation of the hydroelectric power production function. To achieve a more accurate solution of the problem and take into account the nonlinearity of the hydro power production function and head variations, the solution of the original nonlinear optimization problem may be considered. A Frank-Wolfe feasible direction method is applied here as an approximation.

Marguerite Frank and Philip Wolfe proposed a method in 1956 for solving a nonlinear optimization problem which has a convex differentiable objective function but with all linear constraints, this method is named the Frank-Wolfe feasible direction algorithm.

The Frank-Wolfe algorithm is one of the many feasible direction methods and its procedure may be stated as follows: Given the general optimization problem model as

$$\min_{[X]} \quad f([X])$$

Subject to constraints

$$[A] * [X] = [b]$$

$$[0] \leq [X] \leq [U]$$

Where $f([X])$ is a nonlinear convex differentiable objective function. For the specification of this algorithm, this optimization problem objective function is assumed to be differentiable over $\{[X] : [0] \leq [X] \leq [U]\}$. Let $\nabla f([X])$ denote the gradient of the $f([X])$ evaluated at $[X]$. It is a well-known fact that for a convex function $f([X])$ having continuous first derivatives $\nabla f([X])$, the evaluated function value at a point $[X_1]$ never lies below a tangent plane passed through any other point, say $[X_2]$, that is

$$f([X_1]) \geq f([X_2] + \nabla f([X_2]) * ([X_1] - [X_2])$$

as depicted in Diagram 6.2. This result is used to obtain a lower bound of the objective function.

Using the formulae of Taylor's expansion, the problem objective can be linearized around a certain point $[X_0]$ such as:

$$\begin{aligned} f([X]) &= f([X_0]) + \{[X] - [X_0]\} * \frac{\partial f}{\partial [X]}|_{[X]=[X_0]} \\ &= f([X_0]) + \{[X] - [X_0]\} * \nabla f([X_0]) \\ &= \{f([X_0]) - \nabla f([X_0]) * [X_0]\} + \{\nabla f([X_0]) * [X]\} \end{aligned}$$

Since $\{f([X_0]) - \nabla f([X_0]) * [X_0]\}$ is a constant for a certain point $[X_0]$, the minimization of $f([X])$ can be approximated to be the following linear mathematical problem:

$$\min_{[X]} \quad \{\nabla f([X_0]) * [X]\}$$

Subject to the constraints

$$[A] * [X] = [b]$$

$$[0] \leq [X] \leq [U]$$

Hence, given a feasible solution at iteration k such as $[X_k]$, an new feasible solution may be found by solving the following linear programming problem:

$$\min_{[X]} \quad \{\nabla f([X_k]) * [X]\}$$

Subject to

$$[A] * [X] = [b]$$

$$[0] \leq [X] \leq [U]$$

Where $\nabla f([X_k])$ is the derivative of the original nonlinear function at its feasible solution $[X_k]$.

Suppose this nonlinear optimization problem has only linear network constraints, the problem can be solved from a initial feasible flow solution obtained

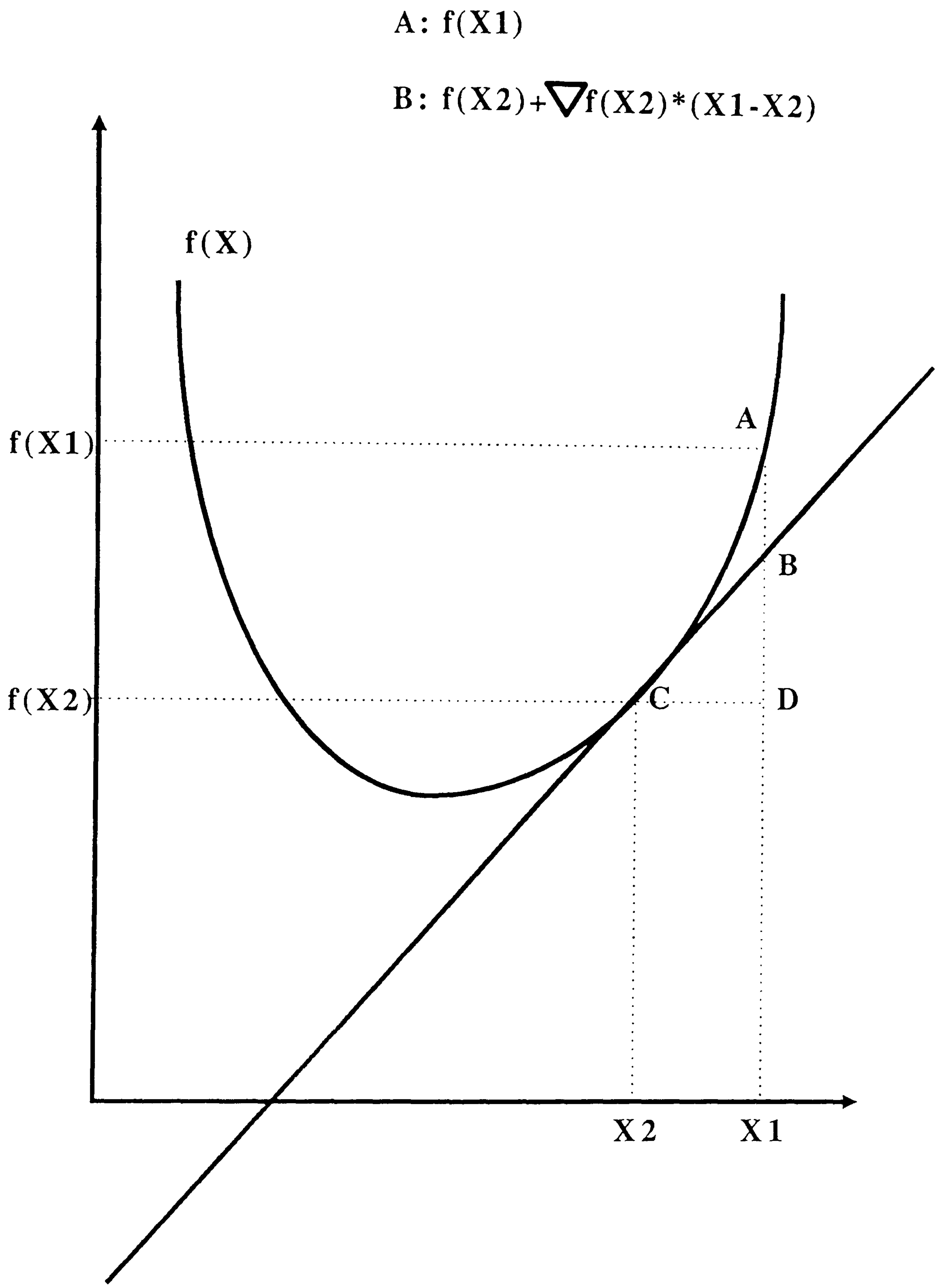


Diagram 6.2. Convex Function Properties

by solving the piecewise linearized model as a linear network programming problem, an improved direction can be found by solving the above linear network flow problem until the nonlinear problem is converged. As a by-product of the solution of this linear network flow problem, a lower bound solution of the nonlinear problem can also be obtained. The process of the Frank-Wolfe feasible direction algorithm can be described as follows:

1. *Initialization.* Let $[X_0]$ be any feasible flow solution as can be obtained by solving the piecewise linearized model through linear network flow algorithms. Set iteration number $k \Leftarrow 0$ and the lower bound of the objective function $\beta \Leftarrow -\infty$ and choose the termination parameter ε with $\varepsilon > 0$.

2. Solve the linear network flow subproblem $\{\min_{[X]} \ (\nabla f([X_k]) * [X])$ Subject to $[A] * [X] = [b]$ and $[0] \leq [X] \leq [U]\}$. Let $[Z_k]$ denote the solution of the above problem. Update the lower bound by setting $\beta \Leftarrow Max \ \{\beta, f([X_k]) - \nabla f([X_k]) * ([Z_k] - [X_k])\}$. If $f([X_k]) - \beta \leq \varepsilon$, terminate with $[X_k]$ as an ε -optimum, otherwise set $k \Leftarrow k+1$, continue.

3. Perform a line search in the chosen direction. Let $[X_k]$ be the flow solution on the line segment between $[X_{k-1}]$ and $[Z_{k-1}]$ having the smallest objective function value. A quadratic interpolation search is proposed as a suitable line search procedure.

4. Update the gradient as $\nabla f([X_k])$ and go back to step 2 with the new gradient factors.

The Frank-Wolfe algorithm is very straightforward and simple. However, without a good initial solution, it may suffer generally from slow convergence and being time-consuming. Fortunately, this algorithm appears to be very suitable for hydro subproblem scheduling since both these two difficulties can be easily overcome through the following considerations:

1. A good initial solution can be readily obtained from the linear network flow algorithm through the piecewise linear approximation approach and this solution is usually very near to its nonlinear optimum solution as shown in the tests.

2. The improving feasible direction is decided by solving a linear network flow problem at each iteration, which is much smaller than the piecewise linear approximation model, and can be solved by the linear network flow algorithm very efficiently as stated, thus the overall computation time is reasonably low.

6.8 LAGRANGIAN RELAXATION TECHNIQUE

In the above sections, the hydro subproblem scheduling was discussed. The problem was firstly formulated as a large scale linear programming problem. The power balance requirement was ignored in this scheduling subproblem since it was assumed to be considered in hydrothermal coordination. In order to solve this hydro subproblem efficiently, network programming methods were used. However, if some security constraints are to be considered in the formulation, due to their non-network structure, the linear network flow algorithms can not be applied directly, also for the generation scheduling problem in a purely hydroelectric power system, the power generation and reserve requirements must be considered. All these requirements result in non-network constraints. To account for these effects, while at the same time exploiting the network structure from the other constraints, a Lagrangian relaxation decomposition technique is applied to this problem for the first time, which is a similar approach to the thermal unit commitment problem in Chapter 5.

The Lagrangian relaxation decomposition method for hydro generation scheduling in a purely hydroelectric generation system can be summarized as follows:

1. Read in hydro generation system data.

2. Preparation for scheduling. Find the possible total maximum hydroelectric generation level, i.e. hydro generation capacity, and the possible total maximum power demand. If the hydro generation capacity is less than maximum power demand, terminate the program with the indication of infeasible solution problem, otherwise start the optimization using Lagrangian relaxation method.
3. Initialization. Set iteration number $k \Leftarrow 0$ and initialize Lagrangian multiplier values.
4. Solve the decomposed hydro scheduling subproblems with a linear network flow algorithm and evaluate the total linear network flow dual function cost value D^k.
5. Find the gradient vector and search ascending direction which is the direction of the steepest ascent, perform a line search to maximize the dual function and update the Lagrangian multipliers.
6. Compare the dual cost values between two iterations, check for convergence. If the dual cost change is inside the convergence criterion and also each power balance equation is satisfied within the predefined percentage margin, the hydro scheduling problem is converged, terminated with the current solution as the feasible and optimal solution, otherwise continue.
7. Update the Lagrangian multipliers and set iteration number $k \Leftarrow k+1$, go to step 4.

6.9 HYDRO GENERATION TEST SYSTEMS

A test system containing 2 of river valleys, 8 hydro power stations with 20 hydro generating units in total has been considered. Each river valley contains 4 reservoirs in cascade as can be seen in Diagram 6.3. All the data for this 2 river valley system, including generators and reservoirs, are given in Tables 6.2 and Table 6.3. Note that the natural water inflows into a reservoir are assumed to be constant during the whole scheduling period. Piecewise linear approximation of hydro power function is applied. Unit slope stands for unit generating efficiency and Q_{max} stands for unit maximum discharge rate. All the discharge rates are in (m^3/h). All the reservoir volumes are in (m^3).

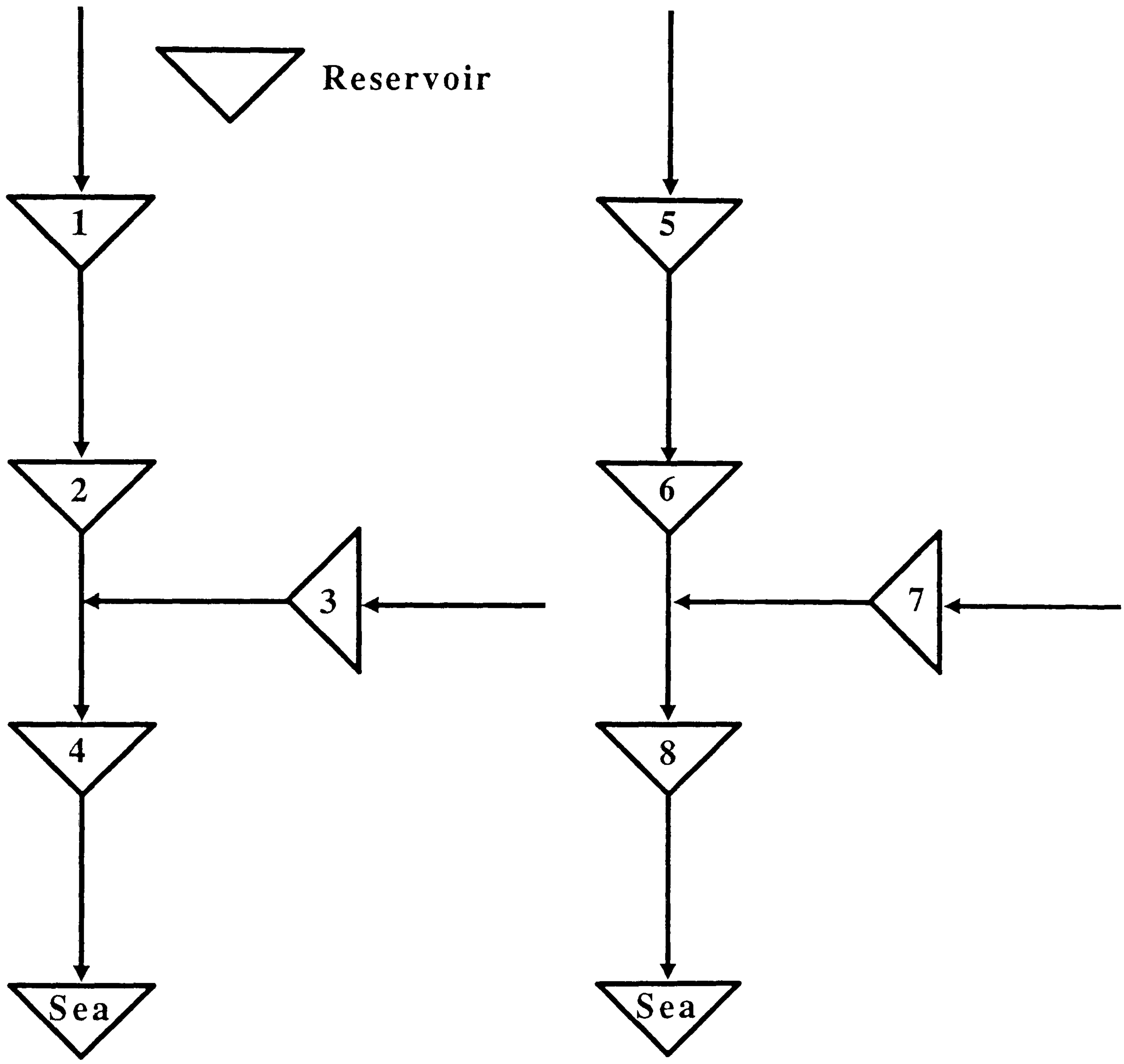


Diagram 6.3. A Multireservoir Hydroelectric Power System

Table 6.2

Hydro generation system data

			MWs/m^3	m^3/s			
River	Plant	Unit	Slope	Q_{max}	$Upstrm_1$	$Upstrm_2$	Downstrm.
1	1	1	0.30	342.0	-	-	2
1	1	2	0.28	270.0	-	-	2
1	2	3	0.23	504.0	1	-	4
1	2	4	0.22	432.0	1	-	4
1	3	5	0.45	324.0	-	-	4
1	3	6	0.44	306.0	-	-	4
1	3	7	0.43	306.0	-	-	4
1	4	8	0.16	468.0	2	3	-
1	4	9	0.15	252.0	2	3	-
1	4	10	0.14	468.0	2	3	-
2	5	11	0.30	342.0	-	-	6
2	5	12	0.28	270.0	-	-	6
2	6	13	0.23	504.0	5	-	8
2	6	14	0.22	432.0	5	-	8
2	7	15	0.45	324.0	-	-	8
2	7	16	0.44	306.0	-	-	8
2	7	17	0.43	306.0	-	-	8
2	8	18	0.16	468.0	6	7	-
2	8	19	0.15	252.0	6	7	-
2	8	20	0.14	468.0	6	7	-

The hydro system considered here is assumed to be a part of a large power system consisting of various power plants such as coal-fired thermal plants, oil-fired thermal plants, gas turbine plants, nuclear plants, etc. For this hydro subproblem scheduling, the system marginal prices are obtained from the optimization of the whole hydrothermal system cost and they correspond to the total system production cost. Since hydroelectric generation has a negligible operational cost, the overall hydrothermal optimization problem is how and when to use, in a given time interval, the water available for hydroelectric generation so that the thermal production cost will be reduced to a minimum. This optimization problem can be decomposed and results in solving the two subproblems independently: a thermal subproblem and a hydro subproblem. The marginal prices are obtained by solving the thermal subproblem, which provides a set of "pseudo" marginal operational costs for hydro generation optimization.

Assuming that the total thermal generation in this system has been decided before the hydro generation scheduling, the total hydro generation must then balance the load demand minus total thermal generation. The hydro generation scheduling problem must then satisfy the remaining load demand, as it must satisfy the power demand were it a purely hydro generation system. There are already applications of Dantzig-Wolfe decomposition technique and partitioning techniques for this problem. In this project, the Lagrangian relaxation decomposition technique is for the first time applied.

Table 6.3

Reservoirs data

	m^3	m^3	m^3	m^3	m^3/s	m^3/s	m^3/s
Reservoir No.	V_{min}	V_{max}	V_{init}	V_{final}	QQ_{max}	QQ_{min}	Inflows
1	46.86	748.29	547.0	548.0	612.0	0.0	360.0
2	62.00	863.77	62.50	63.50	936.0	0.0	18.00
3	980.0	990.00	980.0	980.0	936.0	0.0	720.0
4	65.15	968.18	66.00	68.00	1188.	0.0	18.00
5	46.86	748.29	547.0	548.0	612.0	0.0	360.0
6	62.00	863.77	62.50	63.50	936.0	0.0	18.00
7	980.0	990.00	980.0	980.0	936.0	0.0	720.0
8	65.15	968.18	66.00	68.00	1188.	0.0	18.00

6.10 COMPARISONS AND TEST RESULTS

6.10.1 Test Results of NETFLO, SDRSLP and OUT-OF-KILTER

In the previous sections, several linear programming solution algorithms have been discussed. These algorithms have been developed for the solution of the short-term hydroelectric system scheduling problem as well as to the solution of hydrothermal generation scheduling problem with linear approximation model.

Both the out-of-kilter algorithm and the simplex method on a graph are linear network flow optimization algorithms. For the purpose of comparisons, the NFLP algorithm actually accepts the network model as in the out-of-kilter algorithm except that the supply nodes and demand nodes can be specified in the simplex method on a graph (NFLP) but not in the out-of-kilter method. The network formulation for NFLP is therefore more straightforward and the network itself has less arcs and nodes than the out-of-kilter algorithm, hence the solution of NFLP is more efficient.

The differences in computational process between the simplex method on a graph and the out-of-kilter algorithm are: Firstly, the simplex method on a graph will be processed in two phases: the first phase is searching for a candidate to ensure a feasible circulation; the second phase is searching for an optimal circulation. This corresponds to the general simplex method of searching for a feasible and basic solution. While in the out-of-kilter algorithm, there is no concept such as the basis. The out-of-kilter algorithm actually combines these two phases of operations as a whole process. Secondly, in view of the process of the simplex method on a graph, the kilter number of a variable may not be monotonically non-increasing. An arc that was in kilter in one stage may go out of kilter in some other stages.

Compared with the SDRSLP program, the simplex method on a graph is different in that by exploiting the network structure of the hydro subproblem scheduling, the simplex method on a graph completely eliminates the need for carrying and updating the basis inverse, some operations for finding the basic solution become unnecessary in NFLP program, and the method can be performed on a representation of the network diagram. By exploiting the network structure, the NFLP method is very efficient and fast.

Numerous tests have been made for the comparisons of the results obtained from these algorithms. An one river valley system with 4 reservoirs and 10 hydro generating units was tested using different sets of marginal price data. Then a two river valleys system as described in Table 6.2 and Table 6.3 was tested using same sets of marginal price data. The results have shown that the NETFLO algorithm is more competitive than the out-of-kilter minimal cost algorithm and the fast sparse dual revised simplex method, and is less time-consuming.

The comparisons among the simplex method on a graph (NETFLO), the out-of-kilter algorithm and the sparse dual revised simplex method (SDRSLP) can be seen from the following tables. One set of marginal prices is assumed to be constant over the scheduling period. For the one river valley test system with fixed head, no reserve or other security constraints, the CPU time and the total minimum cost obtained from using these algorithms for weekly and daily planning versus the number of variables are presented in Table 6.4. Since the objective of the hydro subproblem scheduling is to maximize the hydro generation benefits over the scheduling period with the specified marginal price data, the minimum cost value is represented as a negative number as shown in Table 6.4. With another set of marginal prices, the results are shown in Table 6.5. Figure 6.1 shows the marginal prices and the hydro generation schedule obtained. The two river valleys system was tested using the same two sets of marginal price data. See Table 6.6 and Table 6.7 for the results.

The results show that all the three algorithms are robust and both the SDRSLP method and the fast linear network flow algorithm NETFLO are efficient. The CPU time required for the solution of the problem increases only linearly with the size of the problem. The NETFLO algorithm is the most efficient of all, and its problem formulation is quite straight forward and simple. It has been said that this fast network flow NETFLO can be one hundred times faster than standard linear programming.[116.] The NETFLO algorithm

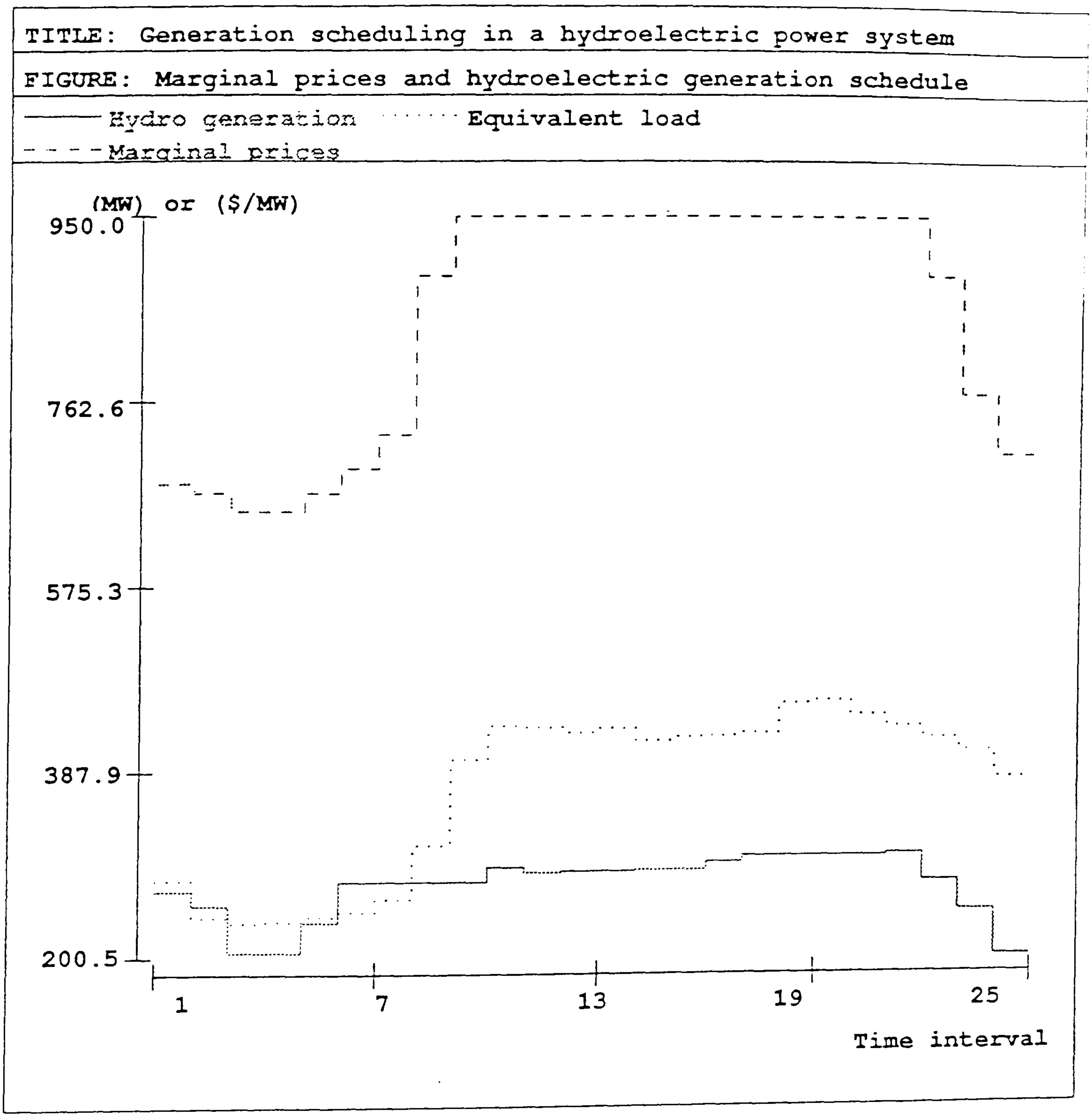

TITLE: Generation scheduling in a hydroelectric power system
FIGURE: Marginal prices and hydroelectric generation schedule
Hydro generation
Equivalent load
Marginal prices
(MW) or ($/MW)
950.0
762.6
575.3
387.9
200.5
1
7
13
19
25
Time interval

Figure 6.1

applied here is about 7 times faster than the SDRSLP method despite the fact that the SDRSLP algorithm uses efficient sparsity techniques. Table 6.8 and Table 6.9 show the test results of using the NETFLO algorithm for daily and weekly scheduling with hourly time intervals and two sets of marginal prices for the one river generation system and two rivers system respectively. Figure 6.2 shows a hydro scheduling result for two days with half-hour time intervals.

Table 6.4

Comparisons of algorithms

Algorithm	No. of variables	CPU time (secs.)	Minimum cost
NETFLO	332	1.64	-163628600
SDRSLP	332	8.81	-163628660
OUT-OF-KILTER	332	17.72	-163628640

Table 6.5

Comparisons of algorithms

Algorithm	No. of variables	CPU time (secs.)	Minimum cost
NETFLO	332	1.74	-731761381
SDRSLP	332	9.54	-731763580
OUT-OF-KILTER	332	32.26	-731763580

Table 6.6

Comparisons of algorithms

Algorithm	No. of variables	CPU time (secs.)	Minimum cost
NETFLO	668	3.26	-327257200
SDRSLP	668	17.35	-327257320
OUT-OF-KILTER	668	35.38	-327257280

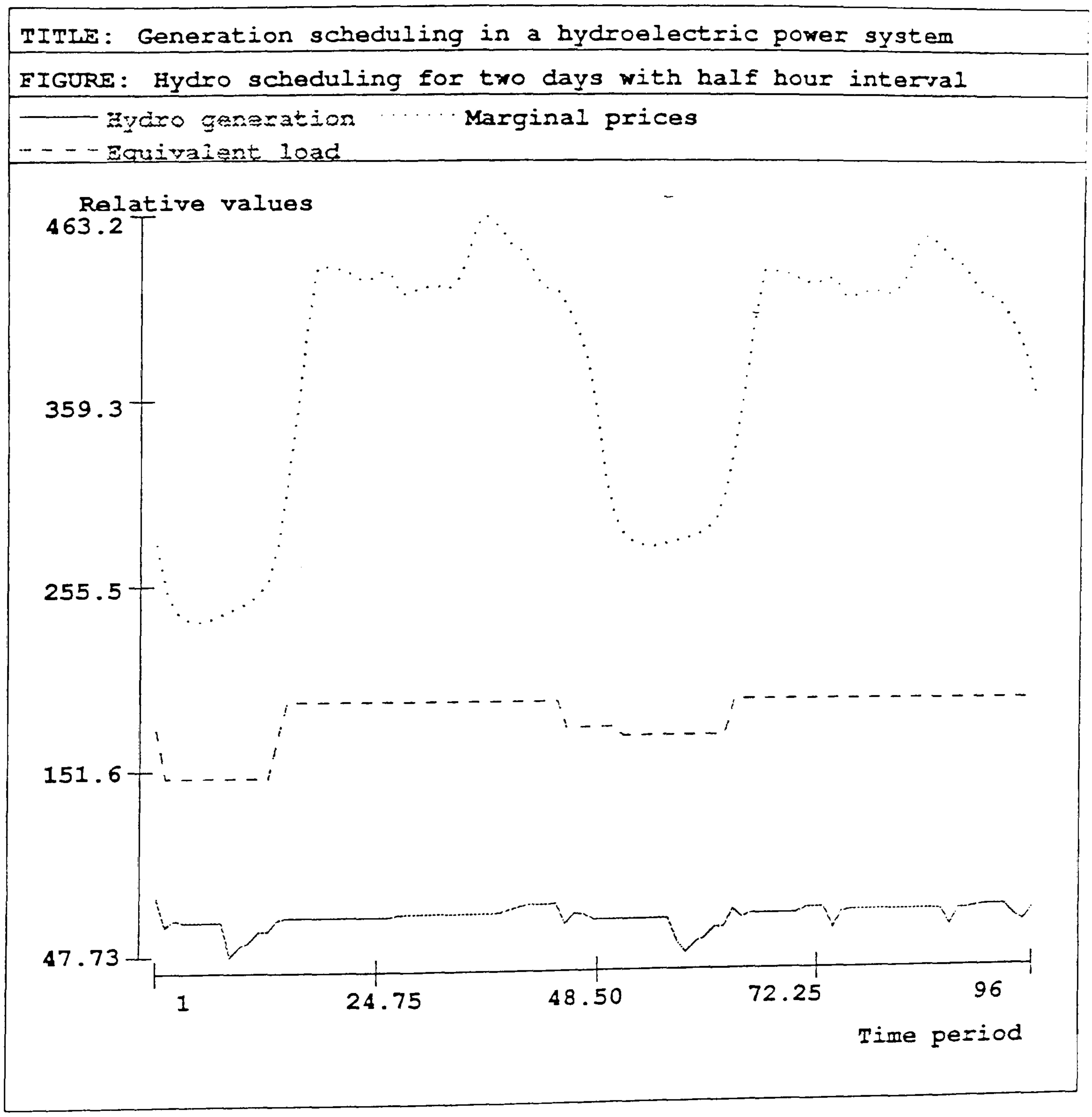

TITLE: Generation scheduling in a hydroelectric power system
FIGURE: Hydro scheduling for two days with half hour interval
Hydro generation
Marginal prices
Equivalent load
Relative values
463.2
359.3
255.5
151.6
47.73
1
24.75
48.50
72.25
96
Time period

Figure 6.2

Table 6.7 Comparisons of algorithms			
Algorithm	No. of variables	CPU time (secs.)	Minimum cost
NETFLO	668	3.49	-1463522762
SDRSLP	668	19.21	-1463527160
OUT-OF-KILTER	668	64.50	-1463527160

Table 6.8 Comparisons between daily and weekly Scheduling One river system (NETFLO)			
Discretized intervals	No. of variables	CPU time (secs.)	Minimum cost
24	332	0.94	-163628600
48	668	3.32	-327270200
72	1004	14.05	-490911800
168	2348	73.46	-1145478200

Table 6.9 Comparisons between daily and weekly Scheduling Two rivers system (NETFLO)			
Discretized intervals	No. of variables	CPU time (secs.)	Minimum cost
24	332	1.76	-7285177
48	668	6.56	-14592255
72	1004	14.79	-21899333
168	2348	77.30	-51127645

A comparison of the corresponding features of NETFLO, SDRSLP and OUT-OF-KILTER algorithms is given in Table 6.10.

Table 6.10

Comparisons of computational process

	SDRSLP	NETFLO	OUT-OF-KILTER
Minimize $[C] * [X]$	Yes	Yes	Yes
Subject to $[A] * [X] = [b]$	Yes	Yes	Yes
And $[X]_{min} \leq [X] \leq [X]_{max}$	Yes	Yes	Yes
$[A]$	Constraint Matrix	Node-Arc Incidence Matrix	Node-Arc Incidence Matrix
$[X]$	Variables	Arc Flows	Arc Flows
$[B]$	Basis	A Spanning Tree with Rooted Node	Kilter Numbers
Pricing	$[C]_B * [B]^{-1} * [N] - [C]_N$	$\pi_{F(j)} - \pi_{T(j)} - C_j$	$\pi_{F(j)} - \pi_{T(j)} - C_j$
Update	Basis	Spanning Tree and Duals	Kilter Numbers

6.10.2 Test Results of Lagrangian Relaxation

The Lagrangian relaxation is used for hydro generation scheduling in a purely hydroelectric power system. Some tests have shown that Lagrangian relaxation can be very efficient for solving the problem, only 7-8 iterations are needed to converge. Table 6.11 shows the input marginal prices, power demand and total hydro generation for the test system represented by Table 6.5 and 6.7, the corresponding graph is given in Figure 6.3. The Lagrangian relaxation algorithm can be very sensitive to the choice of the initial Lagrangian multipliers, hence a good initialization is essential. As can be seen in Figure 6.3, the choice of initial Lagrangian multipliers is quite accurate. This ensures the Lagrangian relaxation algorithm converges in only a few iterations (6 iterations in this test). For this two river test system with 8 reservoirs in cascade and 20 generating units in total, the daily scheduling with 24 time intervals will only take less than 100 seconds CPU time using Lagrangian relaxation. Other results of the application of Lagrangian relaxation are also shown in Figure 6.4 and Figure 6.5. Since there are no integer variables involved in hydro generation scheduling, there is no duality gap between the dual and the primal. This is verified by Figure 6.3, Figure 6.4, Figure 6.5 and Table 6.12 such that the curve of total hydro generation is overlaid upon the curve of load prediction profile, as is the primal cost variation overlaid on the dual cost variation.

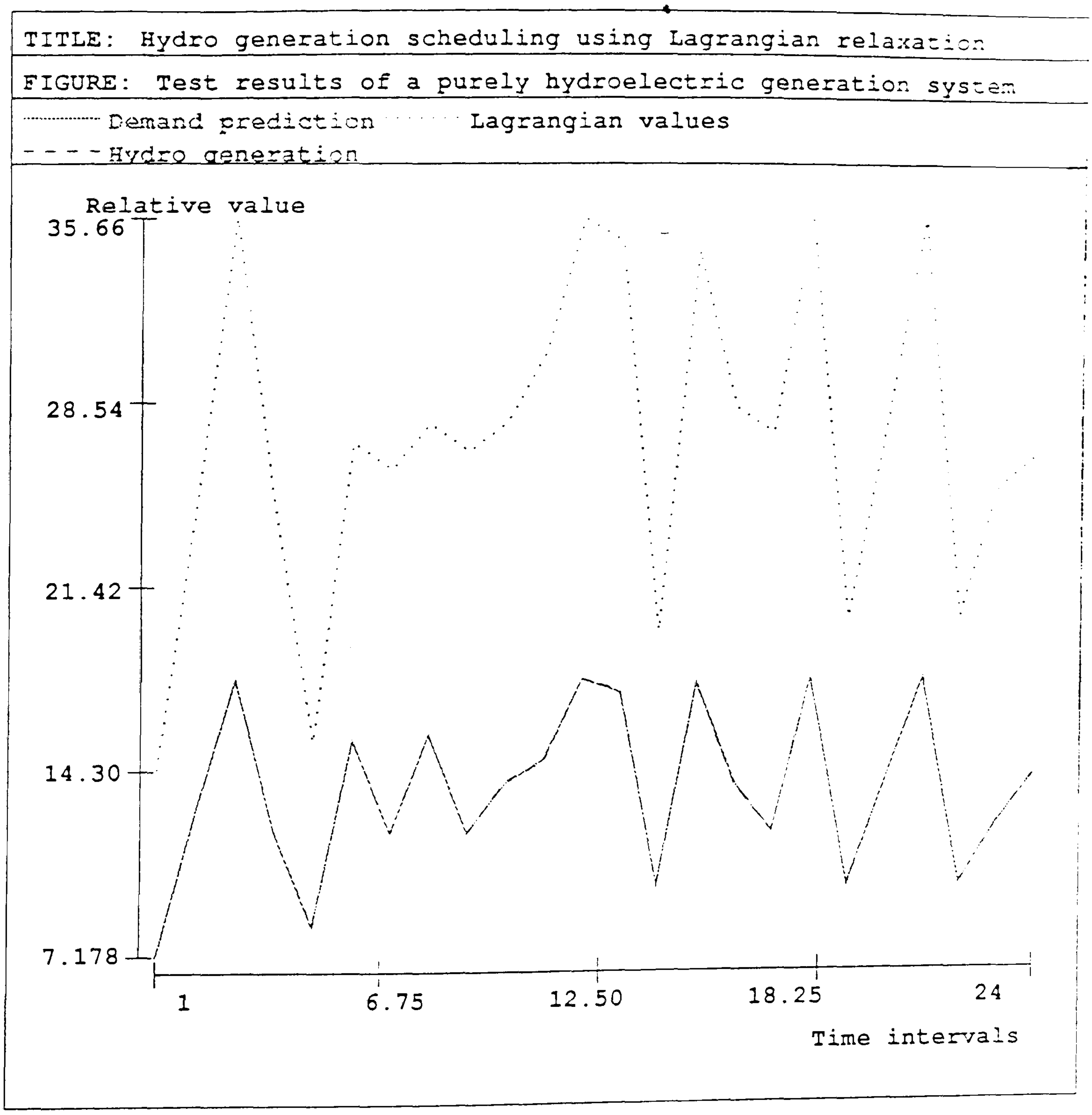

TITLE: Hydro generation scheduling using Lagrangian relaxation
FIGURE: Test results of a purely hydroelectric generation system
Demand prediction
Lagrangian values
Hydro generation
Relative value
35.66
28.54
21.42
14.30
7.178
1
6.75
12.50
18.25
24
Time intervals

Figure 6.3

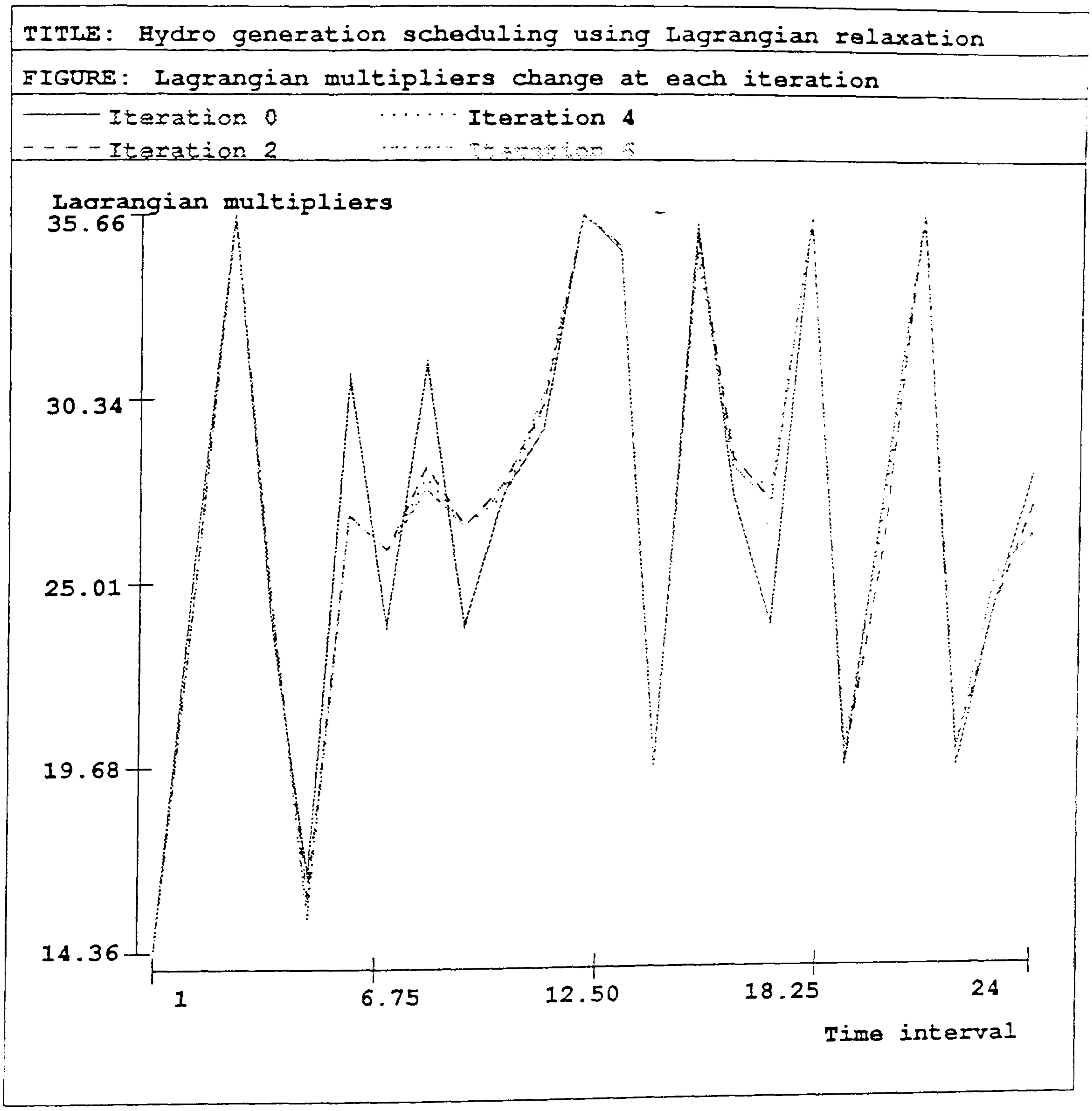

TITLE: Hydro generation scheduling using Lagrangian relaxation
FIGURE: Lagrangian multipliers change at each iteration
Iteration 0
Iteration 4
Iteration 2
Lagrangian multipliers
35.66
30.34
25.01
19.68
14.36
1
6.75
12.50
18.25
24
Time interval

Figure 6.4

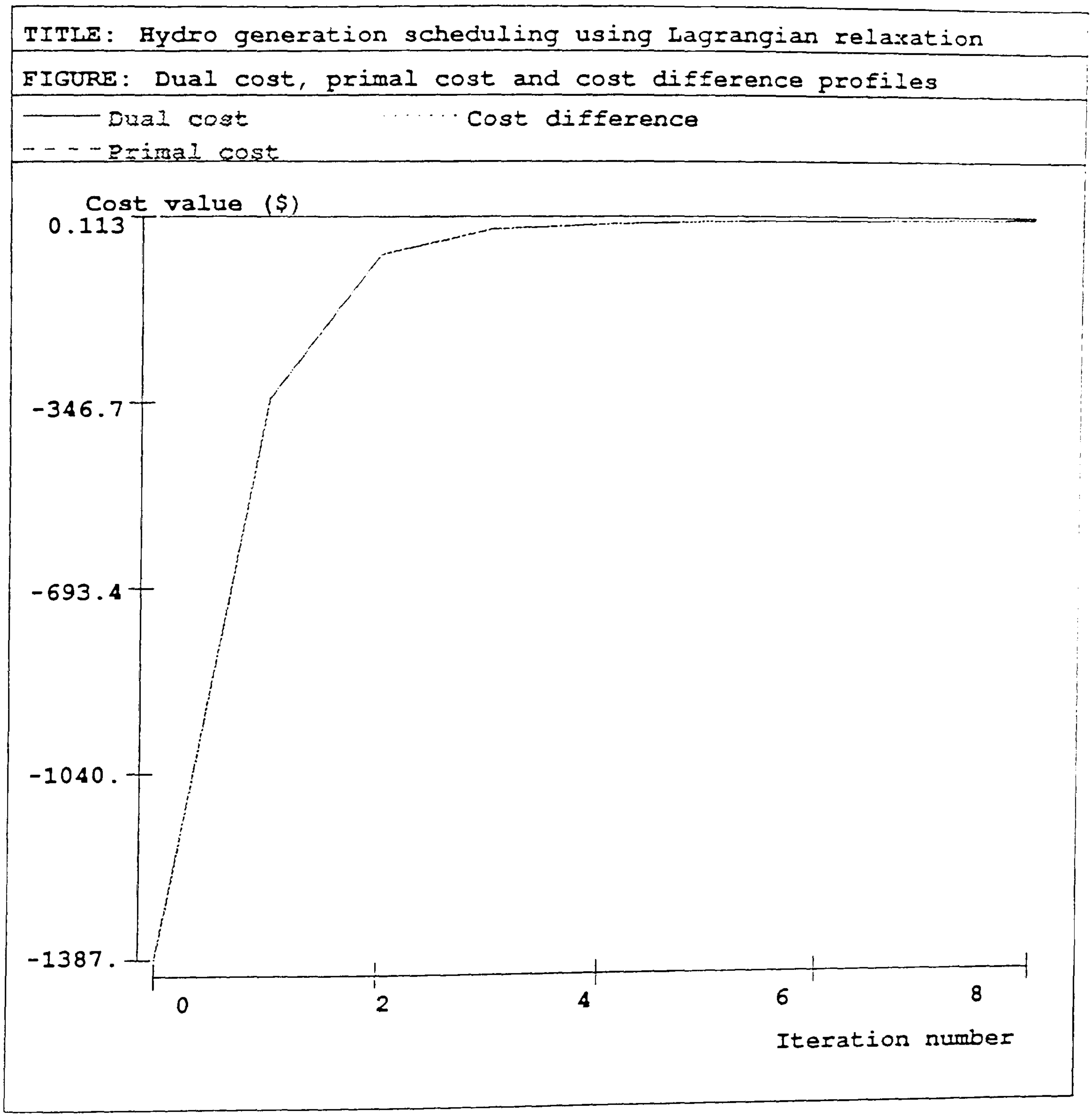

TITLE: Hydro generation scheduling using Lagrangian relaxation
FIGURE: Dual cost, primal cost and cost difference profiles
Dual cost
Cost difference
Primal cost
Cost value ($)
0.113
-346.7
-693.4
-1040.
-1387.
0
2
4
6
8
Iteration number

Figure 6.5

Table 6.11 Hydro generation scheduling using Lagrangian relaxation			
Period	Marginal prices	Total hydro generation	Load demand
1	1435.70	7.178481	7.178481
2	2461.32	12.78338	12.78338
3	3566.24	17.83119	17.83119
4	2455.34	11.97209	11.97209
5	1542.30	8.341088	8.285072
6	2699.20	15.48683	15.48683
7	2596.46	11.88649	11.88649
8	2774.17	15.70283	15.70283
9	2666.69	11.88649	11.88649
10	2774.93	13.88718	13.80474
11	3045.98	14.64156	14.72400
12	3566.24	17.83119	17.83119
13	3478.21	17.39009	17.30909
14	1966.90	9.834489	9.834489
15	3443.87	17.66919	17.66919
16	2832.85	13.69417	13.77661
17	2734.56	11.88649	11.88649
18	3566.24	17.83119	17.83119
19	2014.74	9.781060	9.834489
20	2753.44	13.83613	13.83613
21	3566.24	17.83119	17.83119
22	2024.10	9.834489	9.831676
23	2520.52	12.19713	12.19713
24	2642.71	14.08663	14.08663

Table 6.12 Hydro generation scheduling using Lagrangian relaxation			
Iteration	Dual cost	Primal cost	Cost difference
0	-13870.0	-13869.69	-0.3086
1	-3394.5	-3395.64	1.1353
2	-703.5	-703.77	0.2724
3	-236.0	-235.07	-0.9347
4	-144.5	-143.75	-0.7552
5	-97.5	-97.54	-0.0374
6	-84.5	-84.69	0.0932
7	-66.0	-65.77	-0.2295
8	-48.5	-48.68	0.1830

However, it should be noticed that in general, there does not exist a set of shared marginal prices that will ensure the optimal usage of all the scarce resources if the objective function is not strictly convex. This can be seen in Figure 6.6 where the Lagrangian relaxation technique is used to schedule the hydro production with the same hydro test system but with a new set of load prediction profiles. The result shows that the load demand in some of the time intervals were not satisfied by the total hydro generation. This is due to the complete linearity of the mathematical programming problem for hydro generation scheduling, the fully decentralized master problem and subproblems using the Lagrangian relaxation may not ensure the satisfaction of all the coupling constraints. Instead, the Dantzig-Wolfe decomposition[57.],[123.] may be applied. This is not a fully decentralized scheme as its central unit will take the final optimal decision which can be different from all the proposed ones from the subproblem units, since the master problem solution is a linear combination of the solutions from the subproblem units. Thus, the Lagrangian relaxation technique can not guarantee a satisfactory solution for hydro generation scheduling with a fully linearized model. Further research on this aspect is needed.

6.10.3 Test Results of the Frank-Wolfe Method

Finally, the test results of the application of the Frank-Wolfe feasible direction algorithm are presented. A one river hydroelectric power system based on a Swedish system data is tested. The reservoir and generator data can be found in Tables 6.13-15.

Table 6.14

Reservoirs data

	m^3	m^3	m^3	m^3	m^3/s	m^3/s	m^3/s
Reservoir no.	V_{min}	V_{max}	V_{init}	V_{final}	QQ_{max}	QQ_{min}	Inflows
1	0.00	1275.00	0.00	0.00	295.0	00.00	100.00
2	0.00	1225.00	0.00	0.00	310.0	00.00	0.00
3	0.00	1030.00	0.00	0.00	270.0	00.00	0.00
4	0.00	1100.00	0.00	0.00	325.0	00.00	0.00
5	0.00	1500.00	0.00	0.00	300.0	00.00	0.00
6	0.00	1950.00	0.00	0.00	350.0	00.00	200.00
7	0.00	630.00	0.00	0.00	600.0	00.00	0.00
8	0.00	600.00	0.00	0.00	600.0	00.00	0.00
9	0.00	1330.00	0.00	0.00	600.0	00.00	0.00

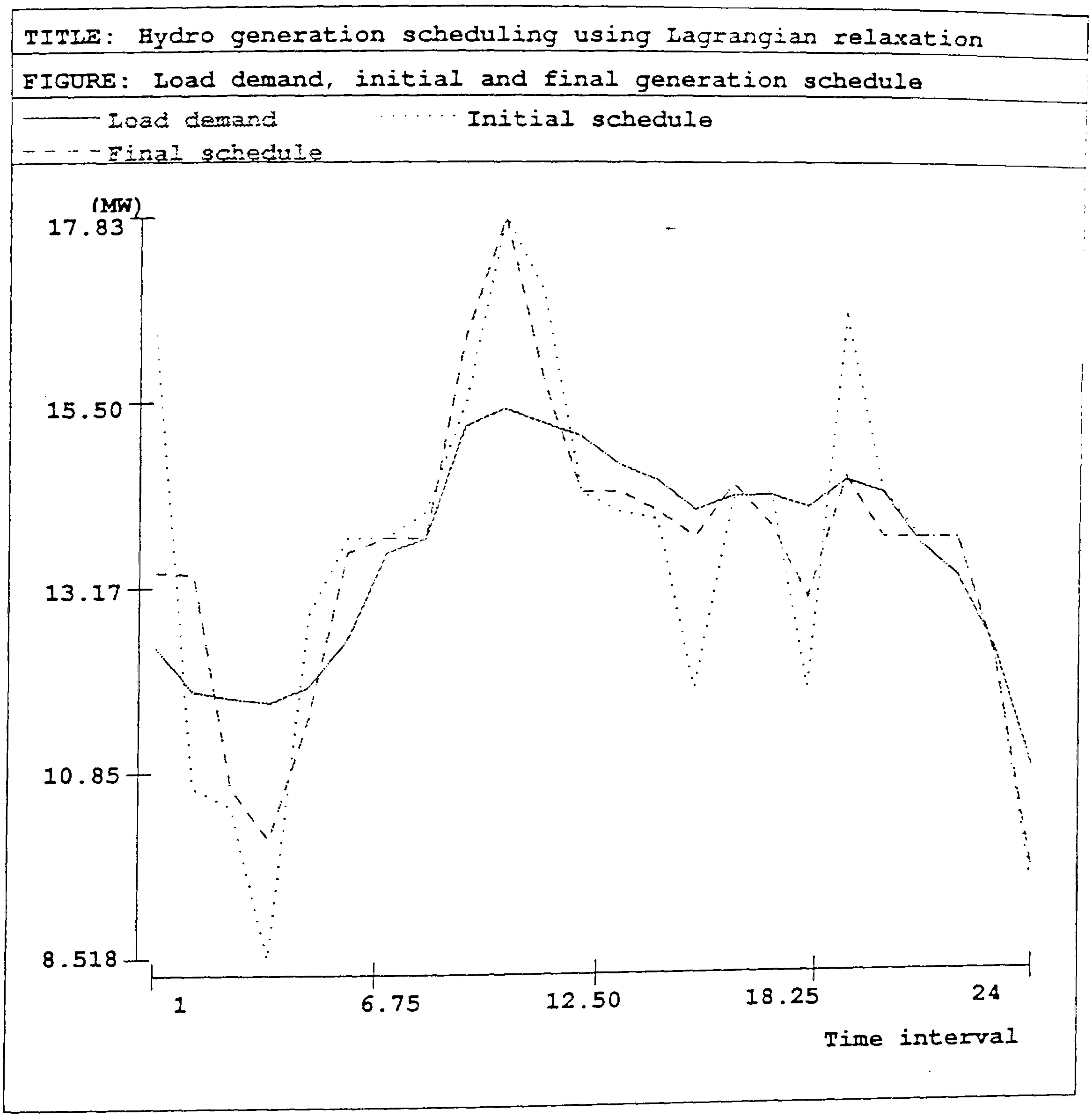

TITLE: Hydro generation scheduling using Lagrangian relaxation
FIGURE: Load demand, initial and final generation schedule
Load demand
Initial schedule
Final schedule
(MW)
17.83
15.50
13.17
10.85
8.518
1
6.75
12.50
18.25
24
Time interval

Figure 6.6

Table 6.13

Hydro generation system data

			MWs/m³	m³/s			
River	Plant	Unit	Slope	Q_{max}	$Upstrm_1$	$Upstrm_2$	Downstrm.
1	1	1	0.207	180.0	-	-	2
1	1	2	0.186	115.0	-	-	2
1	2	3	0.088	200.0	1	-	3
1	2	4	0.064	110.0	1	-	3
1	3	5	0.083	270.0	2	-	4
1	4	6	0.306	95.00	3	-	5
1	4	7	0.283	75.00	3	-	5
1	4	8	0.252	155.0	3	-	5
1	5	9	0.500	110.0	4	-	7
1	5	10	0.470	110.0	4	-	7
1	5	11	0.386	80.00	4	-	7
1	6	12	0.872	90.00	-	-	7
1	6	13	0.871	80.00	-	-	7
1	6	14	0.812	90.00	-	-	7
1	6	15	0.555	90.00	-	-	7
1	7	16	0.201	90.00	5	6	8
1	7	17	0.186	60.00	5	6	8
1	7	18	0.185	120.0	5	6	8
1	7	19	0.135	330.0	5	6	8
1	8	20	0.194	160.0	7	-	9
1	8	21	0.185	140.0	7	-	9
1	8	22	0.178	120.0	7	-	9
1	8	23	0.100	180.0	7	-	9
1	9	24	0.312	100.0	8	-	-
1	9	25	0.310	85.00	8	-	-
1	9	26	0.296	95.00	8	-	-
1	9	27	0.270	85.00	8	-	-
1	9	28	0.190	235.0	8	-	-

Tests have shown that in the application of the Frank-Wolfe algorithm to the hydro subproblem scheduling, it is not generally suitable for setting the convergence criterion as checking for the lower bound β. This is because this criterion may create a form of instability. Instead, the convergence criterion is set to be $|f([X_{k-1}]) - f([X_k]^*)| \leq \epsilon$, where $f([X_{k-1}])$ is the objective value at the previous iteration $k-1$, $f([X_k]^*)$ is the objective value after the line search at iteration k. Results also show that the piecewise linear initialization is quite accurate, and there is no substantial difference between the linearized function schedule with the actual nonlinear function schedule. Thus, only a few iterations are needed for the Frank-Wolfe algorithm to converge, Typically within 1-4 iterations as can be seen in Figure 6.7. For this test system containing 9 reservoirs cascaded and with 28 units in total, the CPU time for a 24 hour schedule is 11.13 seconds. In fact, as the actual piecewise linearization is quite accurate, the initialization takes 4.48 seconds of the CPU time, the actual Frank-Wolfe algorithm with this initial solution only takes 1.5 times the initialization time (6.65 seconds of CPU time) to converge.

Figure 6.7 shows the total hydro generation change against iterations. The initialization is obtained through solving the piecewise linear model. This initialization is quite accurate so that the Frank-Wolfe algorithm needs only a few iterations to converge. The final hydro generation schedule shown in Figure 6.7 is good since it exactly follows the changes of the marginal prices. In hydrothermal scheduling, this means the load demand change may be followed by hydro generation, which results in the thermal generation only needing to cover the base load. Hence a substantial amount of thermal operation cost can be saved.

To conclude, computer programs have been developed for the solution of the hydro generation scheduling problem. The results obtained reveal the efficiency of the minimal network flow algorithm NETFLO. By exploiting the special network structure of reservoir dynamics, the large realistically sized hydro system problem can be solved efficiently. A Frank-Wolfe feasible direction method is used to achieve further accuracy and with piecewise linear initialization only a few additional iterations are needed. Lagrangian relaxation

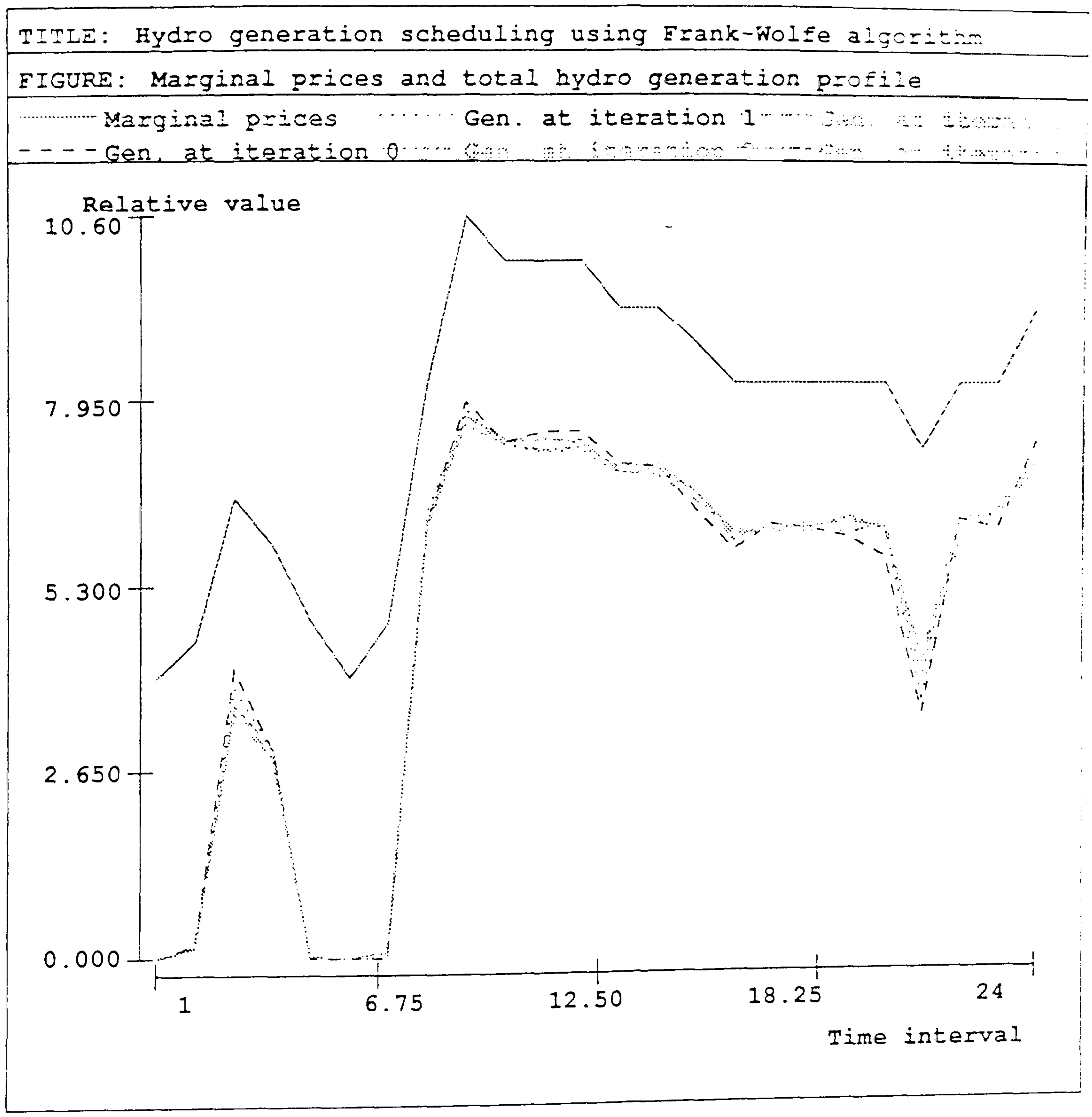

TITLE: Hydro generation scheduling using Frank-Wolfe algorithm
FIGURE: Marginal prices and total hydro generation profile
Marginal prices
Gen. at iteration 1
Gen. at iteration 0
Relative value
10.60
7.950
5.300
2.650
0.000
1
6.75
12.50
18.25
24
Time interval

Figure 6.7

has been applied to deal with the non-network constraints such as the power balance constraints in purely hydro generation scheduling. We believe that this Lagrangian relaxation can also deal efficiently with other security constraints, such as transmission limitations and reserve requirements, in the same way as the power balance constraints. However, more research is needed for the application of the Lagrangian relaxation to the purely hydro generation scheduling problem.

Further work on hydrothermal scheduling will combine the network flow algorithm with thermal unit commitment through a coordination procedure, so that the overall optimal schedule for the hydrothermal system can be achieved.

CHAPTER 7

SOLUTION OF HYDROTHERMAL SCHEDULING

7.1 INTRODUCTION

The short-term generation scheduling problem in a hydrothermal power system, as described in Chapter 4, is a large scale mixed-integer mathematical programming problem. In the two previous chapters the thermal unit commitment problem and the hydroelectric generation scheduling problem have already been discussed and the solution techniques for these two separate problems have also been presented in great detail, together with many test results for various solution algorithms. In this chapter, solution techniques that decompose the entire hydrothermal generation scheduling problem into a hydro scheduling and a thermal scheduling subproblem are considered. As shown in later sections the thermal subproblem becomes very similar to the thermal unit commitment problem. Thus, all the algorithms used in Chapter 5 can be applied to these thermal subproblems. Similarly, all the algorithms used in Chapter 6 can be applied to the hydro subproblems.

The aim of this chapter is to consider the solution techniques required for the hydrothermal generation scheduling problem as a whole, and to discuss the applications of mathematical decomposition and coordination methodologies for hydrothermal generation scheduling.

Much work has been carried out in the short-term hydrothermal generation scheduling area in this project. This generation allocation task is performed in a deterministic manner as the water inflows and the load demand in the short-term operational planning phase are assumed to be deterministic. Usually the period considered is 24 hours (daily scheduling) to a week (weekly scheduling). The consumption of a predetermined amount of water is allocated for each hydraulic generation plant. Most physical and operational constraints have been considered, such as the limit on average reservoir discharge rate and generator output upper and lower limits. The hydrothermal scheduling problem has been solved through different mathematical decomposition and coordination techniques. Two methodologies for hydrothermal coordination have been applied to this problem:

1. Lagrangian relaxation decomposition and coordination.
2. Marginal price decomposition and coordination.

The hydrothermal generation scheduling problem is usually a very large scale mathematical programming problem involving thousands of variables and constraints as well as integer variables. To solve this problem efficiently, the basic idea is to decompose the whole hydrothermal generation scheduling problem into a hydro subproblem and a thermal subproblem, and these two subproblems may be solved iteratively with an intervening coordination procedure. This basic approach is encouraged by the following aspects of the problem:

1. Using the solution techniques discussed in Chapter 6 to exploit the network structure of the hydro subproblem, such as the dual revised simplex linear programming method (SDRSLP), the out-of-kilter algorithm or the network flow simplex method (NETFLO), the hydro subproblem can be solved independently and efficiently. The speed of solution is very fast. The aim of the hydro subproblem becomes simply scheduling the hydro generating units in each river valley to obtain the maximum utilization of the available amount of water resources according to specific Lagrangian multipliers (marginal prices).

2. The thermal subproblem may be solved using the solution techniques discussed in Chapter 5, such as the merit-order scheme, dynamic programming or alternatively the Lagrangian relaxation to further decompose the thermal subproblem. This thermal subproblem is solved in a similar manner to thermal unit commitment problem without hydro. The

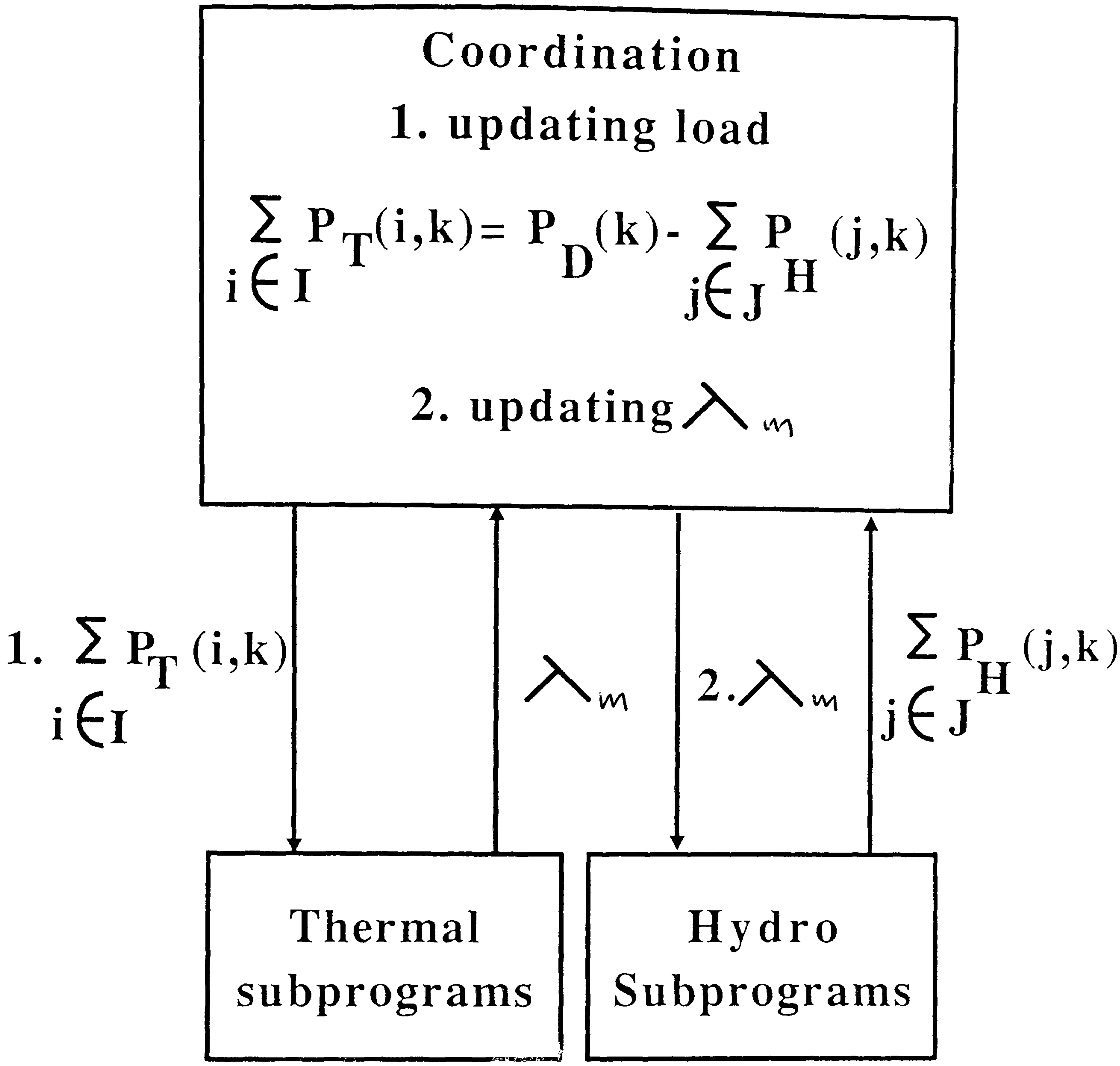
purpose is simply to minimize the total production cost of the thermal generating units under various constraints arising only from the thermal subsystem.

3. The predicted load demand must be satisfied at each time interval. This requirement, in a hydrothermal generation system, is essential and necessitates a coordination procedure between the hydro subproblem and the thermal subproblem. Depending on different coordination strategies, the decomposition and coordination procedure varies. For the marginal price coordination procedure, the load demand is divided between the two subsystems of hydro and thermal so as to obtain as much power utilization as possible from the hydro subsystem while minimizing the production cost of the thermal generating units, and these two subproblems are solved sequentially. For the Lagrangian relaxation pricing mechanism, Lagrangian multipliers are updated at each iteration according to the load demand and generation deviations, and the hydro and thermal subproblems are solved at the same stage using the same set of Lagrangian multipliers. The difference between these two procedures can be seen from Diagram 7.1 and Diagram 7.2.

4. In both coordination procedures, the computational process of hydrothermal scheduling is to solve the hydro subproblem and the thermal subproblem iteratively until no further cost saving can be achieved and the least production cost schedule is determined.

7.2 PROBLEM FORMULATION

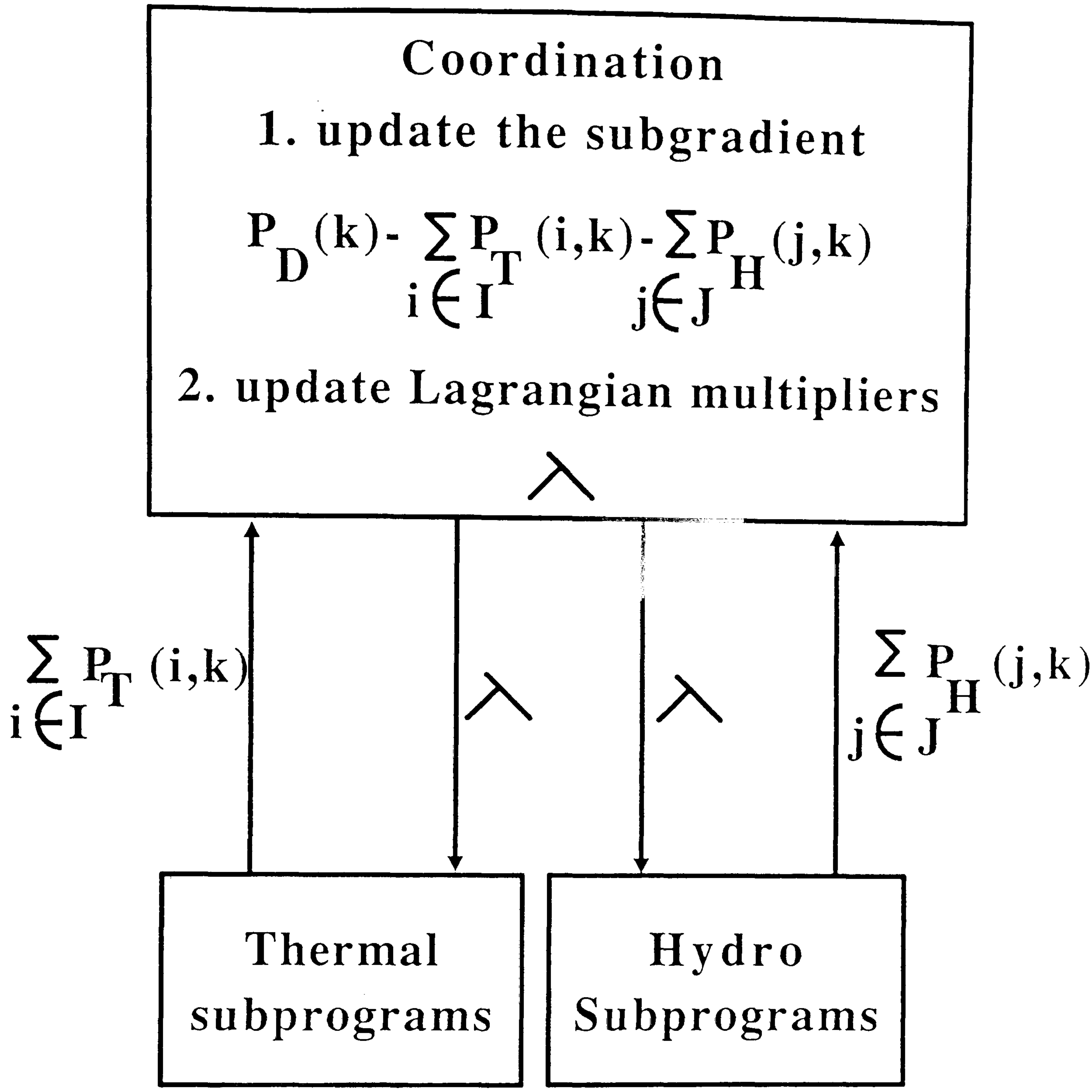
The task of optimal short-term generation scheduling in a mixed hydrothermal power system is aimed at minimizing the total thermal production cost for the whole optimization period of one day or a week while balancing the load demand requirement in each time interval and satisfying all the operating constraints imposed. An optimal generation schedule for each individual hydroelectric and thermal unit in the system in each optimization time interval will be decided.



Based on the concept:

The incremental value of the hydro generation is equal to the incremental cost of the displaced thermal generation.

Diagram 7.1 Marginal Price Decomposition/Coordination



Based on the dual programming theory

Diagram 7.2 Lagrangian relaxation
Decomposition/Coordination

The important characteristics of hydroelectric power generation systems are that hydroelectric power stations have negligible operating costs but the water energy available for a specified time horizon may be limited and normally, reservoirs operate under a highly constrained environment. The optimal schedule for the hydrothermal system is therefore the one which has a minimal production cost for the thermal system. The problem therefore becomes how to use the available water energy to cover part of the load demand at each interval so that the rest of the load will be covered by thermal plant, resulting in a minimum thermal operating cost over the optimization horizon.

To produce an optimal schedule with respect to hydro power production, a trade-off between the immediate use of available hydro energy and the expected value of hydro power in the future must be decided. Whereas for a thermal plant, there will always be a production cost associated with running the plant and generating power, this cost may be very high and part of it could be otherwise saved through replacing the thermal power production by limited hydro power production. The generation scheduling problem in a mixed hydrothermal power system implies that the overall optimal economic effects for a hydrothermal power system must be considered and the optimal coordination of multi-reservoir hydro systems with thermal generating unit operation is therefore necessary.

The solution of this generation scheduling problem for a hydrothermal power system is nonetheless difficult and complicated, primarily because the system contains both hydro and thermal units, which have radically different operating characteristics The difficulties come from the following aspects:

- Large realistic sized power systems are often involved.
- The optimal hydrothermal generation scheduling problem is often modelled as a large scale, complex, nonlinear allocation problem and further complicated by integer variables. Thus, the problem is a large scale mixed-integer programming problem.
- The hydroelectric generation subsystem operates at a negligible marginal cost but has limited available water energy and its total energy production

over a long period is subject to strict limitations implied by reservoir storage and natural inflows.

- Hydroelectric generating units operate in a highly constrained environment, since hydro power stations in the system may be interconnected with cascaded multiple reservoirs.
- In the short-term scheduling problem, the water transport delay may be considered.
- The hydro generation function is a nonlinear function of both discharge rates and reservoir head variations and the thermal production cost is a nonlinear function of the power generated.

To summarize the constraint set, the hydro subsystem model must take into account the specific reservoir operating rules including constraints on discharge rates and reservoir contents set by flood control, irrigation purposes, fisheries requirement and recreational regulations, and constraints on variable water inflows and stream flow limits. The thermal subsystem model must take into account the minimum up time and minimum down time constraints on thermal generating units, generation limits, reserve requirements and maintenance schedules, and also the nonlinearity of the thermal production cost function.

There are two aspects of reducing the total thermal production cost. One is to reduce the startup and shutdown cost of the thermal generating units, another is to reduce the fuel cost of the units. Thermal production cost functions $F_i(P_T(i,k)) = A_i + B_i * P_T(i,k) + C_i * {P_T}^2(i,k)$ imply that thermal fuel cost will quadratically increase as the power output increases. The conclusion is therefore as follows:

> "The total thermal generation profile should remain as flat as possible. The load demand pattern will be followed by the total hydro power generation if possible."

However, in a realistic mixed hydrothermal power system, even with a substantial hydro power capacity, most of the reservoirs in the hydro subsystems may be hydrologically coupled and operate under highly constrained conditions. Therefore the load variation pattern may not necessarily be covered all the time.

The aim of hydrothermal coordination between these two subsystems is then to find the best trade-off that will result in the minimum thermal production cost over the whole scheduling period.

Since the optimal hydrothermal generation scheduling problem is formulated as a complex, large scale mixed-integer programming problem, it is important to develop efficient solution techniques that are capable of solving this large optimization problem with a reasonable computational effort. Much effort has been devoted to developing the solution techniques for purely thermal power systems, purely hydro power systems and mixed hydrothermal power systems with a low percentage of hydroelectric capacity, in order to reduce the complexity of the hydrothermal scheduling problem or simplify the problem formulation. More recently efficient solution techniques for large scale hydrothermal power systems with a substantial amount of hydroelectric capacity have been developed. Unfortunately, so far, no direct application is likely to be practical for the solution of this complicated problem. Instead, recent references on large scale hydrothermal scheduling suggest that the most successful approaches to hydrothermal scheduling problems are the applications of mathematical decomposition techniques. Two types of decomposition approach have been applied for hydrothermal generation scheduling: price directive decomposition and resource directive decomposition. The first solution technique employs Lagrangian multipliers to transfer the coupling constraints (between the hydro and the thermal system) to the objective function. Consequently, the problem is decomposed into a hydro and a thermal subproblem. A master coordinator is used to couple these two subproblems by updating the Lagrangian multipliers at each iteration. The second solution technique decomposes the problem with respect to the continuous and integer variables using Benders decomposition method. The master problem in this decomposition contains integer variables and defines the thermal unit commitment schedule. The subproblem is a continuous hydrothermal economic dispatch problem in which a thermal commitment schedule is already specified, and only continuous variables are involved. Dual decomposition techniques such as Lagrangian relaxation may be applied to further decompose the Benders subproblem with respect to hydro and thermal subsystems. This leads to a multi-level decomposition procedure. In practice, various strategies have been used to solve the short-term hydrothermal generation scheduling problem such as:

1. The thermal unit commitment schedule is assumed to be already known during hydrothermal generation scheduling. The hydrothermal generation scheduling problem then becomes hydrothermal generation allocation or economic dispatch.
2. The thermal unit commitment schedule is first decided by solving the hydrothermal unit commitment problem with specified marginal prices. The total hydro generation is subtracted from the load demand according to the hydro schedule produced and the thermal scheduling subproblem can be solved with the remaining load demand and reserve. The thermal unit commitment obtained schedule is assumed to be specified. The hydrothermal coordination, (i.e. hydrothermal dispatch) continues until no further improvement can be made in the dispatch between hydro and thermal. At the final a stage, the thermal unit commitment schedule is updated once more to achieve a further saving in production cost.
3. The overall coordination approach is to solve the hydrothermal generation scheduling problem as a whole. That is to solve a mixed-integer programme including hydro and thermal generating units and many constraints.

The second approach is a compromise between the first and the third, and is more efficient, practical and suitable for solving large scale realistic problems.

The generation scheduling problem for a hydrothermal power system is, as modelled in Chapter 4, to minimize the total production cost from the thermal subsystem:

$$Min \qquad \sum_{k}^{K} \sum_{i}^{I} \{F_i[P_T(i,k)] + ST_i[X(i,k), U(i,k)]\} \tag{7.1}$$

The variables and constraints involved are summarized as follows:

1. The thermal subsystem constraints:

$$U(i,k) = \begin{cases} 0, & \text{if unit } i \text{ is decided to be 'off'.} \\ 1, & \text{if unit } i \text{ is decided to be 'on'.} \end{cases} \tag{7.2}$$

$$X_s(i,k+1) = \begin{cases} X_s(i,k), & \text{if } X_s(i,k) = 1 \ \& \ U(i,k) = 0 \\ X_s(i,k) + T_{mindown}, & \text{if } X_s(i,k) = 1 \ \& \ U(i,k) = 1 \\ 1, & \text{if } X_s(i,k) = Maxstate \ \& \ U(i,k) = 0 \\ Maxstate, & \text{if } X_s(i,k) = Maxstate \ \& \ U(i,k) = 1 \\ X_s(i,k) + 1, & \text{if } 1 < X_s(i,k) < Maxstate \end{cases} \tag{7.3}$$

$$P_{imin} \leq P_T(i,k) \leq P_{imax} \tag{7.4}$$

$$\Delta P_T(i,k) \leq P_{ramp} \tag{7.5}$$

$$T_{minup} \qquad and \qquad T_{mindown} \tag{7.6}$$

$$F_i(P_T(i,k)) = \begin{cases} 0, & \text{if } U(i,k) = 0 \\ f(i,k), & \text{if } U(i,k) = 1 \end{cases} \tag{7.7}$$

where $f(i,k) = A_i + B_i * P_T(i,k) + C_i * {P_T}^2(i,k)$.

$$\begin{aligned} ST_i(X(i,k), U(i,k)) = {} & C_{coldstart}(i) * \frac{\alpha(i) * T_{down}(i)}{1 + \alpha(i) * T_{down}(i)} \\ & + C_{shutdown}(i) \end{aligned} \tag{7.8}$$

2. The coupling constraints:

$$P_D(k) - \sum_i^I P_T(i,k) - \sum_j^J P_H(j,k) \leq 0 \tag{7.9}$$

3. The hydro subsystem constraints:

$$V(j,k+1) - V(j,k) + Q(j,k) - \sum_m^M Q(m,k) = R(j,k) \tag{7.10}$$

$$V_{jmin} \leq V(j,k) \leq V_{jmax} \tag{7.11}$$

$$Q_{jmin} \leq Q(j,k) \leq Q_{jmax} \tag{7.12}$$

$$P_H(j,k) = \sum_n^N C_{jn} * U_n(j,k) \tag{7.13}$$

7.3 APPLICATIONS OF LAGRANGIAN RELAXATION METHOD

The Lagrangian relaxation technique has been applied as one of the decomposition and coordination procedures. The application of this decomposition technique has made it possible to exploit the special structure arising from different subproblems (namely, hydro and thermal subproblems), of the large number of variables and constraints involved.

As with the thermal unit commitment problem discussed in Chapter 5, in the short-term hydrothermal generation scheduling problem, the coupling constraints between hydro and thermal generation subsystems are the power balance equation and the reserve requirements respectively.

$$P_D(k) - \sum_{i}^{I} P_T(i,k) - \sum_{j}^{J} P_H(j,k) \leq 0 \tag{7.14}$$

$$P_R(k) - \sum_{i}^{I} \Delta P_T(i,k) - \sum_{j}^{J} \Delta P_H(j,k) \leq 0 \tag{7.15}$$

Where $P_D(k)$ is the expected average demand during time interval k, $\sum P_H(j,k)$ is the total hydro generation and $\sum P_T(i,k)$ is the total thermal generation. $P_R(k)$ is the specified threshold reserve chosen to ensure that with a high probability the power demand will be covered even if some units fail to generate or the actual demand varies from the expected demand during time interval k.

Assuming that the reserve requirements, with the existence of hydroelectric power generation subsystem, will be easily satisfied. This type of coupling constraints is not considered explicitly. The only coupling constraints are the power balance requirements. The algorithm uses a Lagrangian relaxation methodology in the usual way to decompose the original hydrothermal unit commitment optimization problem into a series of smaller problems. The power balance constraints can be relaxed and adjoined to the original objective function of

$$\min_{\{P_T(i,k),X(i,k),U(i,k)\}} \sum_{k}^{K} \sum_{i}^{I} \{F_i[P_T(i,k)] + ST_i[X(i,k),U(i,k)]\} \tag{7.16}$$

and Lagrangian multipliers $\lambda(k), \quad k \in K$ can be added to each power balance constraint. The Lagrangian function then becomes

$$
\begin{aligned}
L[\lambda, P_T(i,k), U(i,k), X(i,k), P_H(j,k)] = \min_{\{P_T(i,k), X(i,k), U(i,k)\}} & \{\sum_k^K \sum_i^I F_i[P_T(i,k)] \\
& + \sum_k^K \sum_i^I ST_i[X(i,k), U(i,k)]\} \\
& + \sum_k^K \lambda(k) * P_D(k) \\
& - \sum_k^K \lambda(k)[\sum_i^I P_T(i,k) - \sum_j^J P_H(j,k)]
\end{aligned} \tag{7.17}
$$

subject to the other constraints included.

The dual problem becomes

$$
\max_{\{\lambda \geq 0\}} \{ \min_{\{P_T(i,k), X(i,k), U(i,k)\}} L[\lambda, P_T(i,k), P_H(j,k), X(i,k), U(i,k)]\} \tag{7.18}
$$

Subject to other independent hydro and thermal constraints. The objective of the inner problems or the subproblems is the minimization of the Lagrangian dual function value with respect to the variables $U(i,k)$, $X(i,k)$, $P_T(i,k)$ and $P_H(j,k)$. The objective of the outer problem or the master coordination problem is the maximization of the Lagrangian dual function with respect to variables $\lambda(k)$. The dual function $L[\lambda, P_T(i,k), P_H(j,k), X(i,k), U(i,k)]$ can be rewritten as:

$$
\begin{aligned}
L[\lambda, P_T(i,k), P_H(j,k), X(i,k), U(i,k)] = \min_{\{P_T(i,k), X(i,k), U(i,k)\}} & \{\sum_k^K \sum_i^I F_i[P_T(i,k)] \\
& + \sum_k^K \sum_i^I ST_i[X(i,k), U(i,k)] \\
& - \sum_k^K \lambda(k) \sum_i^I P_T(i,k)\} \\
& - \sum_k^K \lambda(k) \sum_j^J P_H(j,k) \\
& + \sum_k^K \lambda(k) P_D(k)
\end{aligned} \tag{7.19}
$$

As in the thermal unit commitment problem, after the elimination of the coupling constraints by the introduction of Lagrangian multipliers $\lambda(k)s$, the inner minimization subproblem, with specified $\lambda(k)$, can be decomposed into a hydro subproblem and a thermal subproblem. These two subproblems are further decomposable and the minimization problem becomes the solution of many hydro and thermal subproblems as follows:

1. Thermal subproblems exist with respect to individual thermal units. For each individual thermal unit i, there is a decomposed unit commitment subproblem such as:

$$\min_{\{P_T(i,k),X(i,k),U(i,k)\}} \sum_{k}^{K}\{F_i[P_T(i,k)]+ST_i[X(i,k),U(i,k)]-\lambda(k)*P_T(i,k)\} \tag{7.20}$$

2. Hydro subproblems exist with respect to each river valley. For each river valley containing cascaded reservoirs, no further decomposition can be made. Therefore, for each individual river valley, there is a decomposed hydro scheduling subproblem such as:

$$\min_{\{V(j,k),Q_n(j,k)\}} \sum_{k}^{K} -\lambda(k) * \sum_{j}^{J}\sum_{n}^{N} C_{jn} * Q_n(j,k)) \tag{7.21}$$

A master or a coordination procedure is used to couple the hydro and the thermal subproblems through updating Lagrangian multipliers at each iteration. This decomposition and coordination program structure is shown in Diagram 7.3. The solution of thermal subproblems is similar to the solution of the thermal unit commitment problem using the Lagrangian relaxation dual methodology and the algorithms applied in Chapter 5 can be employed for the solution of the thermal subproblem here. The hydro subproblems can be solved in the same way as described in Chapter 6. Two different solution approaches are proposed for the dual master coordination, namely:

1. The feasible direction methods. These techniques will take a gradient direction for the dual maximization and possibly take an appropriate search step or perform a line search in the chosen direction. The

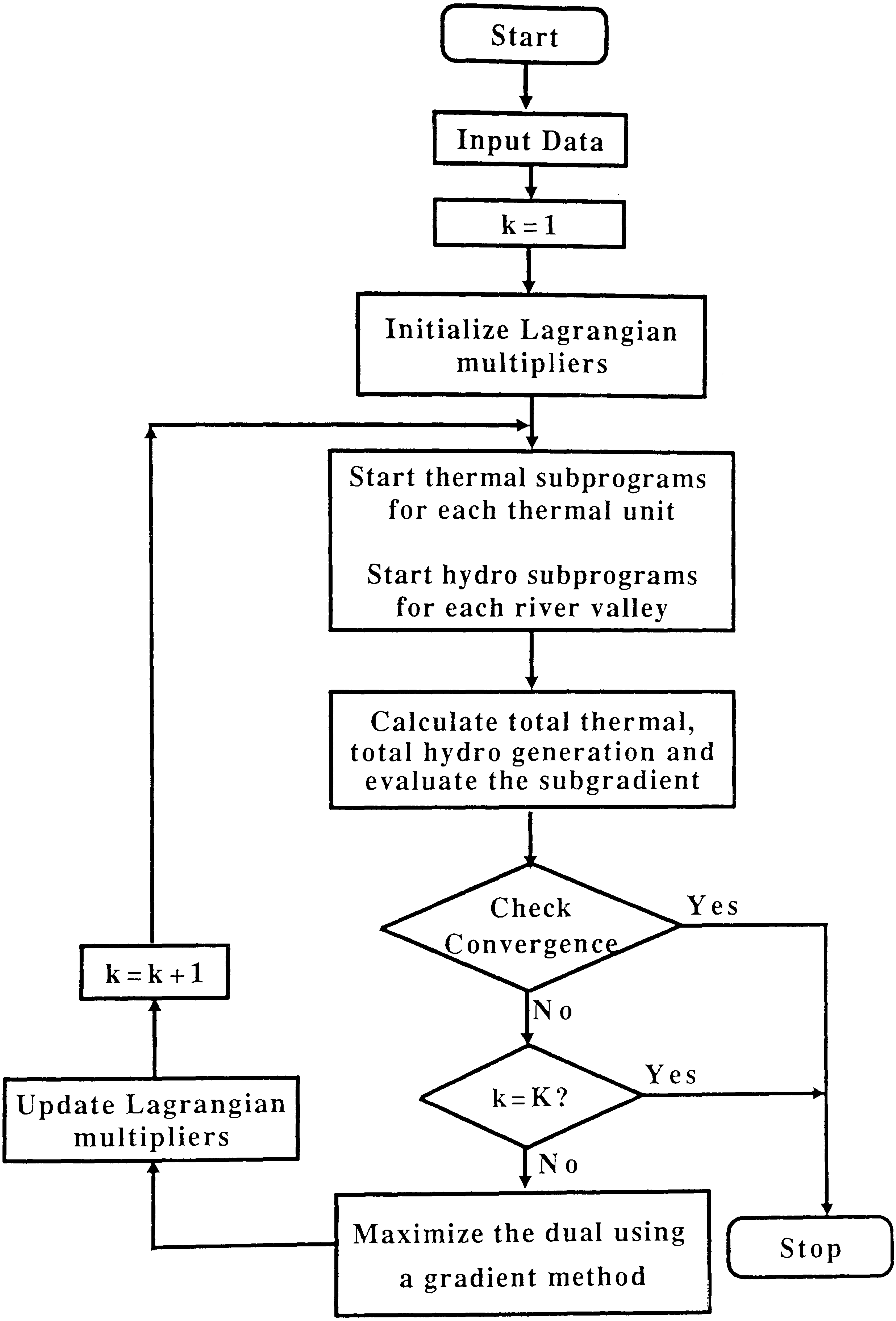


Diagram 7.3 Flow Chart of Hydrothermal Scheduling Using Lagrangian Relaxation

sub-gradient optimization technique is one of the methods within this approach.

2. Another approach is the tangential approximation technique.

Feasible direction methods proceed towards the optimum using the gradient or the sub-gradient, whereas the tangential approximation method makes a linear approximation of the dual objective. Several algorithms for master coordination and updating the Lagrangian multipliers have been considered in this Lagrangian relaxation approach, such as:

1. The sub-gradient optimization algorithm.
2. The steepest descent gradient method.
3. A quasi-Newton gradient method (DFP)
4. A maximum entropy approach.

7.3.1 The sub-gradient optimization algorithm

The sub-gradient optimization algorithm is a very simple, approximate ascent algorithm for unconstrained non-differentiable concave programming problems. There are two mathematical assumptions associated with the application of this algorithm. If these two assumptions are satisfied, this algorithm will converge to a local optimal solution. These assumptions are:

1. The norms of the sub-gradients encountered by the algorithm are uniformly bounded.
2. The value of the maximal objective function is known a-priori.

The first assumption is a fairly weak assumption, and it is easily satisfied in the hydrothermal scheduling problem. The second assumption is, however, difficult to satisfy since it is difficult to know the optimum beforehand. Therefore, the theoretical convergence of the algorithm can not be guaranteed in practice. However, this algorithm is still very popular because it has worked well in practice as will be demonstrated in test solutions of the hydrothermal scheduling problem.

The sub-gradient optimization algorithm generates the solution for $\lambda^{i+1}(k)$ of the dual variables by using the rule:

$$\lambda^{i+1}(k) = \lambda^{i}(k) + \alpha^{i} * g^{i}(k), \quad i = 0, 1, 2, \ldots \quad iteration \quad number \tag{7.22}$$

where $g^{i}(k)$ is any sub-gradient of the dual function $L(\lambda(k))$ at $\lambda^{i}(k)$, and there are three alternative forms of the sub-gradient updating formula:

1. This form was first proposed by Polyak et al.[161.] with

$$\alpha^{i} = \frac{\beta^{i} * (L'(\lambda(k)) - L(\lambda^{i}(k))}{||g^{i}(k)||^{2}} \tag{7.23}$$

   Where β is a scalar which satisfies $0 < \epsilon_1 < \beta^{i} \leq 2 - \epsilon_2$ and $\epsilon_2 > 0$. $L'(\lambda(k))$ is an estimate of the optimal dual function value $L^{*}(\lambda(k))$.
2. To avoid the difficulty associated with the first formulae of finding good values of $L'(\lambda(k))$, another procedure was proposed by Held [100.] with

$$\alpha^{i} = \frac{1}{a + b * i} \tag{7.24}$$

   where a and b are scalars taking positive values and i is the iteration number.
3. The third procedure is very similar to the second with only a change in the choice of α^{i},

$$\alpha^{i} = \frac{c}{||g^{i}(k)||} \tag{7.25}$$

   where c is a positive scalar. This procedure was first proposed by Shor,[100.]. All the three proposals were discussed by Held.[100.].

Here the first two formulae are used. The direction finding procedure of the sub-gradient algorithm is to select any sub-gradient at the solution $\lambda^{i}(k)$ and no efforts are made to discover if the dual function $L(\lambda^{i}(k))$ actually increases in the sub-gradient direction or not. Similarly, the step length α^{i} is determined solely by the formula above, no line search is performed, and no further effort is made to try to discover if the dual function is increased. This is why the sub-gradient optimization algorithm is only *approximately* ascending. However, the sub-gradient optimization algorithm has been proven both mathematically and practically to generate a sequence of points converging to a local optimal solution. Since no efforts are made to find a proper ascending direction and to maximize along the chosen direction by performing a line search, much computation time may be saved. The efficiency of this sub-gradient algorithm thus depends upon the overall number of iterations required for convergence.

It is certainly possible to use dL to define a direction of steepest ascent for any point which does not solve the problem, and develop a close analogue of the method of steepest ascent for the dual problem, as was applied in some work. Several such approaches has been reported, but the process of finding the entire gradient vector in order to get a locally "best" direction imposes too heavy a computational requirement, instead, the approximation of the sub-gradient $G^i(k)$ is preferred.[100.]

Another aspect in which the algorithm differs from the usual steepest ascent algorithm is in making no effort to maximize $L(\lambda^i(k))$ along the chosen direction. The reason is because to perform a line search will entail a very heavy computational burden. Also the choice of the search direction in this algorithm hardly makes it worthwhile and advisable to perform a line search. Instead, the choice of the step length depends only very modestly on the behaviour of the dual function.

In this algorithm, the sequence of $L(\lambda^i(k))$ either converges to the best value of $L(\lambda(k))$ if the optimal value of $L^*(\lambda(k))$ is known a-priori or a point $\lambda^i(k)$ is obtained such that $L(\lambda^i(k)) \geq L^*(\lambda(k))$ if an over estimate is used. β^i's is chosen as follows:

$$\beta^i = \begin{cases} \frac{2}{1+i}, & \text{if } i \leq 3 \\ 0.5, & \text{otherwise.} \end{cases} \tag{7.26}$$

The main difference between the algorithm adopted here and the conventional sub-gradient algorithm is in the convergence criterion. Due to the unit commitment feature of having integer variables, to ensure a feasible solution of the unit commitment schedule, not only should the dual function convergence criterion be set up, but also the power balance constraints must be checked. If these constraints can not be satisfied, the process must be continued, otherwise there will be no feasible solution to the generation scheduling despite a lower bound of the solution for the primal problem being produced.

The physical meaning of the sub-gradient in hydrothermal scheduling actually represents the shortage or the surplus of the total power generation over the power demand during different time intervals. The Lagrangian multipliers in this scheduling problem represent the shared marginal prices of each hydro and thermal unit at different time intervals. In the sub-gradient algorithm, they are modified at each iteration according to this sub-gradient with a proper step length α such as:

$$\lambda^{i+1}(k) = \lambda^{i}(k) + \alpha * g^{i}(k) \tag{7.27}$$

The pricing mechanism of the Lagrangian relaxation decomposition technique for hydrothermal scheduling can be interpreted as: if there is a shortage of total power generation during a certain time interval, the marginal price or the Lagrangian multiplier for this time interval will be increased; whereas if there is a surplus of total power generation during a time interval, the Lagrangian multiplier will be decreased. At the next iteration the power supplied will try to balance the demand for each time interval and an improved solution can be found, until no further improvement can be achieved by changing the marginal prices.

The process of the sub-gradient optimization algorithm can be summarized in the following steps:

1. Set iteration number $i \Leftarrow 0$. Calculate the initial function value.
2. Compute the gradient vector and initial search direction $g^{i}(k)$.
3. Compute $\lambda^{i+1}(k)$ with

$$\lambda^{i+1}(k) = \lambda^{i}(k) + \alpha^{i} * g^{i}(k)$$

where

$$\alpha^{i} = \frac{\beta^{i} * (L'(\lambda(k)) - L(\lambda^{i}(k))}{\|g^{i}(k)\|^{2}}$$

with $0 < \epsilon_1 < \beta^i \leq 2 - \epsilon_2$ and $\epsilon_2 > 0$ or with

$$\alpha^i = \frac{1}{a + b * i}$$

4. Compute the new function value and gradient $g^{i+1}(k)$.
5. If convergence is considered to be attained, set the optimal solution $\lambda^* = \lambda^{i+1}$, terminate the process.
6. Set iteration $i \Leftarrow i + 1$, go to step 3.

7.3.2 The steepest descent (ascent) gradient algorithm

The optimization of the hydrothermal generation scheduling problem with integer variables, as described, is solved by Lagrangian relaxation, and the maximization of the master dual function is achieved approximately by the sub-gradient optimization algorithm. As with the approach used for the thermal unit commitment discussed in Chapter 5, the convergence of the discrete problem leads to the solution of a continuous problem in which only real variables are involved. In this case, since the generation dispatch can be very fast, the steepest ascent algorithm may be used for a more accurate maximization of the continuous dual function. Hence, a line search is performed along the chosen direction to maximize the dual value. A quadratic interpolation line search is used.

The optimization of the continuous problem in this case is to check whether or not the power supply satisfies the load demand for all the time intervals. If not, the Lagrangian multipliers $\lambda(k)$ are updated by maximizing the dual function using the steepest ascent gradient algorithm. A line search is performed to find the optimal step in order to maximize the dual function. The program then continues with updated $\lambda(k)$ until no improvement can be made in maximizing the dual function and all the coupling constraints are satisfied within the convergence criterion.

The process of the steepest ascent method can be summarized in the following steps:

1. Set iteration number $k \Leftarrow 0$. Calculate the initial function value.

2. Compute the gradient vector and initial search direction $g(\lambda^k)$.
3. Perform a quadratic interpolation line search to find an optimal step length α^k with

$$L[\lambda^k + \alpha^k g(\lambda^k)] = \max_{\alpha} L[\lambda^k + \alpha g(\lambda^k)]$$

4. Compute the new function value and gradient $g(\lambda^{k+1})$.
5. If convergence is considered to be attained, set the optimal solution $\lambda^* = \lambda^{k+1}$, terminate the process.
6. Set iteration $k \Leftarrow k + 1$, go to step 3.

7.3.3 A quasi-Newton gradient method (DFP)

The common contribution of hydroelectric generation and thermal generation towards the power demand in the hydrothermal scheduling problem leads to the appearance the coupling constraints in the optimization problem. Fortunately, these constraints are small in number. By applying one of the price directive decomposition techniques (Lagrangian relaxation), the coupling constraints can be treated in the dual problem associated with the dual prices. By iteratively adjusting the dual prices, the coupling constraints can be satisfied eventually. However, computational experience shows a few weaknesses of using the conventional sub-gradient optimization or the steepest ascent gradient method for the maximization of the dual function in the Lagrangian relaxation:

1. To avoid the instability created by an improper high optimal dual function value estimate, this estimate should be chosen not far away from the exact dual optimal value. This again creates the difficulty of slow convergence rate for the sub-gradient optimization algorithm and more experimentation may be needed to lead to good heuristic estimates.
2. Since the hydrothermal scheduling problem contains integer variables from the thermal unit commitment and the hydro subproblems are piecewise linear, the overall dual problem is not everywhere differentiable. Even after the integer variables are specified in the discrete problem, the hydro subproblems in the continuous problem still produce non-differentiability. Hence, the steepest ascent gradient algorithm which performs a line

search along the chosen sub-gradient direction will create a heavy computational burden while achieving little improvement, slow convergence and computation time are still its main drawbacks.

3. It has been observed that it is difficult to obtain a feasible solution for the primal problem even from the solution of the continuous problem. Tests have shown that at the end of the iteration, the price adjustment, even if it is very small, will produce oscillations in the solutions. It oscillates around the coupling constraints without finding an exact feasible solution. This is mainly caused by the complete linearity of the hydro problem, since the primal problem of hydrothermal scheduling is not strictly convex and the full decentralization using the dual approach will result in some infeasibility. Further intervention is thus necessary. To avoid the loss of quality of the optimization, cautions must be taken in the termination criterion of the solution program for the dual problem. The feasible hydrothermal scheduling solution is obtained later by adjusting the thermal generation optimally for the remaining load demand obtained from the total demand minus the total hydro production. Results show that since the infeasibility of the coupling constraints from the solution of the dual is not very significant, feasibility can be achieved easily by performing a thermal economic generation allocation. Tests have shown that the difference between the total thermal production cost after the adjustment and the optimal dual value is very small, normally only about 0.1 - 0.3%.

To avoid oscillation and speed up the convergence, a *quasi-Newton* gradient method is proposed, which has a super-linear convergence rate while avoiding the need to compute and invert the *Hessian matrix* $H(x)$ by using an approximation of the inverse Hessian matrix. Here the method of Davidon-Fletcher-Powell (DFP)[207.] is applied which is a quasi-Newton method as well as a conjugate direction method.

Given any function $f(\mathrm{x})$, take some particular point x_0 as the origin of the coordinate system with coordinates x, then this function can be approximated by the first three items of its Taylor series, that is;

$$
\begin{aligned}
f(\mathrm{x}) &= f(\mathrm{x}_0) + \sum_i \frac{\partial f}{\partial x_i} x_i + \frac{1}{2} \cdot \sum_{i,j} \frac{\partial^2 f}{\partial x_i \partial x_j} x_i x_j + ... \\
&\approx \mathrm{c} - \mathrm{b} \cdot \mathrm{x} + \frac{1}{2} \cdot \mathrm{x} \cdot \mathbf{A} \cdot \mathrm{x}
\end{aligned}
\tag{7.28}
$$

Where

$$
\mathrm{c} \equiv f(\mathrm{x}_0) \qquad \mathrm{b} \equiv -\nabla f \mid_{\mathrm{x}_0} \qquad [\mathbf{A}]_{ij} \equiv \left. \frac{\partial^2 f}{\partial x_i \partial x_j} \right|_{\mathrm{x}_0} \tag{7.29}
$$

The matrix $\mathbf{A}$ whose components are the second partial derivatives of the function is called the *Hessian matrix* of the function at x_0. In the approximation of the above equation, the gradient of f is easily calculated as:

$$
\nabla f = \mathbf{A} \cdot \mathrm{x} - \mathrm{b} \tag{7.30}
$$

The Newton methods usually utilise the full second partial derivative matrix in defining the gradient direction. However, full Newton gradient methods may be too time consuming to be practically acceptable.

The purpose of quasi-Newton methods is to accumulate information from successive line minimization and to build up, iteratively, a good approximation to the inverse Hessian matrix $[\mathbf{A}^{-1}]$ so that K such line search minimization will lead to the exact minimum of a quadratic form in K dimensions. The method will also be super-linearly (quadratically) convergent for more general smooth functions. Thus, the property holds:

$$
\lim_{i \to \infty} \mathbf{H}_i = \mathbf{A}^{-1} \tag{7.31}
$$

Where $\mathbf{H}_i$ is the approximation of the inverse Hessian matrix.

Since the minimum point x^* satisfies

$$
\mathbf{A} \cdot \mathrm{x}^* = \mathrm{b} \tag{7.32}
$$

and any current point x_i will have

$$
\mathbf{A} \cdot \mathrm{x}_i = \nabla f(\mathrm{x}_i) + \mathrm{b} \tag{7.33}
$$

Thus,

$$\mathbf{x}_{i+1} - \mathbf{x}_i = \mathbf{A}^{-1} \cdot (\nabla f_{i+1} - \nabla f_i) \tag{7.34}$$

Where $\nabla f_i = \nabla f(\mathbf{x}_i)$. The updating formula for the solution of the variables is then:

$$\mathbf{x}_{i+1} - \mathbf{x}_i = \mathbf{H}_{i+1} \cdot (\nabla f_{i+1} - \nabla f_i) \tag{7.35}$$

and the updating formula for the approximation of the inverse Hessian matrix is:

$$\begin{aligned} \mathbf{H}_{i+1} &= \mathbf{H}_i + \mathit{correction} \\ &= \mathbf{H}_i + \frac{(\mathbf{x}_{i+1} - \mathbf{x}_i) \cdot (\mathbf{x}_{i+1} - \mathbf{x}_i)^T}{(\mathbf{x}_{i+1} - \mathbf{x}_i)^T \cdot (\nabla f_{i+1} - \nabla f_i)} \\ &- \frac{[\mathbf{H}_i \cdot (\nabla f_{i+1} - \nabla f_i)] \cdot [\mathbf{H}_i \cdot (\nabla f_{i+1} - \nabla f_i)]^T}{(\nabla f_{i+1} - \nabla f_i)^T \cdot \mathbf{H}_i \cdot (\nabla f_{i+1} - \nabla f_i)} \end{aligned} \tag{7.36}$$

where the dividends are "outer" or "direct" products of the two vectors, i.e. matrices, the divisors are "inner" products of two vectors, i.e. values and **T** denotes the transpose of a column vector.

The process of the DFP quasi-Newton method for solving the dual problem with the dual variables λ can be summarized in the following steps:

1. Set iteration number $k \Leftarrow 0$. Calculate the initial function value.
2. Initialize the inverse Hessian matrix $\mathbf{H}$ as the unit matrix $\mathbf{1}$.
3. Compute the initial gradient vector and the initial search direction

$$\mathbf{g}^k = \mathbf{g}(\lambda^k)$$

4. Compute the new (conjugate) search direction $\mathbf{p}^k = -\mathbf{H}^k \cdot \mathbf{g}^k$.
5. Perform a line search to find an optimal step length α^k with

$$L(\lambda^k + \alpha^k \mathbf{p}^k) = \max_{\alpha} L(\lambda^k + \alpha \mathbf{p}^k)$$

6. Save the old function value and the old gradient. Set

$$\lambda^{k+1} = \lambda^k + \alpha^k \mathbf{p}^k$$

7. Compute the new function value and gradient $\mathbf{g}(\lambda^{k+1})$.
8. Compute $\mathbf{s}^k$, $\mathbf{y}^k$ from

$$\mathbf{s}^k = \alpha^k \mathbf{p}^k$$

and

$$\mathbf{y}^k = \mathbf{g}^{k+1} - \mathbf{g}^k$$

9. If convergence is considered to be attained, set the optimal solution $\lambda^* = \lambda^{k+1}$, terminate the process.
10. Compute $\mathbf{z}^k$ from

$$\mathbf{z}^k = \mathbf{H}^k \cdot \mathbf{y}^k$$

11. Compute the new $\mathbf{H}^{k+1}$ from

$$\mathbf{H}^{k+1} = \mathbf{H}^k + \frac{\mathbf{s}^k \cdot \mathbf{s}^{k^T}}{\mathbf{s}^{k^T} \cdot \mathbf{y}^k} - \frac{\mathbf{z}^k \cdot \mathbf{z}^{k^T}}{\mathbf{y}^{k^T} \cdot \mathbf{z}^k}$$

12. Set iteration $k \Leftarrow k + 1$, go to step 3.

7.4 A MAXIMUM ENTROPY APPROACH FOR COORDINATION

In recent years, *entropy* has emerged as a very important and powerful concept in a widely variety of different fields. Entropy is most commonly known in the physics and engineering fields in connection with the second law of thermo-dynamics, i.e. *the entropy law* , which states that the entropy, or the amount of disorder, in any closed conservative thermo-dynamic system, tends to be a maximum. The Shannon entropy measure and Jaynes's Maximum Entropy Principle have been applied in the context of civil engineering for problems with uncertainty. The entropy can be used to deduce the desired results when only limited information is available.

As far as optimization is concerned, the optimization process is itself a deductive process. Given some implicit or explicit function $f(x)$ of variables x_i, $i = 1, 2, 3, ...N$, and some constraint functions $g_i(x)$, the process of locating a minimum value of $f(x)$ commences with no numerical information at all. An initial point is chosen and information is calculated about the objective value and constraint functions, typically their numerical values and gradient at this initial point. This numerical information is then used in some deterministic mathematical programming algorithm to infer where the next trial point should be placed so as to get closer to the constrained optimum of the problem. The new trial solution generates more information from which another trial point is obtained and eventually the solution is reached by this process of gathering better and better information.

Almost all the conventional optimization algorithms use some form of geometrical estimation to generate a sequence of improved trial points. The functions in the problem are interpreted as geometrical hyper-surfaces with contours, slopes and gradients. Actually, sufficient information is not available to be able to plot the geometry except for some simple problems. However, it is convenient to imagine these hyper-geometrical shapes exist since it is helpful for visualizing what a numerical search algorithm is performing and to think about different search strategies as well as to develop new solution algorithms based on knowledge of the geometry.

Instead of using the currently available but incomplete numerical information about the problem in a geometrical inference process, the same information can be used in some sort of entropic inference process. Some useful work on the application of this principle for constrained nonlinear programming problems has been carried out.[132,186.] The idea of entropic inference can be described as follows:

Given a general inequality constrained nonlinear programming problem such as:

$$\min_{[X]} \quad f([X])$$

Subject to

$$g_i([X]) \leq [0], \quad i = 1, 2, 3, ...M$$

$$[X] = X_j, \quad j = 1, 2, 3, ...N$$

The equivalent surrogate form of this problem can be written as:

$$\min_{[X]} \quad f([X])$$

Subject to

$$\sum_{i}^{M} \lambda_i * g_i([X]) \leq [0]$$

$$[X] = X_j, \quad j = 1, 2, 3, ...N$$

in which λ_i, $i = 1, 2, 3, ...M$ are non-negative surrogate multipliers that satisfy a normality condition

$$\sum_{i}^{M} \lambda_i = 1$$

These surrogate multipliers are in many respects similar to normalized Lagrangian multipliers. It can be shown that optimum values exist for the vector of surrogate multipliers λ^* such that optimum vector $[X]^*$ which solve the surrogate problem with λ^* will also solve the original problem. The difficulty arises in that the optimum values of surrogate multipliers are unknown and must be found.

The surrogate multipliers can be interpreted as probabilities which must satisfy the normality condition

$$\sum_{i}^{M} \lambda_i = 1$$

and the expected value condition which forms the single constraint of the surrogate problem.[186.] Values of surrogate multipliers can be obtained by the Maximum Entropy Principle. This leads to a two phase solution procedure for the original problem through the surrogate problem which works as follows:

1. Set iteration number $k \Leftarrow 0$. It is assumed that all M constraints have an equal probability of being active at the problem solution, thus the surrogate multipliers are assigned the value of $\lambda_i{}^0 = 1/M$, $i = 1, 2, 3, ...M$. This corresponds to the maximum entropy solution when no information other than the normality condition is available.
2. Solve the surrogate problem

$$\min_{[X]} \quad f([X])$$

subject to

$$\sum_{i}^{M} \lambda_i * g_i([X]) \leq [0]$$

over variable vector $[X]$ with these value of $\lambda_i{}^0$ to give $[X]^0$.
3. Values of all the constraint functions are then evaluated at $[X_0]$, which provide information that can be used to find improved values of the surrogate multipliers λ^i through $g^0{}_i([X_0])$.

4. By solving the maximum entropy problem

$$\max_{\lambda} \quad S = -K * \sum_{i}^{M} \lambda_i{}^1 * Ln\lambda_i{}^1$$

Subject to

$$\sum_{i}^{M} \lambda_i{}^1 = 1, \quad i = 1, 2, 3, ...M$$

$$\sum_{i}^{M} \lambda_i{}^1 * g_i([X_0]) = \epsilon$$

Where ϵ is an error term reflecting the fact that the constraint function value $g_i([X_0])$ has been used in place of $g_i([X_1])$ which is not yet available. ϵ should be small, positive and decrease towards zero as iterations proceed. Values of $\lambda_i{}^1$ which solve the above maximum entropy problem are given by the following result:

$$\lambda^1{}_i = \frac{EXP(\beta * g_i([X_0])/K)}{\sum_i^M EXP(\beta * g_i([X_0])/K)}, \quad i = 1, 2.......M$$

in which β is the Lagrangian multiplier for the expected value constraint in the above optimization problem. Since ϵ is not uniquely known, and K is any positive constant, β/K may be considered as a control parameter for updating the surrogate multipliers. For ϵ to display the desired convergence characteristics, β/K must be positive and increase towards infinity with successive iterations.

5. Check convergence, if not achieved, go back to step 2 with the new set of surrogate multipliers, otherwise terminate the program.

However, despite the efficiency that has been claimed, tests have shown that there are three main difficulties associated with this approach. Firstly, the control parameter β/K is only limited as positive increasing to infinity, but how to choose it is the most difficult problem. Secondly, the process of updating surrogate multipliers will cause instability as well as floating point overflow problems. Finally, the choice of the Lagrangian multiplier is not easy to specify. For the example set by Li Xingsi,[132.] the test results are shown in Table 7.1 below. (k is the iteration number)

Hydrothermal scheduling for a small hydrothermal system was tested using this approach. The results show that this approach is very unstable with respect to the choice of control parameters and Lagrangian multiplier. The physical meaning of this approach is to create an overall Lagrangian multiplier for all the constraints $G_i([X])$, $i = 1, ...M$ and then distribute this Lagrangian multiplier price to each constraint by an amount of its surrogate multiplier proportion value, thus to update the normalized surrogate multipliers and the Lagrangian multiplier to achieve the optimal solution. Since the approach was very sensitive, the results were not very good, and the convergence was very difficult to obtain, this research hence has not been undertaken further. However, further development of this approach may find a way of deciding the control parameters and Lagrangian multiplier value, and further work may be carried out in the future.

7.5 MARGINAL PRICE COORDINATION

In this hydrothermal coordination procedure, the entire hydrothermal problem is decomposed into the two subproblems: the hydro subproblem and the thermal subproblem. The hydro and thermal generation scheduling are performed separately with an intervening coordination procedure. The system incremental costs or the marginal generation prices are defined to be the factors that coordinate the hydro generation and the thermal generation. A marginal price coordination procedure is similar to a Lagrangian relaxation coordination apart from the fact that the marginal price coordination does not use Lagrangian multipliers $\lambda(k)$ for thermal subproblem scheduling. In marginal price coordination, the total thermal generation is updated to match the remaining load, that is the difference between the total load demand and the total hydro generation evaluated from the hydro subproblem scheduling according to a set of specified marginal prices $\lambda(k)$. The thermal subproblem is then solved as an independent thermal unit commitment problem with the amount of the remaining load as the demand requirement at each time interval. The

Table 7.1

β	Minimum cost	Iterations	λ_1	λ_2	λ_3	X_1	X_2	X_3	G_1	G_2	G_3
$= k$	202.76	20	0.533	0.000	0.467	0.217	0.172	0.132	0.0019	-.4786	-.0022
$= e^k$	202.76	3	0.536	0.000	0.464	0.217	0.173	0.131	0.0012	-.4786	-.0014
$= .1 * e^k$	202.67	5	0.538	0.000	0.462	0.217	0.173	0.131	0.0009	-.4786	-.0006
$= .2 * e^k$	202.10	4	0.533	0.002	0.464	0.217	0.173	0.132	0.0027	-.4781	-.0007
$= .3 * e^k$	202.72	4	0.538	0.000	0.462	0.217	0.173	0.131	0.0007	-.4786	-.0006
$= .4 * e^k$	202.77	4	0.535	0.000	0.465	0.217	0.173	0.131	0.0014	-.4786	-.0016
$= .5 * e^k$	202.75	4	0.529	0.000	0.471	0.218	0.172	0.132	0.0031	-.4785	-.0034
$= .6 * e^k$	202.38	3	0.535	0.001	0.464	0.217	0.173	0.132	0.0019	-.4783	-.0008
$= .7 * e^k$	202.62	3	0.537	0.001	0.462	0.217	0.173	0.131	0.0011	-.4785	-.0008
$= .8 * e^k$	202.72	3	0.538	0.000	0.462	0.217	0.173	0.131	0.0009	-.4786	-.0008
$= .9 * e^k$	202.75	3	0.537	0.000	0.463	0.217	0.173	0.131	0.0009	-.4787	-.0010

marginal prices $\lambda(k)$ are adjusted and updated according to the corresponding thermal incremental (marginal) prices. The hydro subproblem is then solved again using these specified marginal prices. These two subproblems are solved iteratively until the problem reaches its optimum.

The idea is based on the concept that the incremental value of hydro generation is actually equal to the incremental cost of displaced thermal generation. This concept was first discussed by Seymore[170.] and Lasdon and Waren[125.]. As a result, the system incremental costs indicate the incremental values of hydro generation in each scheduling time interval and the cost of thermal generation can then be minimized through the hydro generation scheduling using the specified incremental prices. This hydro generation scheduling will maximize the total worth of the available water resources for the hydro generation according to the defined incremental costs, so as to achieve a minimization of thermal generation cost and an overall optimal solution for hydrothermal generation scheduling. The efficiency of this coordination procedure is thus dependent upon the speed and the accuracy of both the hydro subproblem and the thermal problem as well as the number of iterations that the coordination takes.

The hydro subproblem scheduling approach adopted here is the network flow algorithms. The hydroelectric scheduling problem, as presented in Chapter 6, is a capacitated network flow problem that can be very efficiently solved with available network optimization algorithms. The thermal scheduling approach is the same as that applied in the thermal unit commitment problem. Efficient thermal unit commitment and dispatch algorithms may be used to solve the thermal subproblem. Here, to avoid the difficulty of choosing the optimal dual estimate at each iteration and the complexity of the Lagrangian relaxation program, the CCDP algorithm is used for thermal subproblem scheduling. This coordination procedure is simple and straightforward, by iteratively solving the hydro and thermal subproblems, the coordination results in an optimal hydrothermal scheduling solution as the objective. However, the overall coordination by this approach is found to be very time consuming compared with the

Lagrangian relaxation dual maximization coordination, as shown in the later results.

7.6 COMPUTATIONAL EXPERIENCE AND TEST RESULTS

7.6.1 Introduction

All the algorithms described in the thesis have been programmed in Fortran 77 on a DEC VAX 8600 computer. All the tests have been undertaken on this computer. The main purpose of this project is to determine practically acceptable and efficient mathematical programming approaches to solve economic operational planning and generation scheduling problems in large scale mixed hydro and thermal power systems.

In the earlier stage of this project, the algorithms for thermal unit commitment and hydro generation scheduling were developed independently. The algorithms which were derived are used individually for unit commitment in a thermal generation system and for hydroelectric generation scheduling. The overall study has concentrated on developing algorithms to deal with the hydrothermal unit commitment scheduling problem so that the unit commitment schedule and the preliminary dispatch schedule can be determined optimally and efficiently. The algorithms are mainly for operational planning and scheduling purpose.

7.6.2 Lagrangian Relaxation Tests

A Lagrangian relaxation technique has been implemented to solve large scale hydrothermal scheduling problems involving integer variables. For problems with integer variables such as hydrothermal scheduling containing the thermal unit commitment problem, the maximum of the dual function value in Lagrangian relaxation is not necessarily of good value to the primal problem. Instead, it usually only provides a lower bound for the optimal schedule of the primal problem. It has been discovered that an optimal dual cost estimate higher than the actual maximized value of the dual function actually works better than the exact optimal dual cost value, in that the convergence using the higher estimate is faster. This effect is similar to that discussed by Held,[100.]

and this higher cost estimate can be obtained more easily through previous test experience. So far, no difficulties have been encountered with the choice of the optimal estimate.

However, even though the duality gap between the primal and the dual problem at its optimum is small for very large scale problems, an efficient way of generating the near-optimal yet feasible primal solution must be found. To tackle this problem, a procedure is proposed which has been found to be very effective and robust. The solution obtained is always very near to the exact optimum while, importantly, *feasible.*

This new procedure proposed for hydrothermal scheduling is summarized as follows. Firstly, the sub-gradient optimization method is used for solving the discrete problem. If there are no changes on thermal unit commitment schedule, and the feasibility checks are satisfied, the thermal unit commitment schedule can be fixed. The reason for setting all the feasibility convergence checks together with checking the changes on thermal unit commitment schedule, comes from the fact that if the convergence criterion for the discrete (mixed-integer programming) problem is only no changes on thermal unit commitment schedule, the fixed unit commitment schedule may not ensure a feasible yet optimal or near-optimal solution. However, in case the feasibility is ensured, the fixed thermal unit commitment schedule does save much time on performing dynamic programming programs.

The feasibility convergence checks set for the sub-gradient optimization of the discrete problem are as follows:

- $$\left| \frac{L(\lambda^i(k)) - L(\lambda^{i+1}(k))}{L(\lambda^i(k))} \right| \leq \epsilon_1$$

  to ensure the optimality of the dual problem.

- $$\sum_i P_{imin} + \sum_j P_{jmax} \leq P_D(k)$$

to ensure that adjustment in the generation dispatch will be possible for each time interval, where i belongs to the set of "on" thermal units and j belongs to the set of all hydro units at time interval k.

- $$\sum_i P_{imax} + \sum_j P_{jmax} \geq P_D(k)$$

  to ensure that there will be enough generation capacity to make possible the adjustment in generation dispatch at each time interval, where i belongs to the set of "on" thermal units and j belongs to the set of all hydro units at time interval k.

- $$\left| \frac{P_D(k) - \sum_{i \in I} P_T(i,k) - \sum_{j \in J} P_H(j,k)}{P_D(k)} \right| \leq \epsilon_2$$

  to ensure the optimality of the primal solution.

To ensure a near-optimal solution, another convergence criterion can be added to compare the dual cost values in the latest two iterations. This criterion should not be very tight; usually (0.5%-0.05%) is more than enough to save computation time. Alternatively, the convergence criterion can be set to check the changes in Lagrangian multipliers during the latest two iterations (say 0.5%).

Previous tests have shown that for purely hydroelectric generation scheduling, if the Lagrangian relaxation methodology is applied together with a piecewise linear hydro scheduling model, there will be some degree of infeasibility. This result has been presented in Chapter 6. For purely thermal unit commitment, after the thermal unit commitment schedule is fixed and feasible solutions are ensured, the thermal generation dispatch will have no duality gap at all. Similarly for hydrothermal coordination, an infeasibility will be created by the hydro subproblem scheduling using piecewise linear models. This problem will be further discussed later. To ensure near-optimality of the primal problem and feasibility, a convergence criterion is added to check the individual deviation between power demand and total generation at each time interval and if the deviation is too large, the optimization is continued.

The efficiency of the sub-gradient optimization method and the optimal value obtained by using the sub-gradient optimization method (as tested), are very much dependent upon the initial values of Lagrangian multipliers and the optimal value estimate of the dual function value used. For the short-term hydrothermal scheduling problem, either there may always be good initial values available or otherwise they can be computed heuristically, as discussed in Chapter 5, according to the problem characteristics.

The dual function maximization of the continuous problem can be carried out by many proposed methods, as discussed. The golden section line search and quadratic interpolation line search are used to find the optimal step length for maximizing the Lagrangian function.

The first comparison of hydrothermal scheduling with marginal price coordination and Lagrangian relaxation was performed on a set of hypothetical system data. The computational performance of the Lagrangian relaxation solution scheme was first tested on a hydrothermal system consisting of 8 hydro stations with 20 units in total. The data of the hydro subsystem are shown in Table 6.2 and Table 6.3, and the data of the thermal subsystem consisting of 12 generating units are given in Table 7.2 and Table 7.3 below. Note all the costs are in ($)s.

Table 7.2

Thermal generators data: number of units = 12

Unit	P_{min}	P_{max}	A_i	B_i	C_i	$C_{coldstart}$	T_{start}	$C_{shutdown}$
1	0.5	3.0	29.0	190.0	100.0	113.0	2.0	13.5
2	0.5	2.5	29.0	200.0	150.0	113.0	1.5	11.0
3	0.2	1.7	25.0	210.0	170.0	101.0	1.0	10.0
4	0.1	1.5	15.0	210.0	170.0	85.0	0.5	8.5
5	0.1	1.5	15.0	210.0	170.0	85.0	0.5	8.5
6	0.1	1.5	15.0	210.0	170.0	85.0	0.5	8.5
7	0.5	3.0	29.0	190.0	100.0	113.0	2.0	13.5
8	0.5	2.5	29.0	200.0	150.0	113.0	1.5	11.0
9	0.2	1.7	25.0	210.0	170.0	101.0	1.0	10.0
10	0.1	1.5	15.0	210.0	170.0	85.0	0.5	8.5
11	0.1	1.5	15.0	210.0	170.0	85.0	0.5	8.5
12	0.1	1.5	15.0	210.0	170.0	85.0	0.5	8.5

Table 7.3

Thermal generators data: number of units = 12 (continued)

Unit	T_{minup}	$T_{mindown}$	Status	T_{change}	I_{ramp}	D_{ramp}	G_{init}
1	3.0	3.0	1	06/02/1985.23:00:00	0.040	0.040	0.5
2	3.0	3.0	0	06/02/1985.23:00:00	0.030	0.030	0.0
3	2.0	2.0	1	06/02/1985.23:00:00	0.014	0.014	0.2
4	1.0	1.0	1	06/02/1985.23:00:00	0.010	0.010	0.1
5	1.0	1.0	1	06/02/1985.23:00:00	0.010	0.010	0.1
6	1.0	1.0	1	06/02/1985.23:00:00	0.010	0.010	0.1
7	3.0	3.0	1	06/02/1985.23:00:00	0.040	0.040	0.5
8	3.0	3.0	1	06/02/1985.23:00:00	0.030	0.030	0.5
9	2.0	2.0	1	06/02/1985.23:00:00	0.014	0.014	0.2
10	1.0	1.0	1	06/02/1985.23:00:00	0.010	0.010	0.1
11	1.0	1.0	1	06/02/1985.23:00:00	0.010	0.010	0.5
12	1.0	1.0	0	06/02/1985.23:00:00	0.010	0.010	0.0

The per unit base of the test system is chosen to be 100.00 (MW). The total thermal generation capacity is then 23.40 ($P.U.$) and it is assumed that no units must be "on" or must be "off". The total hydroelectric capacity is 19.60 ($P.U.$). Thus the ratio of generation capacity is 45.58% hydro and 54.42% thermal. A 24 hour load forecast was constructed as in Table 7.4 below.

Table 7.4 Load prediction data Created at: 07/02/1985.23:00:00		
Interval no.	Absolute time	Load demand (P.U. value)
1	07/02/1985.23:00:00	18.7000
2	08/02/1985.00:00:00	18.1000
3	08/02/1985.01:00:00	18.0000
4	08/02/1985.02:00:00	18.0000
5	08/02/1985.03:00:00	18.2000
6	08/02/1985.04:00:00	18.8000
7	08/02/1985.05:00:00	20.3000
8	08/02/1985.06:00:00	23.0000
9	08/02/1985.07:00:00	25.6000
10	08/02/1985.08:00:00	25.8000
11	08/02/1985.09:00:00	25.6000
12	08/02/1985.10:00:00	25.4000
13	08/02/1985.11:00:00	25.0000
14	08/02/1985.12:00:00	24.8000
15	08/02/1985.13:00:00	24.4000
16	08/02/1985.14:00:00	24.6000
17	08/02/1985.15:00:00	24.6000
18	08/02/1985.16:00:00	24.4000
19	08/02/1985.17:00:00	24.8000
20	08/02/1985.18:00:00	24.6000
21	08/02/1985.19:00:00	23.8000
22	08/02/1985.20:00:00	23.0000
23	08/02/1985.21:00:00	21.8000
24	08/02/1985.22:00:00	20.2000

A generation schedule for this 24 hour scheduling period was thus obtained for the above test system data. Three approaches for maximizing the dual function were first tested, namely, the sub-gradient optimization method with the dual optimal value estimate, the steepest ascent method and the conventional DFP quasi-Newton method. Tests have shown that the DFP quasi-Newton method is certainly more efficient than the steepest ascent algorithm due largely to its super-linear convergence rate, and the result obtained by the DFP method is certainly slightly better and stable. However, since the conventional DFP quasi-Newton method needs to perform a line search to maximize the dual function value along the chosen conjugate direction, the computation of the line search at each dual problem iteration is very time-consuming, and the computational burden is very heavy.

To take advantage of the super-linear convergent rate of the conventional DFP quasi-Newton method, while taking into the consideration the fact that the dual function is not everywhere differentiable, a modified sub-gradient optimization method is proposed here for the first time which combines the conventional sub-gradient optimization method with the DFP quasi-Newton ascending direction. To avoid the difficulty of obtaining a good estimated value for the dual optimum, the formulae

$$\alpha^i = \frac{1}{a + b * i}$$

is used. A similar method can be found as a "variable metric" method proposed by Aoki.[6.],[7.] The major differences between the "variable metric" method proposed and the modified method here are in the choice of the updating formulae of the inverse Hessian matrix approximation.

The proposed modified quasi-Newton sub-gradient optimization algorithm can be described as follows:

1. Set iteration number $k \Leftarrow 0$. Calculate the initial function value.
2. Initialize the inverse Hessian matrix $\mathbf{H}$ as the unit matrix $\mathbf{1}$.
3. Compute the initial gradient vector and initial search direction

$$\mathbf{g}^k = \mathbf{g}(\lambda^k)$$

4. Compute $\mathbf{p}^k = -\mathbf{H}^k \cdot \mathbf{g}^k$.

5. Save the old function value and the old gradient. Set

$$\lambda^{k+1} = \lambda^k + \alpha^k \mathbf{p}^k$$

with

$$\alpha^i = \frac{1}{a + b * i}$$

6. Compute the new function value and gradient $\mathbf{g}(\lambda^{k+1})$.
7. Compute $\mathbf{s}^k$, $\mathbf{y}^k$ from

$$\mathbf{s}^k = \alpha^k \mathbf{p}^k$$

and

$$\mathbf{y}^k = \mathbf{g}^{k+1} - \mathbf{g}^k$$

8. If convergence is considered to be attained, set the optimal solution $\lambda^* = \lambda^{k+1}$, terminate the process.
9. Compute $\mathbf{z}^k$ from

$$\mathbf{z}^k = \mathbf{H}^k \cdot \mathbf{y}^k$$

10. Compute the new $\mathbf{H}^{k+1}$ from

$$\mathbf{H}^{k+1} = \mathbf{H}^k + \gamma\left(\frac{\mathbf{s}^k \cdot \mathbf{s}^{k^T}}{\mathbf{s}^{k^T} \cdot \mathbf{y}^k} - \frac{\mathbf{z}^k \cdot \mathbf{z}^{k^T}}{\mathbf{y}^{k^T} \cdot \mathbf{z}^k}\right)$$

where γ is a positive scalar which satisfies $0 < \gamma \leq 1$.
11. Set iteration $k \Leftarrow k + 1$, go to step 3.

The comparison of these four approaches in term of approximate CPU time, and the total minimum production cost obtained can be seen in Table 7.5. The load demand curve and the final hydro and thermal generation schedule are given for these approaches respectively in Figure 7.1, Figure 7.2, Figure 7.3 and Figure 7.4. For the steepest ascent method, the conventional DFP quasi-Newton method and the modified DFP quasi-Newton sub-gradient optimization method in maximizing the dual function of the continuous problem, the dual cost change, the primal cost change, the difference between the dual cost change and the primal cost, and the maximum individual power balance deviation are presented in Table 7.6, Table 7.7 and Table 7.8 respectively. The primal and dual cost changes of these four algorithms are also shown in Figure 7.5, Figure 7.6, Figure 7.7 and Figure 7.8. The Lagrangian multiplier changes in solving the continuous the conventional and the modified DFP method are shown in Figure 7.9 to Figure 7.14. Tests show that despite the small difference between the primal and dual cost value in the sub-gradient optimization approach, the feasible hydro and thermal generation schedules produced are actually not as good as those of the other three approaches. The reason for this is that, even though the total difference between the infeasible primal and the dual is small, the individual power and generation deviations are still substantial, thus the actual adjusted hydro and thermal generation schedule is not as smooth. Figure 7.9 to Figure 7.14 show the rapid disappearance of the oscillations in the Lagrangian multipliers.

Table 7.5

Comparisons of algorithms

Algorithm	Iter. No	CPU time (secs.)	Dual cost	Minimum cost
Sub-gradient	76	180.0	75039.8	75124.70
Steepest ascent	29	411.01	74954.4	75101.01
DFP	17	221.90	74979.8	75101.66
Modified DFP	31	90.50	74981.1	75106.81

The results show that all the feasible solutions obtained are not far from the dual optimal value, so the duality gap between the primal solution and the dual solution is actually very small, only around 0.11%- 0.2%. Tests also show that the modified DFP method is very efficient compared with the steepest ascent method and the conventional DFP quasi-Newton method, while robust and stable compared with the sub-gradient optimization algorithm. The schedule obtained by the modified DFP method is satisfactory while the result obtained by the sub-gradient optimization algorithm may have a less smooth hydro and thermal generation schedule. However, following a suitable choice of control parameters, such as a good estimate of the dual optimal value, the sub-gradient optimization, which avoids the heavy computational burden of a line search, may offer a good alternative to the modified DFP method, and is certainly better than the steepest ascent and the conventional DFP algorithm.

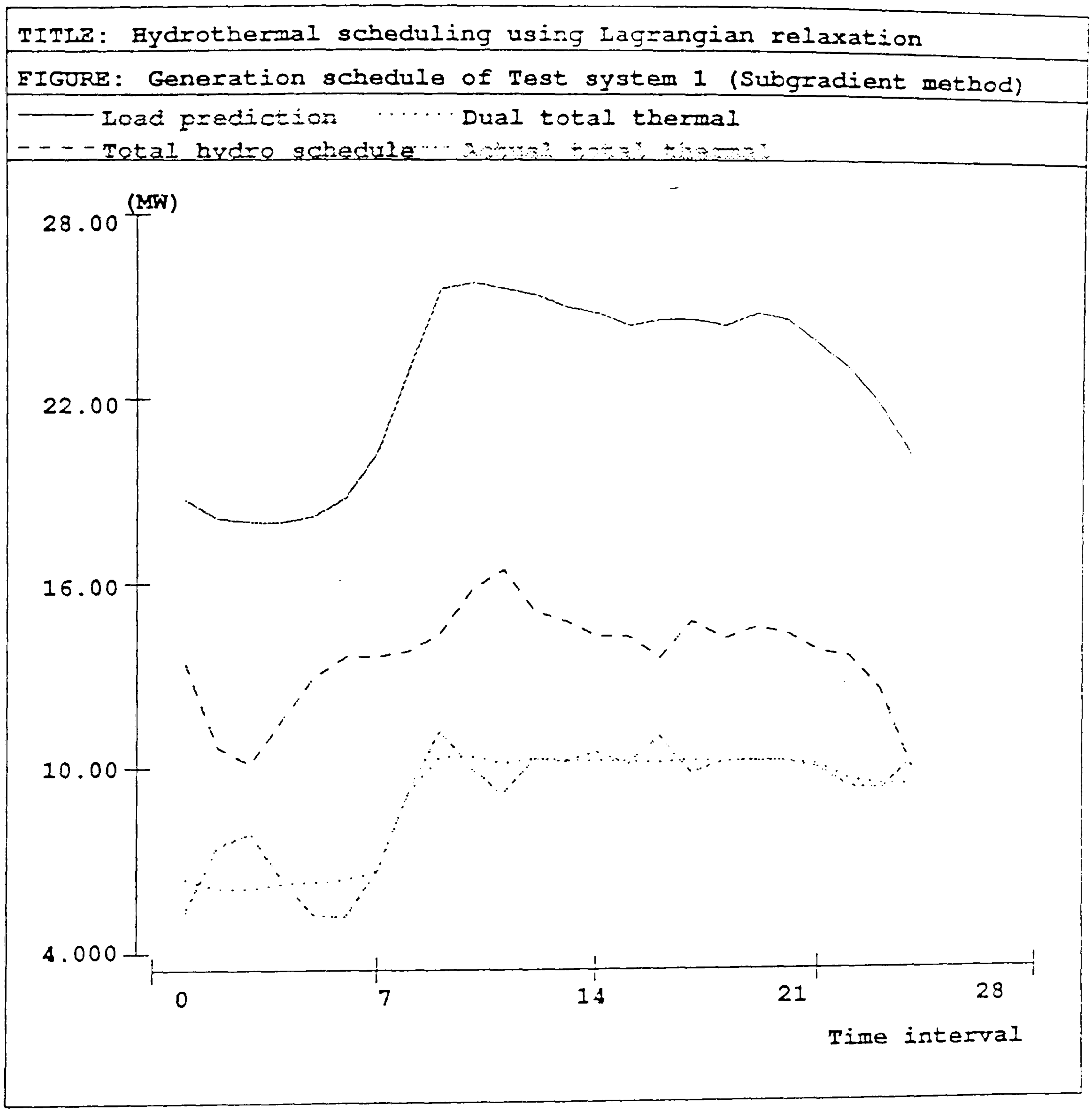
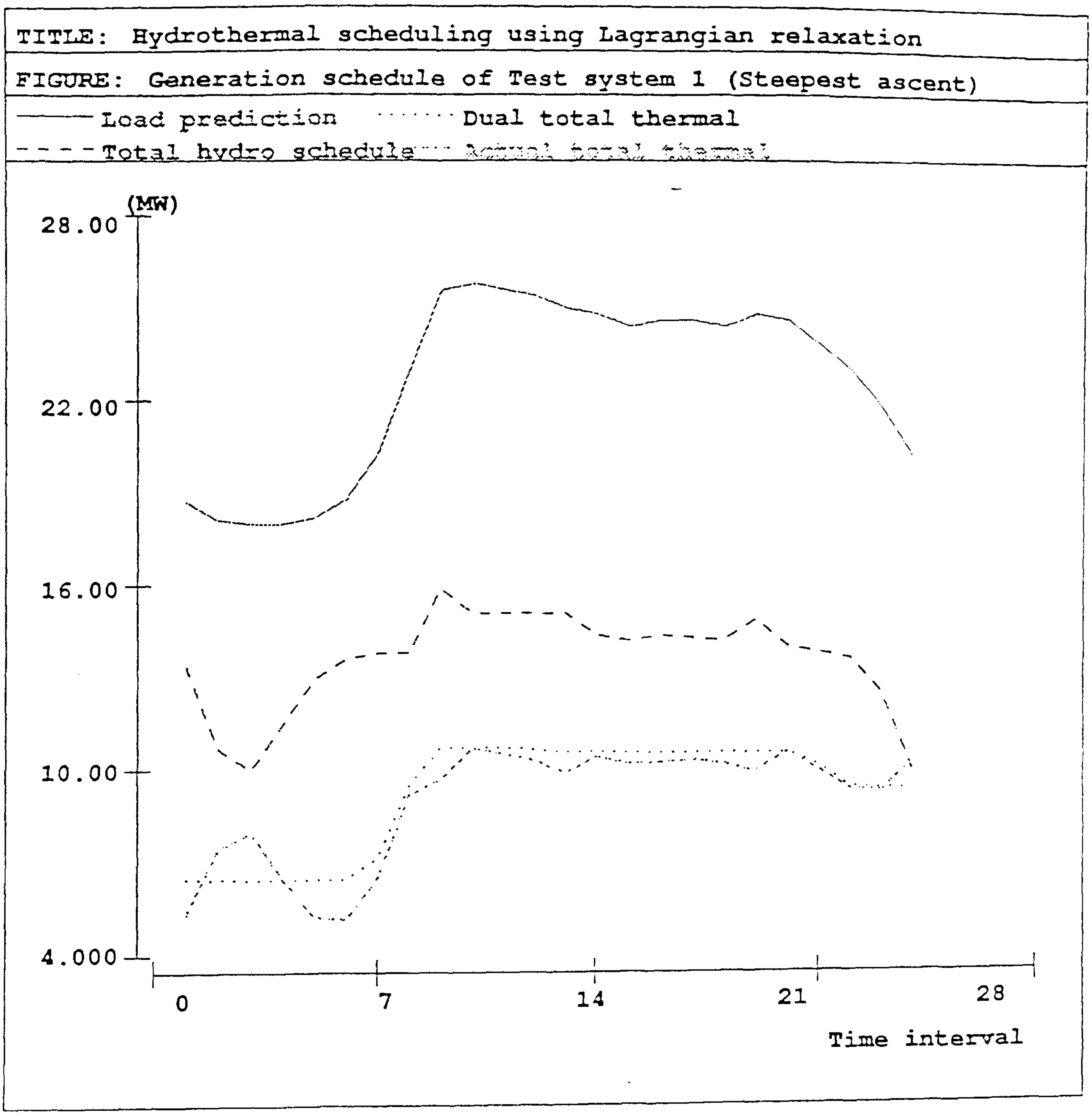

TITLE: Hydrothermal scheduling using Lagrangian relaxation
FIGURE: Generation schedule of Test system 1 (Subgradient method)
Load prediction
Dual total thermal
Total hydro schedule
(MW)
28.00
22.00
16.00
10.00
4.000
0
7
14
21
28
Time interval

Figure 7.1



TITLE: Hydrothermal scheduling using Lagrangian relaxation
FIGURE: Generation schedule of Test system 1 (Steepest ascent)
Load prediction
Dual total thermal
Total hydro schedule
(MW)
28.00
22.00
16.00
10.00
4.000
0
7
14
21
28
Time interval

Figure 7.2

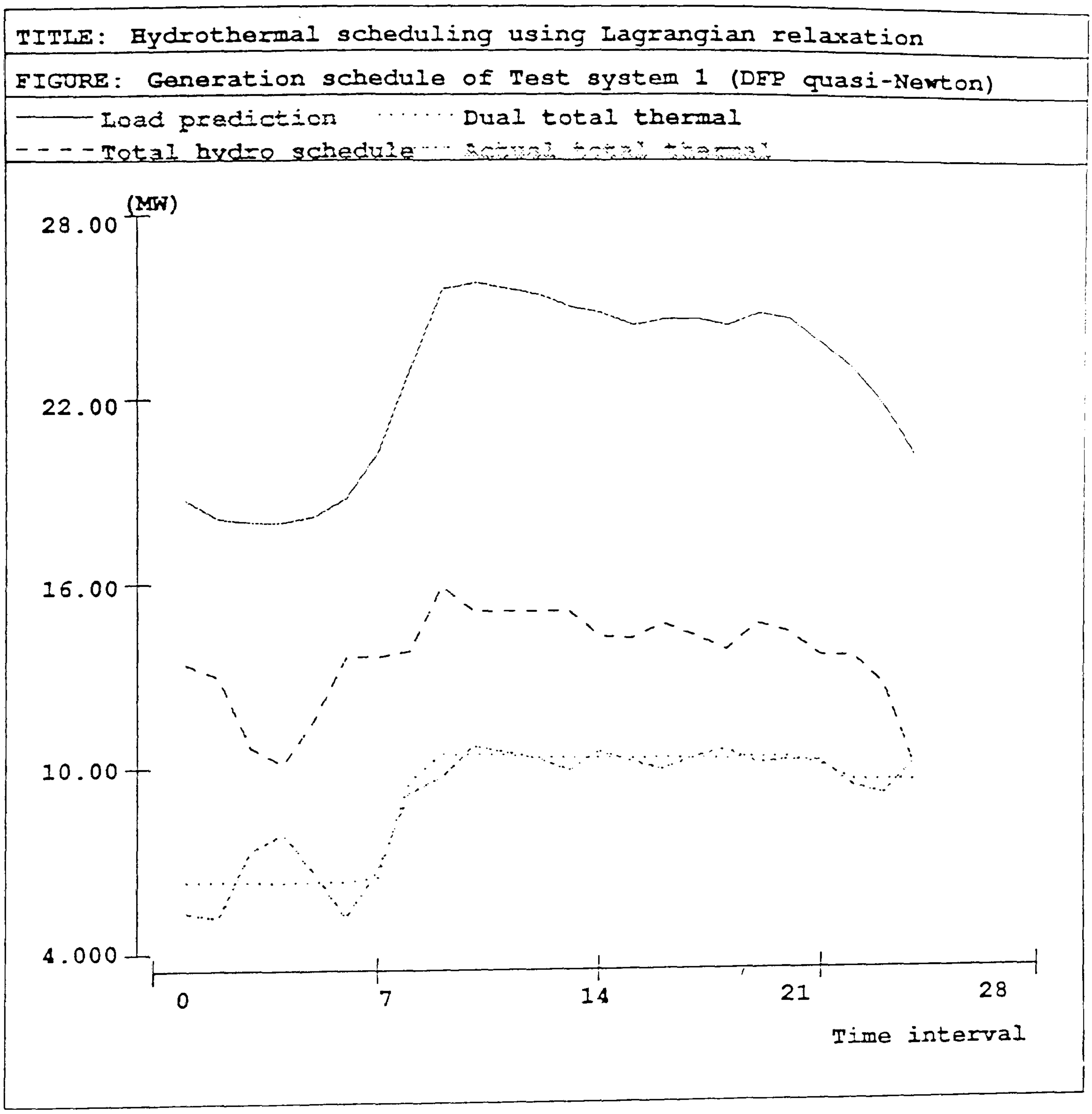

TITLE: Hydrothermal scheduling using Lagrangian relaxation
FIGURE: Generation schedule of Test system 1 (DFP quasi-Newton)
Load prediction
Dual total thermal
Total hydro schedule
(MW)
28.00
22.00
16.00
10.00
4.000
0
7
14
21
28
Time interval

Figure 7.3

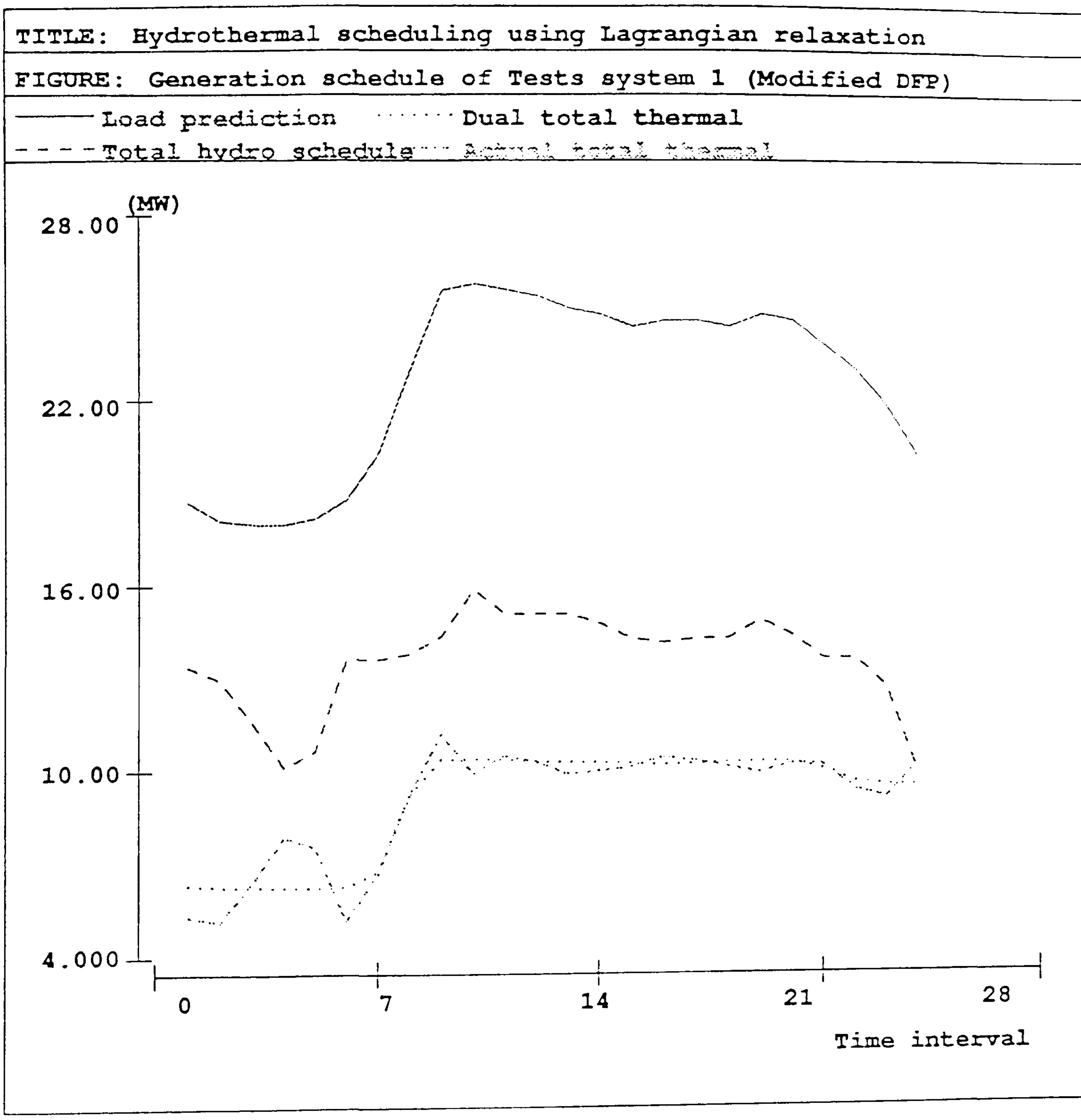

TITLE: Hydrothermal scheduling using Lagrangian relaxation
FIGURE: Generation schedule of Tests system 1 (Modified DFP)
Load prediction
Dual total thermal
Total hydro schedule
Actual total thermal
(MW)
28.00
22.00
16.00
10.00
4.000
0
7
14
21
28
Time interval

Figure 7.4

Table 7.6

The steepest ascent method test

Iter. No.	Dual	Primal	$\sum \lambda_i g_i(x)$	$\mid MAX g_i(x) \mid$ (%)	No. $\mid (g_i(x) \mid > 10(\%)$	No. $\mid (g_i(x) \mid < 5(\%)$	No. $\mid (g_i(x) \mid < 1(\%)$
1	73460.9	85535.8	-12074.9	40	10	12	0
2	74461.2	82220.1	-7758.9	20	7	10	1
3	74690.7	81059.4	-6368.7	16	5	15	4
4	74772.6	80053.3	-5280.7	17	3	13	7
5	74855.3	79303.8	-4448.5	9	0	15	6
6	74897.4	78747.2	-3849.8	16	1	18	5
7	74916.2	78441.4	-3525.1	10	1	17	8
8	74920.6	78296.4	-3375.8	16	1	18	5
9	74932.5	78184.8	-3252.3	8	0	20	4
10	74936.2	78052.0	-3115.9	16	1	20	7
11	74940.5	77975.9	-3035.4	16	1	17	6
12	74943.2	77930.6	-2987.3	15	1	20	6
13	74946.0	77893.0	-2947.0	15	1	21	9
14	74946.9	77872.0	-2925.1	15	1	19	4
15	74946.9	77849.5	-2902.5	15	1	18	9
16	74947.2	77833.7	-2886.5	15	1	20	4

Table 7.6 (continued)

The steepest ascent method test

Iter. No.	Dual	Primal	$\sum \lambda_i g_i(x)$	$\mid MAX g_i(x) \mid$ (%)	No. $\mid (g_i(x) \mid > 10$(%)	No. $\mid (g_i(x) \mid < 5$(%)	No. $\mid (g_i(x) \mid < 1$(%)
17	74948.7	77814.5	-2865.8	16	1	18	7
18	74948.7	77798.5	-2849.8	15	1	18	7
19	74949.4	77778.4	-2829.0	14	1	20	9
20	74950.2	77760.1	-2809.9	12	1	18	5
21	74950.2	77745.3	-2795.1	15	3	16	5
22	74950.6	77730.1	-2779.4	14	1	20	9
23	74951.4	77712.6	-2761.2	15	1	19	5
24	74951.5	77697.4	-2746.0	12	1	19	7
25	74951.5	77668.9	-2717.4	15	1	18	6
26	74952.2	77646.5	-2694.3	12	1	17	5
27	74953.4	77621.6	-2668.3	14	1	18	7
28	74953.4	77601.5	-2648.2	16	1	19	9
29	74954.4	77582.6	-2628.2	8	0	19	9

Table 7.7

The DFP quasi-Newton method test

Iter. No.	Dual	Primal	$\sum \lambda_i g_i(x)$	$\mid MAXg_i(x) \mid (\%)$	No. $\mid (g_i(x) \mid > 10(\%)$	No. $\mid (g_i(x) \mid < 5(\%)$	No. $\mid (g_i(x) \mid < 1(\%)$
1	73460.9	85535.8	-12074.9	40	10	12	0
2	74461.2	82220.1	-7758.9	20	7	10	1
3	74746.0	79863.8	-5117.8	17	8	11	1
4	74835.2	78641.9	-3806.7	15	3	11	4
5	74874.0	78898.8	-4024.8	11	1	15	4
6	74888.7	78535.8	-3647.1	16	2	18	6
7	74922.8	77841.1	-2918.3	16	1	16	5
8	74939.9	77583.3	-2643.4	9	0	16	5
9	74948.1	77254.4	-2306.3	12	1	16	6
10	74955.9	76892.0	-1936.1	14	3	16	6
11	74959.5	76697.7	-1738.2	15	3	16	8
12	74966.0	76252.7	-1286.6	15	3	18	11
13	74970.8	76019.5	-1048.7	15	2	18	7
14	74974.0	75882.3	-908.4	15	1	19	4
15	74978.5	75736.2	-757.7	11	1	18	11
16	74979.3	75712.8	-733.5	11	1	17	10
17	74979.8	75687.7	-707.9	8	0	19	9

Table 7.8

The modified DFP quasi-Newton method test

Iter. No.	Dual	Primal	$\sum \lambda_i g_i(x)$	$\mid MAXg_i(x) \mid (\%)$	No. $\mid (g_i(x) \mid > 10(\%)$	No. $\mid (g_i(x) \mid < 5(\%)$	No. $\mid (g_i(x) \mid < 1(\%)$
1	73094.5	99447.3	-26352.9	47	7	6	1
2	73887.8	89324.2	-15436.5	35	11	7	0
3	74173.3	90856.7	-16683.4	21	6	9	1
4	74347.4	88648.2	-14300.8	23	6	10	2
5	74603.5	79827.2	-5223.7	36	7	10	4
6	74720.7	80072.2	-5351.5	17	7	13	2
7	74801.9	79285.3	-4483.4	17	7	16	2
8	74857.7	78295.9	-3438.2	17	4	15	4
9	74877.0	77679.5	-2802.5	16	5	16	5
10	74891.7	77254.0	-2362.4	15	5	17	4
11	74903.8	76705.0	-1801.2	15	4	16	5
12	74916.1	76229.9	-1313.8	15	4	17	7
13	74928.7	75638.1	-709.4	15	3	17	7
14	74939.5	75161.3	-221.9	14	3	15	6
15	74946.3	74842.0	104.4	11	2	14	6
16	74952.2	74617.5	334.8	11	2	15	6

Table 7.8 (continued)

The modified DFP quasi-Newton method test

Iter. No.	Dual	Primal	$\sum \lambda_i g_i(x)$	$\mid MAX g_i(x) \mid (\%)$	No. $\mid (g_i(x) \mid > 10(\%)$	No. $\mid (g_i(x) \mid < 5(\%)$	No. $\mid (g_i(x) \mid < 1(\%)$
17	74956.2	74416.6	539.7	11	4	12	7
18	74961.3	74334.0	627.3	14	4	15	7
19	74965.3	74333.4	631.9	13	4	17	7
20	74967.6	74371.6	596.0	11	1	20	6
21	74970.2	74457.9	512.3	10	1	20	6
22	74974.3	74659.5	314.8	10	1	19	9
23	74975.8	74801.1	174.7	13	2	17	8
24	74976.6	74913.7	62.9	13	2	19	9
25	74977.3	74970.1	07.2	13	1	20	8
26	74977.9	75054.7	-76.8	13	4	15	8
27	74978.4	75096.8	-118.3	13	2	18	9
28	74979.0	75137.4	-158.4	13	2	18	9
29	74979.8	75183.1	-203.2	14	2	18	9
30	74980.5	75241.5	-261.0	10	1	18	9
31	74981.0	75274.9	-293.8	9	0	19	10

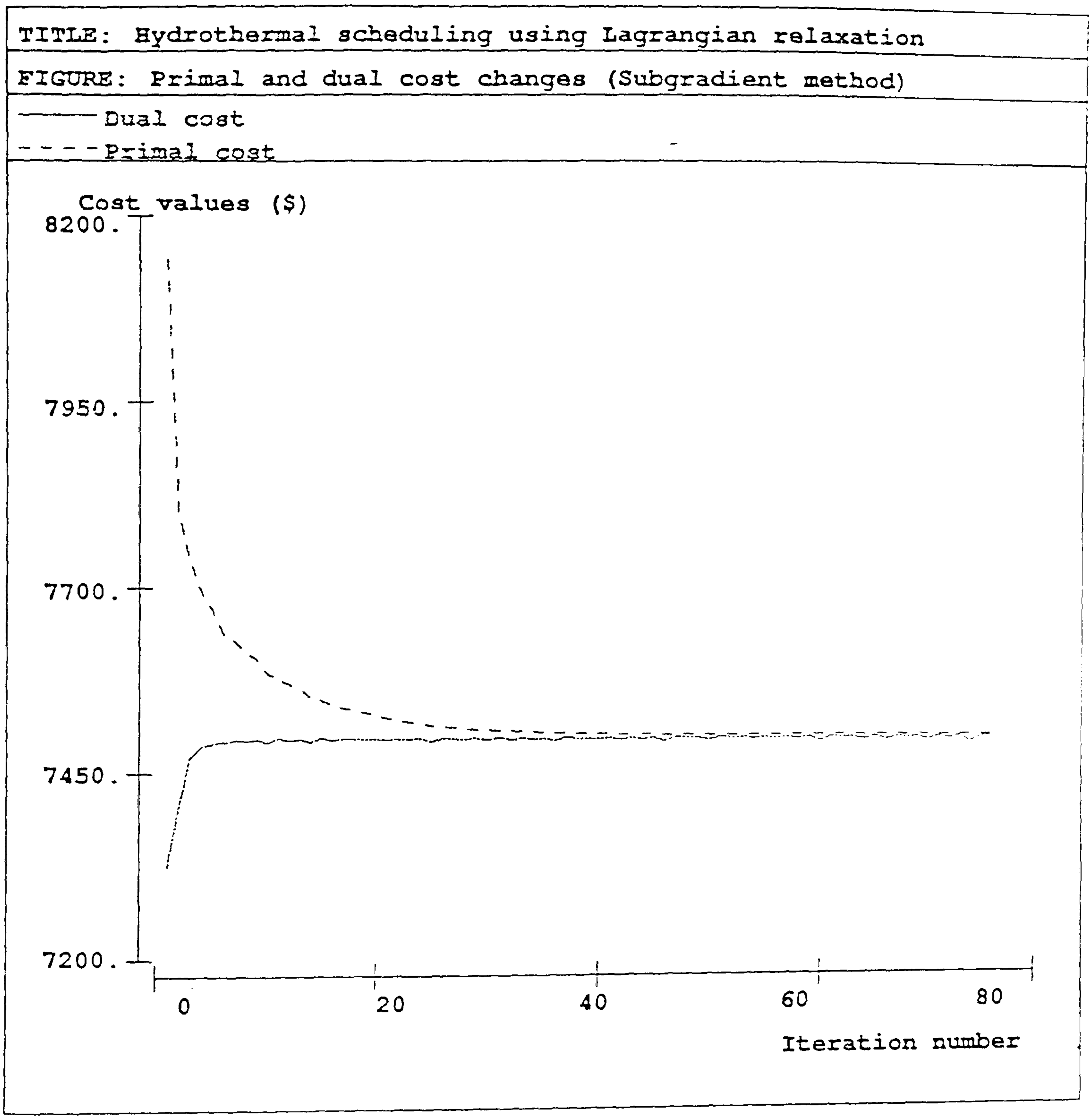

TITLE: Hydrothermal scheduling using Lagrangian relaxation
FIGURE: Primal and dual cost changes (Subgradient method)
Dual cost
Primal cost
Cost values ($)
8200.
7950.
7700.
7450.
7200.
0
20
40
60
80
Iteration number

Figure 7.5

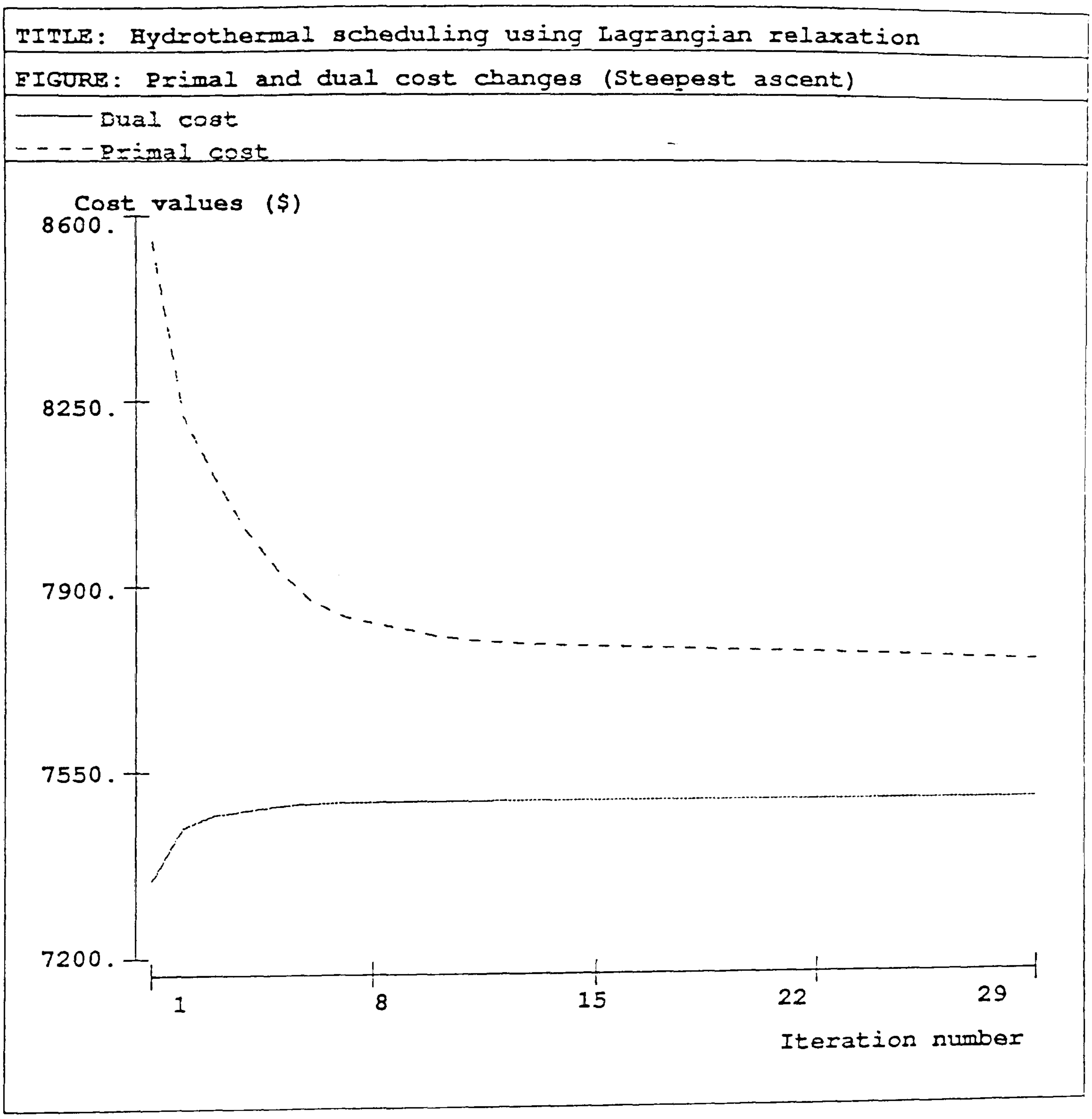

TITLE: Hydrothermal scheduling using Lagrangian relaxation
FIGURE: Primal and dual cost changes (Steepest ascent)
Dual cost
Primal cost
Cost values ($)
8600.
8250.
7900.
7550.
7200.
1
8
15
22
29
Iteration number

Figure 7.6

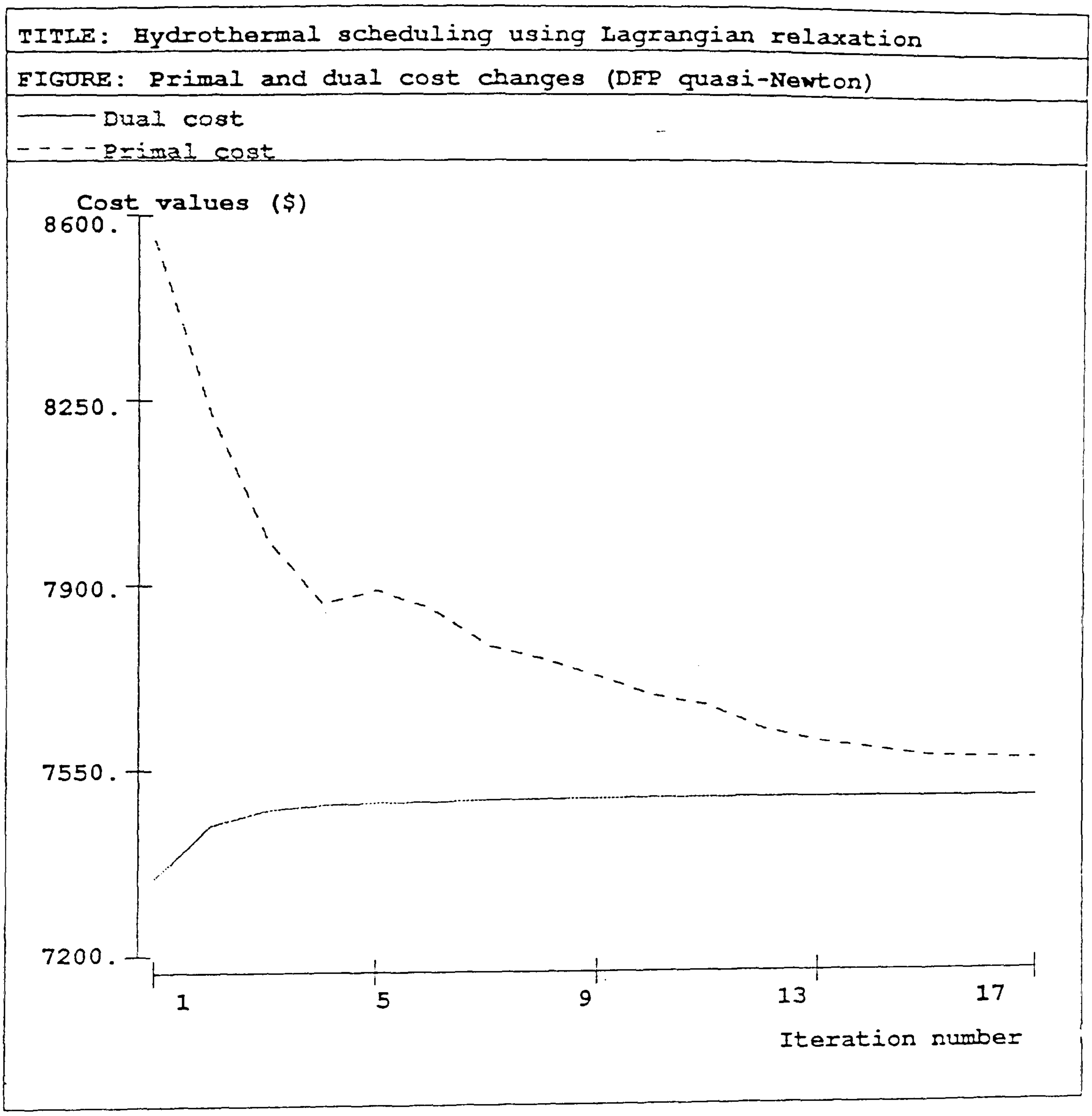

TITLE: Hydrothermal scheduling using Lagrangian relaxation
FIGURE: Primal and dual cost changes (DFP quasi-Newton)
Dual cost
Primal cost
Cost values ($)
8600.
8250.
7900.
7550.
7200.
1
5
9
13
17
Iteration number

Figure 7.7

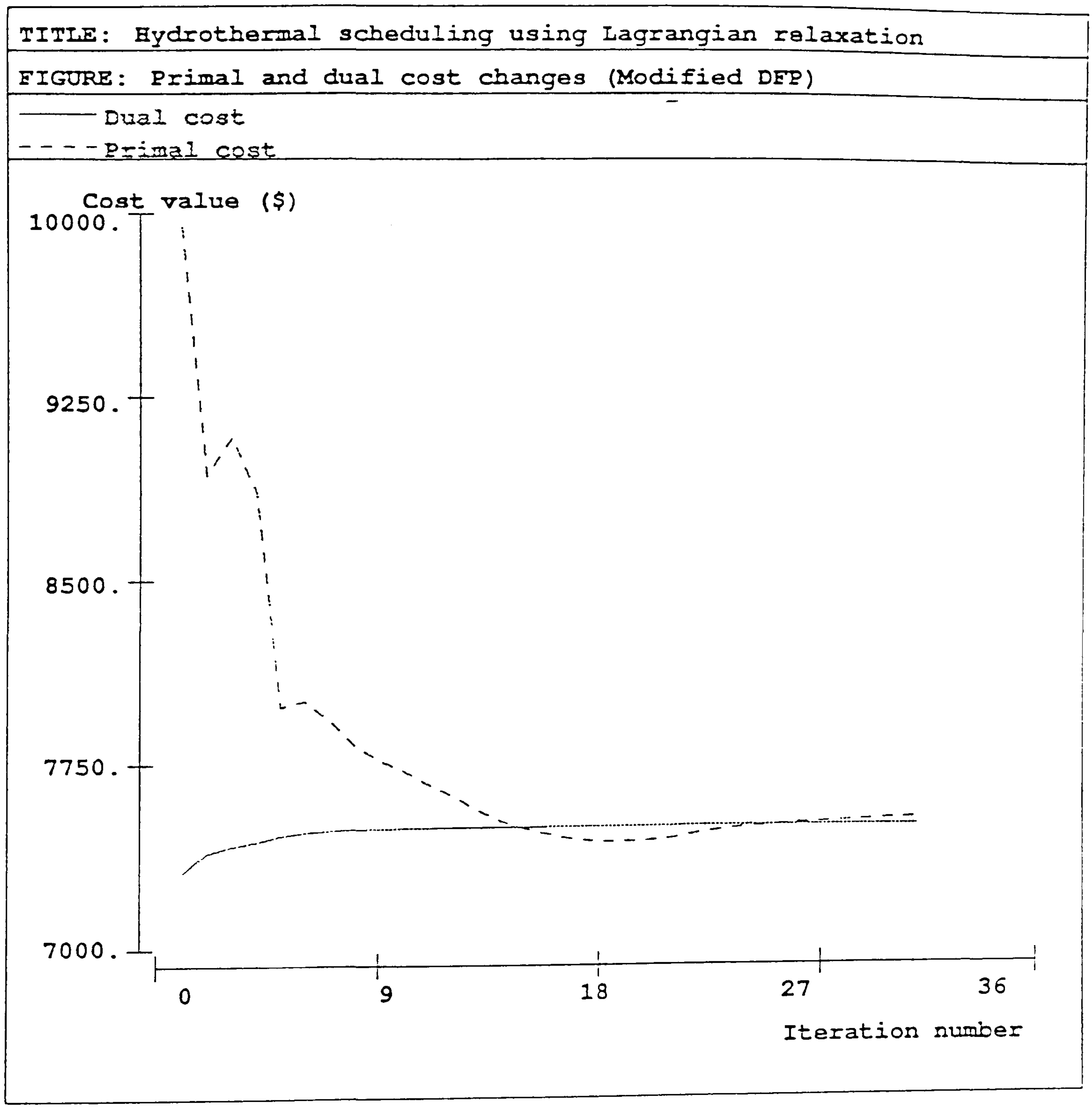

TITLE: Hydrothermal scheduling using Lagrangian relaxation
FIGURE: Primal and dual cost changes (Modified DFP)
Dual cost
Primal cost
Cost value ($)
10000.
9250.
8500.
7750.
7000.
0
9
18
27
36
Iteration number

Figure 7.8

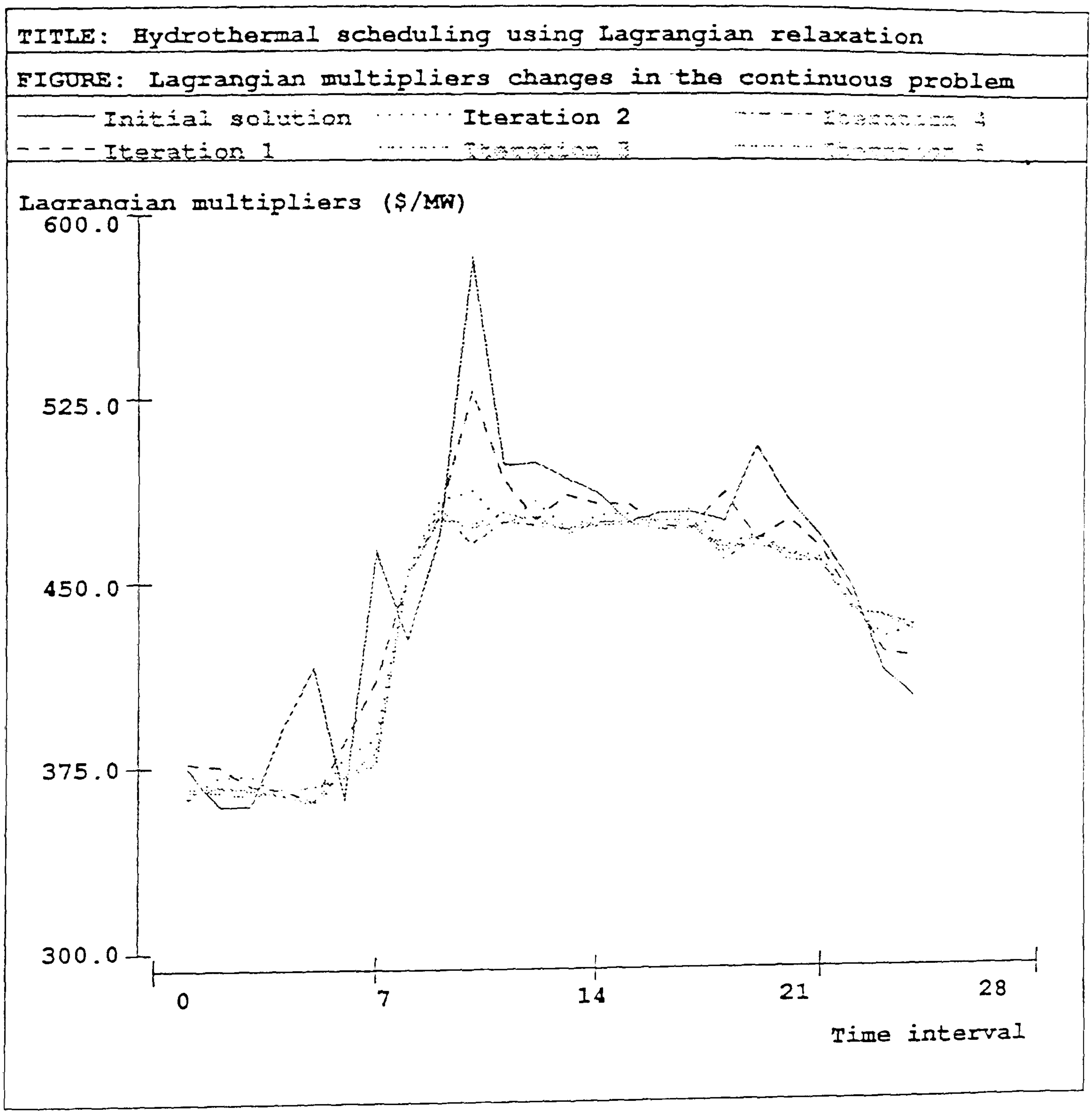

TITLE: Hydrothermal scheduling using Lagrangian relaxation
FIGURE: Lagrangian multipliers changes in the continuous problem
Initial solution
Iteration 2
Iteration 1
Lagrangian multipliers ($/MW)
600.0
525.0
450.0
375.0
300.0
0
7
14
21
28
Time interval

Figure 7.9

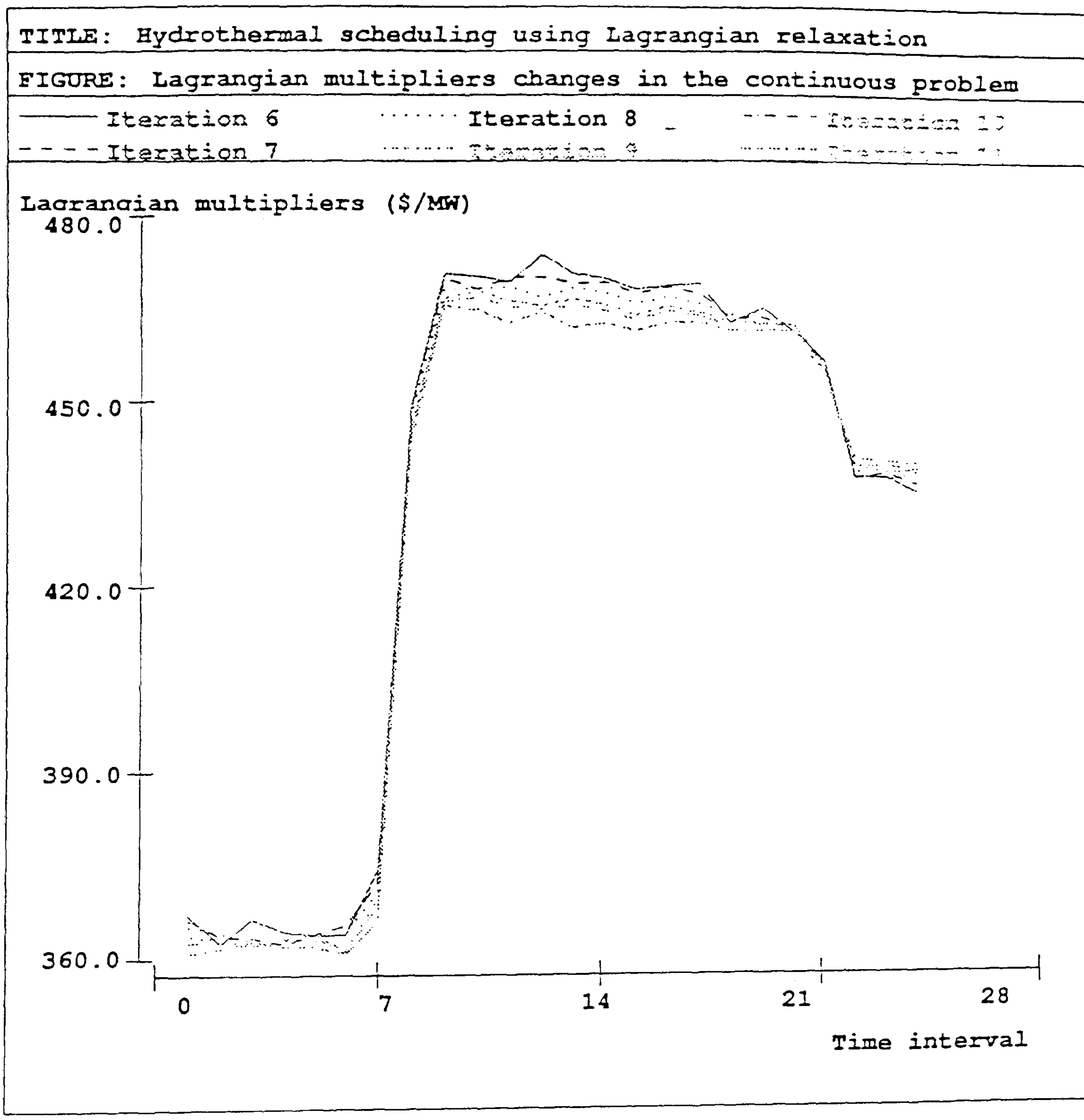

TITLE: Hydrothermal scheduling using Lagrangian relaxation
FIGURE: Lagrangian multipliers changes in the continuous problem
Iteration 6
Iteration 7
Iteration 8
Lagrangian multipliers ($/MW)
480.0
450.0
420.0
390.0
360.0
0
7
14
21
28
Time interval

Figure 7.10

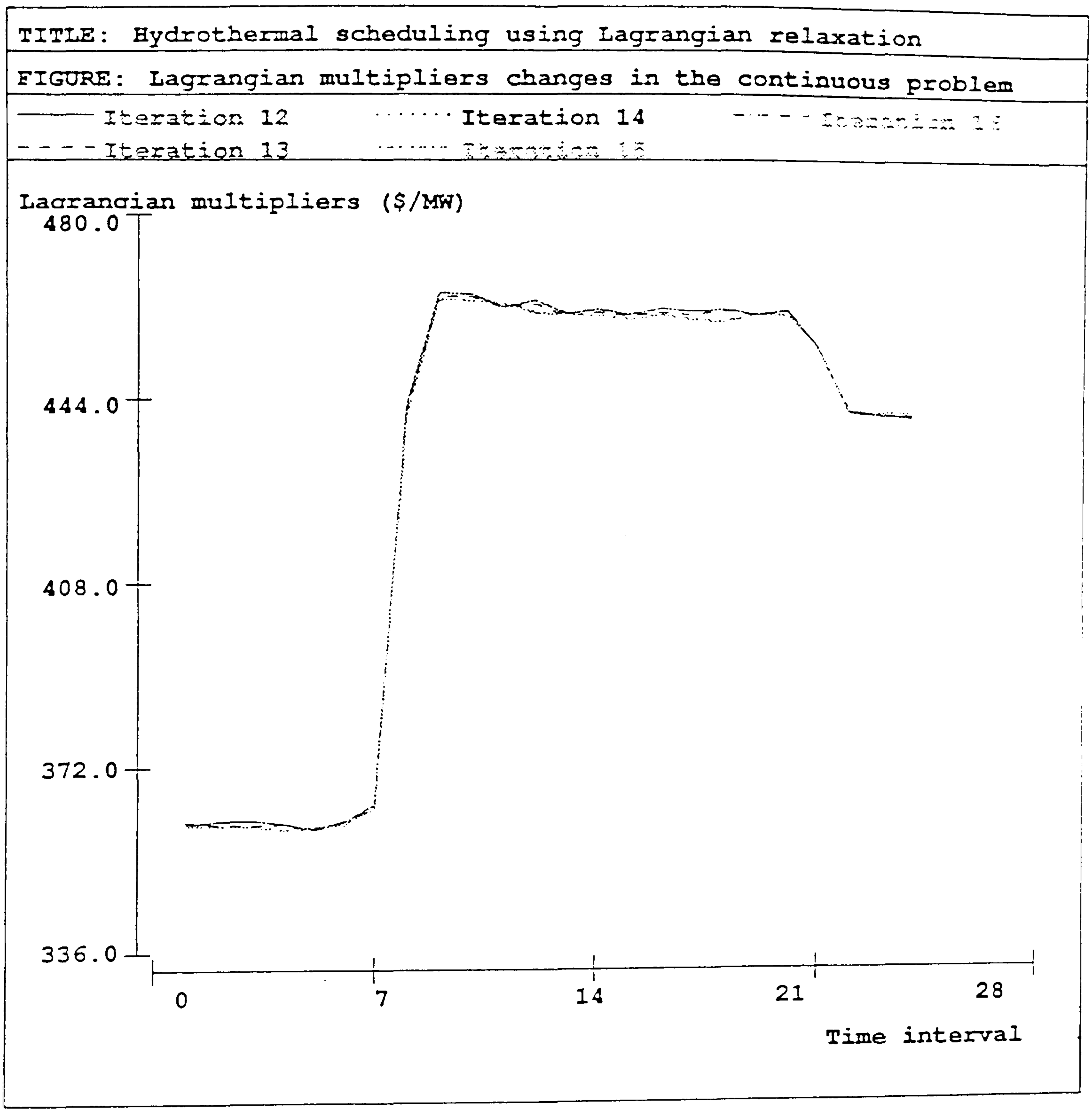

TITLE: Hydrothermal scheduling using Lagrangian relaxation
FIGURE: Lagrangian multipliers changes in the continuous problem
Iteration 12
Iteration 14
Iteration 13
Lagrangian multipliers ($/MW)
480.0
444.0
408.0
372.0
336.0
0
7
14
21
28
Time interval

Figure 7.11

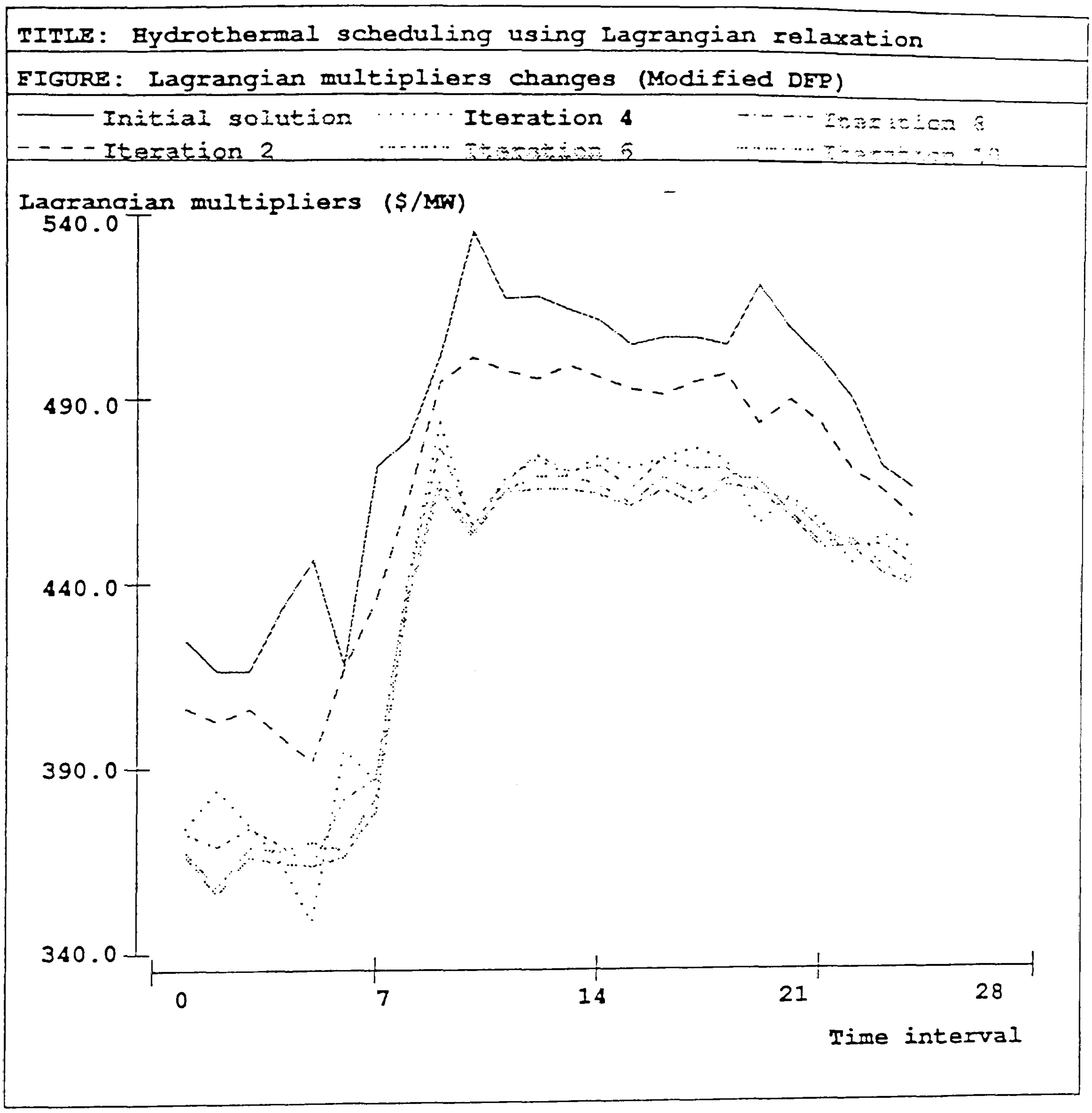

TITLE: Hydrothermal scheduling using Lagrangian relaxation
FIGURE: Lagrangian multipliers changes (Modified DFP)
Initial solution
Iteration 4
Iteration 2
Lagrangian multipliers ($/MW)
540.0
490.0
440.0
390.0
340.0
0
7
14
21
28
Time interval

Figure 7.12

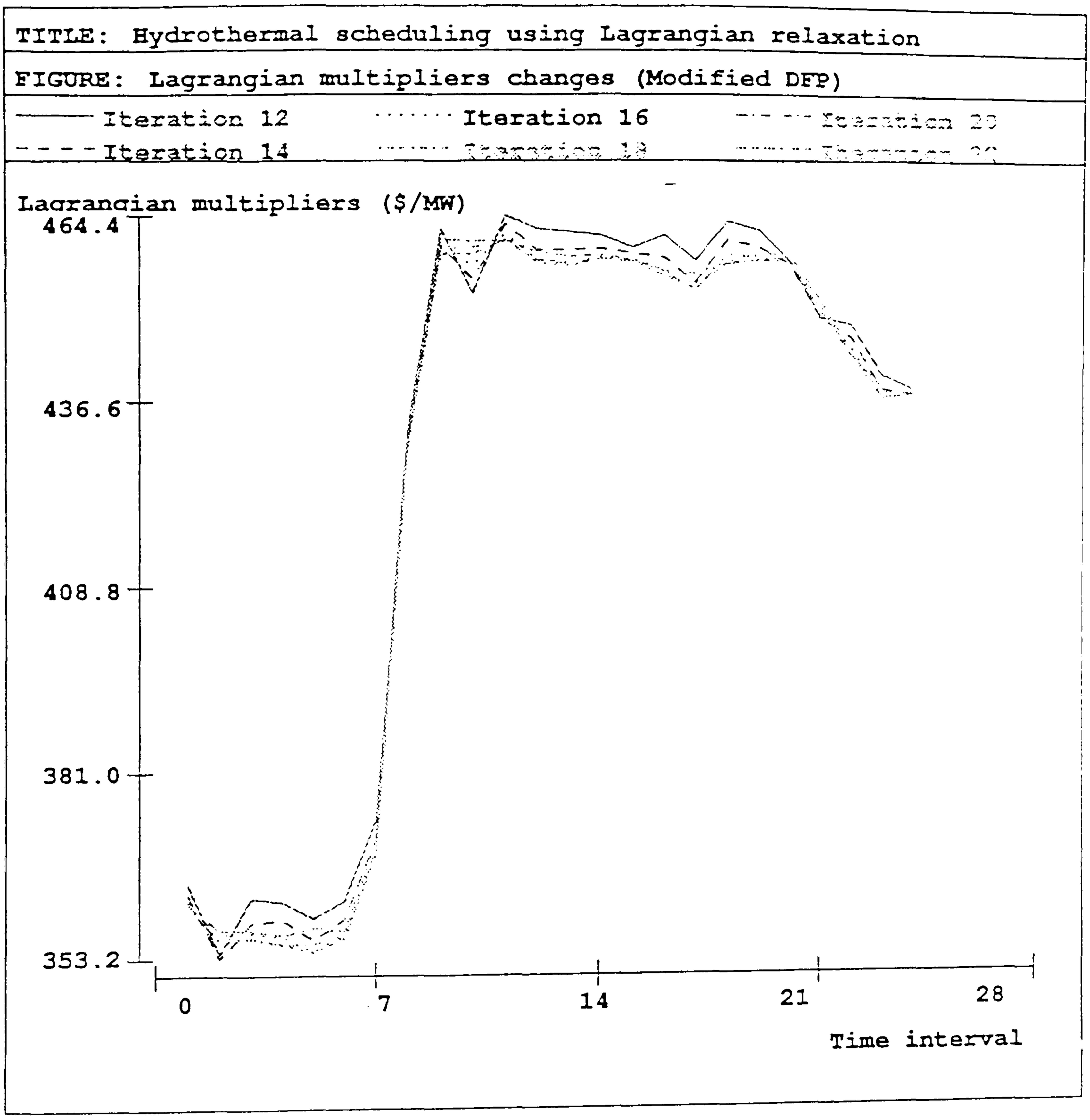

TITLE: Hydrothermal scheduling using Lagrangian relaxation
FIGURE: Lagrangian multipliers changes (Modified DFP)
Iteration 12
Iteration 16
Iteration 14
Lagrangian multipliers ($/MW)
464.4
436.6
408.8
381.0
353.2
0
7
14
21
28
Time interval

Figure 7.13

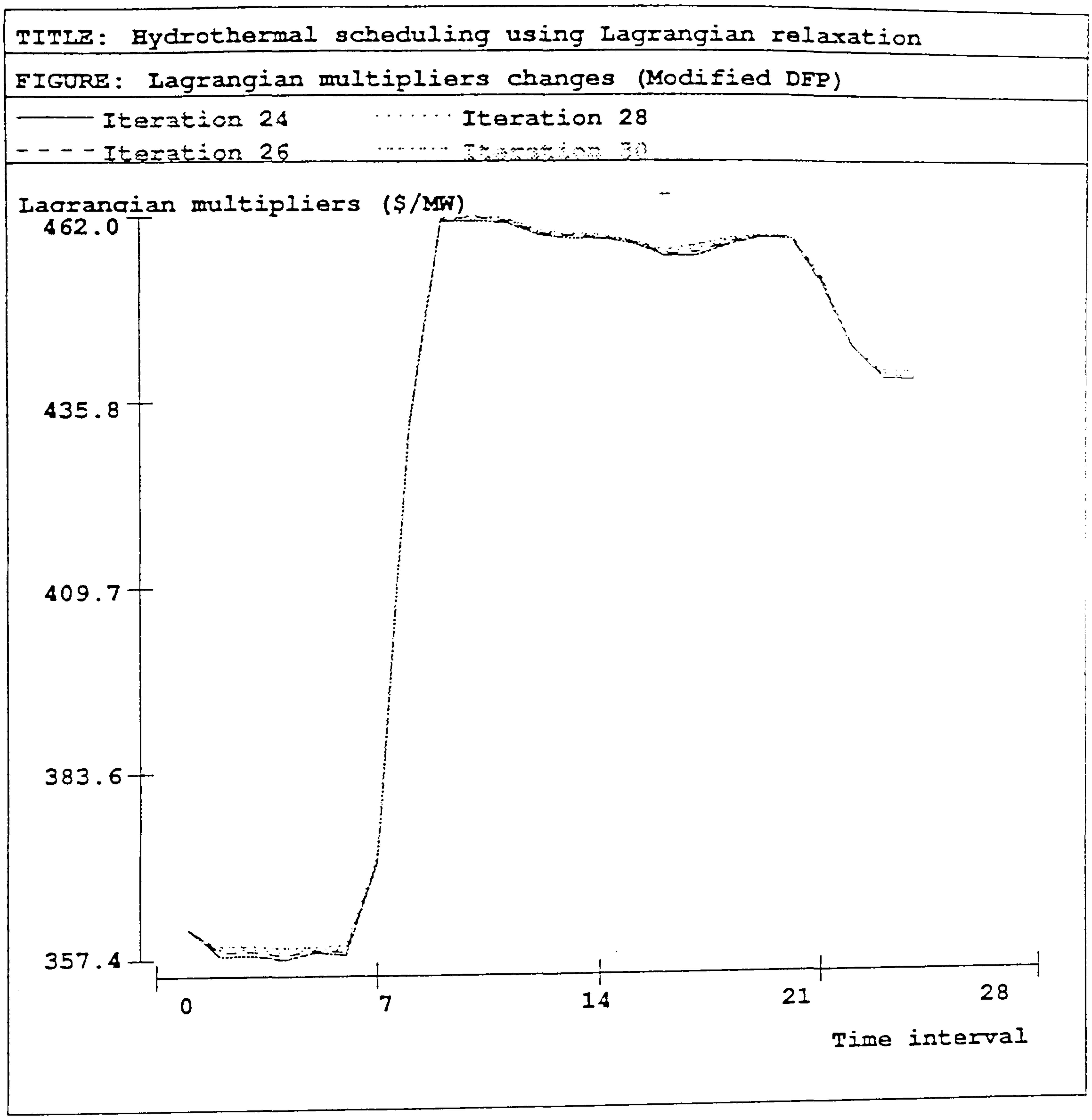

TITLE: Hydrothermal scheduling using Lagrangian relaxation
FIGURE: Lagrangian multipliers changes (Modified DFP)
Iteration 24
Iteration 28
Iteration 26
Lagrangian multipliers ($/MW)
462.0
435.8
409.7
383.6
357.4
0
7
14
21
28
Time interval

Figure 7.14

Tests also show that to ensure a near-optimal solution with small individual power balance deviations, it is not sufficient to maximize the dual function value, in addition, a smooth hydro generation schedule which results in the smallest power balance deviations must be found. For this reason, the modified DFP sub-gradient algorithm seems the most convenient and simple.

To summarize the findings for these four approaches, the modified DFP quasi-Newton approach is the most efficient and robust. The conventional DFP method guarantees a satisfactory near-optimal solution, although the convergence may sometimes be slower than the modified DFP and the conventional sub-gradient optimization algorithm. However, to achieve efficiency, the modified sub-gradient optimization algorithm seems the most attractive. In situations where insufficient information is available to define the control parameters, the modified DFP quasi-Newton method is better suited than the sub-gradient algorithm. On the other hand, the availability of a good dual optimal value estimate for the sub-gradient optimization allow this algorithm to achieve efficiency and stability.

The total production costs obtained from tests of these four approaches are shown in Table 7.9.

Table 7.9

Total production cost comparison

Algorithm	Iter. No.	Fuel ($)	Startup ($)	Shutdown ($)	Total Cost ($)
Sub-gradient	76	74936.30	188.41	0.00	75124.70
Steepest ascent	29	74912.61	188.41	0.00	75101.02
DFP convention	17	74913.25	188.41	0.00	75101.66
Modified DFP	31	74918.41	188.41	0.00	75106.81

The linear network flow algorithm (NETFLO) has been used for the hydro scheduling subproblem, which maximizes the hydro generation benefits over the scheduling period, according to specified Lagrangian multipliers that are interpreted as marginal prices. The above Figure 7.1, Figure 7.2, Figure 7.3 and Figure 7.4 have demonstrated that the hydro resource is scheduled in a load following manner. Since the load demand in a hydro dominated system is followed mainly by the hydro generation with the thermal covering the base load, the application of this Lagrangian relaxation approach is very successful.

However, as shown in Figure 7.1 to Figure 7.4, the hydro generation schedule does not follow the slight variations in the load demand and this creates a certain degree of infeasibility. This infeasibility is caused by the fact that the hydro model applied is piecewise linear and the power balance equation is considered via Lagrangian relaxation. The hydro subproblem resulting from Lagrangian relaxation does not include any constraints to ensure that the power demand will be satisfied by the hydro and thermal generation. Furthermore, a variation in marginal prices corresponding with the load variations over the scheduling period, despite being small, may cause the hydro generation schedule to move from one best operating point to another, but the total resulting generation schedule will not necessarily satisfy the load demand exactly. However, this difficulty with unbalanced load demand may be overcome by adjusting the thermal schedule finally to achieve feasibility with respect to the power balance.

Another difficulty is that instability may be caused by the linear hydro optimization model. Since the hydro subproblem does not have an explicit cost function, its optimization function is completely determined by the Lagrangian multipliers or marginal prices, which results in a linear programming problem for the hydro subproblem. Thus, the optimum of the hydro subproblem can be flat as also described in some previous work.[38.],[126.] This means that a number of hydro solutions will result in almost the same objective function value for hydro optimization. So a small change in the value of marginal prices may cause the hydro solution to change considerably even though the corresponding thermal schedule does not change much. As a result, instability or oscillation may occur in the coordination of the hydro and the thermal subsystems.

However, a close examination of the cause of the instability reveals that a new way of setting up a proper convergence criterion can avoid the problem of oscillation as described in Section 7.6.2 and this criterion has been found to work well. That is, the proper hydro schedule which minimizes the total thermal production cost while balancing the load demand with the total thermal generation may be found. The individual deviation of total generation and demand should be a minimum given that there are choices of different hydro schedules. Thus a good hydro schedule can be obtained which will result in a near-optimal solution after the adjustment of the thermal schedule to achieve feasibility.

For the first test hydrothermal system, which has a substantial percentage of hydro generation capacity, to solve the local hydraulic subproblem the CPU computational time is 1.68 seconds, and 1.02 seconds is needed to solve the local thermal subproblem. The remainder of the CPU time is for executing the coordination procedure. It is true that the CPU time needed for the hydro subproblem may be substantially reduced by improvement of the FORTRAN code, especially through storing the optimal base of the linear program during the iterations.

For a hydrothermal power system with a small share of hydroelectric capacity, the hydro generation is use mainly for peak shaving purpose. The Lagrangian multipliers will be much less sensitive to change the hydro generation schedule. The impact on the change of Lagrangian multipliers is mainly from the thermal generation and the feasibility of the generation schedule is ensured by thermal generation which is the major resource in the system. The second hydrothermal test system with only a hydro station and a small percentage of hydro generation capacity is tested for this purpose. The test was done for one day using hourly time intervals. Figure 7.15 shows the resulting hydro and thermal generation with a reference level to indicate the peak shaving affect. It is very clear that even with a small amount of hydro generation capacity, the total production cost can be reduced considerably by peak shaving operation of hydro power stations. In this test, to satisfy the same load demand over the scheduling period, the total production cost without hydro is 76394.85, with peak shaving from the small amount of hydro generation, the total production cost is reduced to 72110.75.

To test the capability of the Lagrangian relaxation technique for a large scale hydrothermal system with a high percentage of hydro generation capacity, the data from a practical large scale hydroelectric system (a Swedish state power system[38.],[90.]) are used together with the 12 thermal unit test subsystem data. This hydrothermal system contains 5 river valleys in total. The first river has 15 hydro power stations with cascaded reservoirs, the second 10, the third 11, the forth 9 and the fifth 5, so the total number of hydro power stations is 50. With the piecewise linear approximation hydro production model for each hydro power station, there are 45 hydro units to be considered on the first river, 32 units on the second, 28 units on the third, 25 units on the forth and 9 units on the fifth river, so the total number of hydro generating units is 139. The thermal subsystem contains 12 thermal generating units in total with a generation capacity of $3540MW$, the thermal system data is similar to that used in the first test system except the total generation capacity is different and the total hydro generation capacity is $7588.8MW$. The details of the hydro subsystem data can be found in Refs. [38.] and [90.]. Since the system is very large, it is very time-consuming and hence not practical to perform a line search at each iteration for the dual function maximization. In this case, only the conventional sub-gradient method and the modified DFP method are efficient. The results of using the four approaches for the Lagrangian relaxation dual maximization are presented in Table 7.10 below. The test results show that the modified DFP sub-gradient optimization algorithm is the most efficient of all the four approaches and the hydro and thermal generation schedules obtained by the conventional sub-gradient method and the modified DFP method shown in Figure 7.16 and Figure 7.17 are very satisfactory.

Table 7.10

Comparisons of algorithms

Algorithm	Iter. No	CPU time (secs.)	Dual cost	Minimum cost
Sub-gradient	10	204.14	152096.31	153414.72
Steepest ascent	6	1014.60	151485.60	153619.13
DFP	4	680.90	151007.30	153454.68
Modified DFP	8	173.04	151950.10	153408.00

7.6.3 Tests of Marginal Price Coordination

In the marginal price coordination procedure, the hydro generation and the thermal generation scheduling subproblem are *sequentially* optimized until some convergence criterion is satisfied, i.e. the thermal production cost can not be further reduced. The thermal system is acting as a *slack* system and is scheduled to satisfy the remaining load demand after the hydro scheduling, and the CCDP algorithm is used for the solution of the thermal subproblem. The hydro generation scheduling is guided by the marginal prices obtained from the thermal generation scheduling. This iterative process then continues until convergence occurs.

The computational experience has shown that both marginal price coordination and the Lagrangian relaxation algorithm converge very quickly for small hydrothermal power systems. However, the Lagrangian relaxation converges much faster than marginal price coordination for large scale hydrothermal systems since the thermal subproblem can be further decomposed in the Lagrangian relaxation procedure. The optimum obtained from the Lagrangian relaxation method is of a much better quality. The comparisons can be seen firstly from the test of the hydrothermal power system with a small amount of hydro generation for peak shaving purposes. Figure 7.18 shows the generation schedule obtained when using the marginal price coordination. It is quite clear that compared with Figure 7.15, the result obtained using the Lagrangian relaxation is better. The first test system, with 12 thermal units and two rivers system, has also been tested using the marginal price coordination. The hydro and generation schedule obtained by marginal price coordination is shown in Figure 7.19. The total thermal production cost for this schedule is \$ 75113.80, a little higher than the result of using the Lagrangian relaxation, with a total fuel cost equal to \$ 74925.39, the total startup cost equals \$ 188.41 and there was no shutdown cost. The CPU time using the marginal prices is 132.45 seconds, which is slightly longer than the CPU time of using the modified DFP method in the Lagrangian relaxation procedure.

As shown in Figure 7.20, the marginal prices obtained through marginal price coordination will be reduced considerably and also vary less during the day due to the possibilities of hydraulic generation modulation and the marginal prices are smoothed out by the availability of hydraulic power.

It has been discovered that some practical difficulties are associated with the marginal price coordination procedure, especially concerning convergence and efficiency. The quality of the results of the marginal price coordination is very dependent on the choice of the step length α $(0 < \alpha < 1)$ used in updating the marginal prices for the hydro scheduling subproblem. If α is chosen to be too small, convergence may be very slow, whereas when α is chosen to be too large, convergence may be difficult to achieve because of instability or oscillation problems. Hence, a careful choice of α is very important. From test experience, it has been found that α depends on the hydrothermal system characteristics. A perfect choice of α for a particular system is not easy to find, but a general guide can be provided from the system characteristics. Generally speaking, for a hydrothermal system with a small percentage of hydro generation capacity, α can be chosen to be larger (about 0.6), whereas when the hydro generation proportion is high, to avoid the instability or oscillation problems, the parameter α should be chosen to be a small number (about 0.1). Tests also show that the improvement in cost obtained by using the Lagrangian relaxation decomposition compared with the marginal price coordination approach is about 0.1-0.44%.

7.6.4 Conclusions of the Tests

To conclude, as far as the two decomposition-coordination procedures are concerned, the only advantage of the marginal price coordination procedure over the Lagrangian relaxation procedure is that it can be terminated at any time with a good feasible solution for the hydro and thermal generation schedules. On the other hand, the Lagrangian relaxation procedure must converge in order to provide a feasible and near-optimal solution. However, many tests have shown that since the marginal price coordination procedure is based on some heuristic rules, a near-optimal solution is not guaranteed by this approach. The quality of the results obtained from the marginal price coordination procedure is not as good as those achieved by the Lagrangian relaxation procedure where the hydro and thermal subproblems are scheduled *in parallel* with shared prices, namely, the Lagrangian multipliers. Furthermore, the marginal price coordination procedure may have the convergence problems in many practical situations. The conclusion can be drawn from the computational experience that the Lagrangian relaxation procedure is much more advantageous than the marginal price coordination procedure due to its efficiency, optimality and fast convergence.

For the Lagrangian relaxation procedure, several gradient approaches have been proposed to solve the maximization problem of the dual function. The modified DFP quasi-Newton sub-gradient optimization method has been found to be the most efficient and robust.

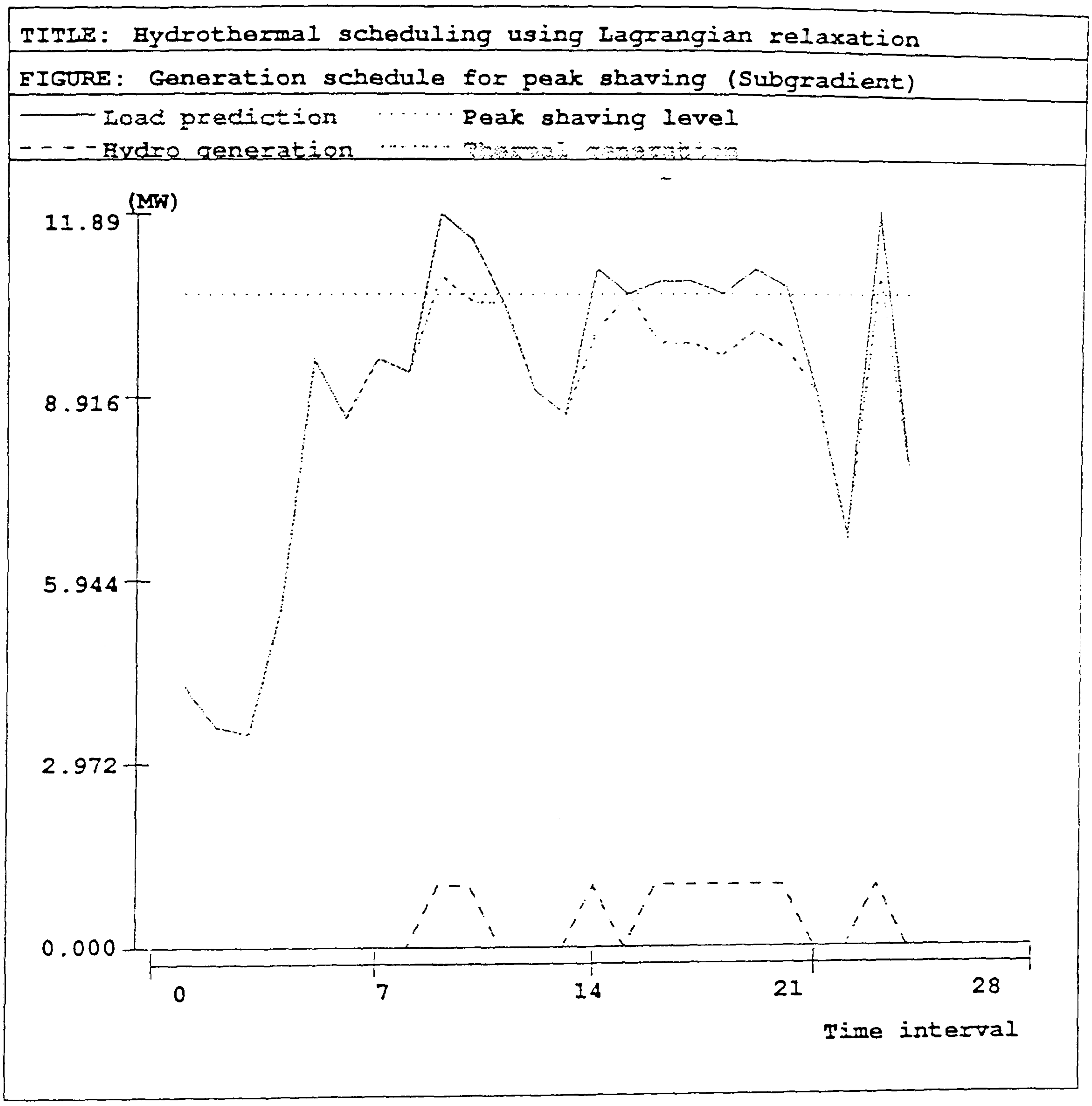

TITLE: Hydrothermal scheduling using Lagrangian relaxation
FIGURE: Generation schedule for peak shaving (Subgradient)
Load prediction
Peak shaving level
Hydro generation
(MW)
11.89
8.916
5.944
2.972
0.000
0
7
14
21
28
Time interval

Figure 7.15

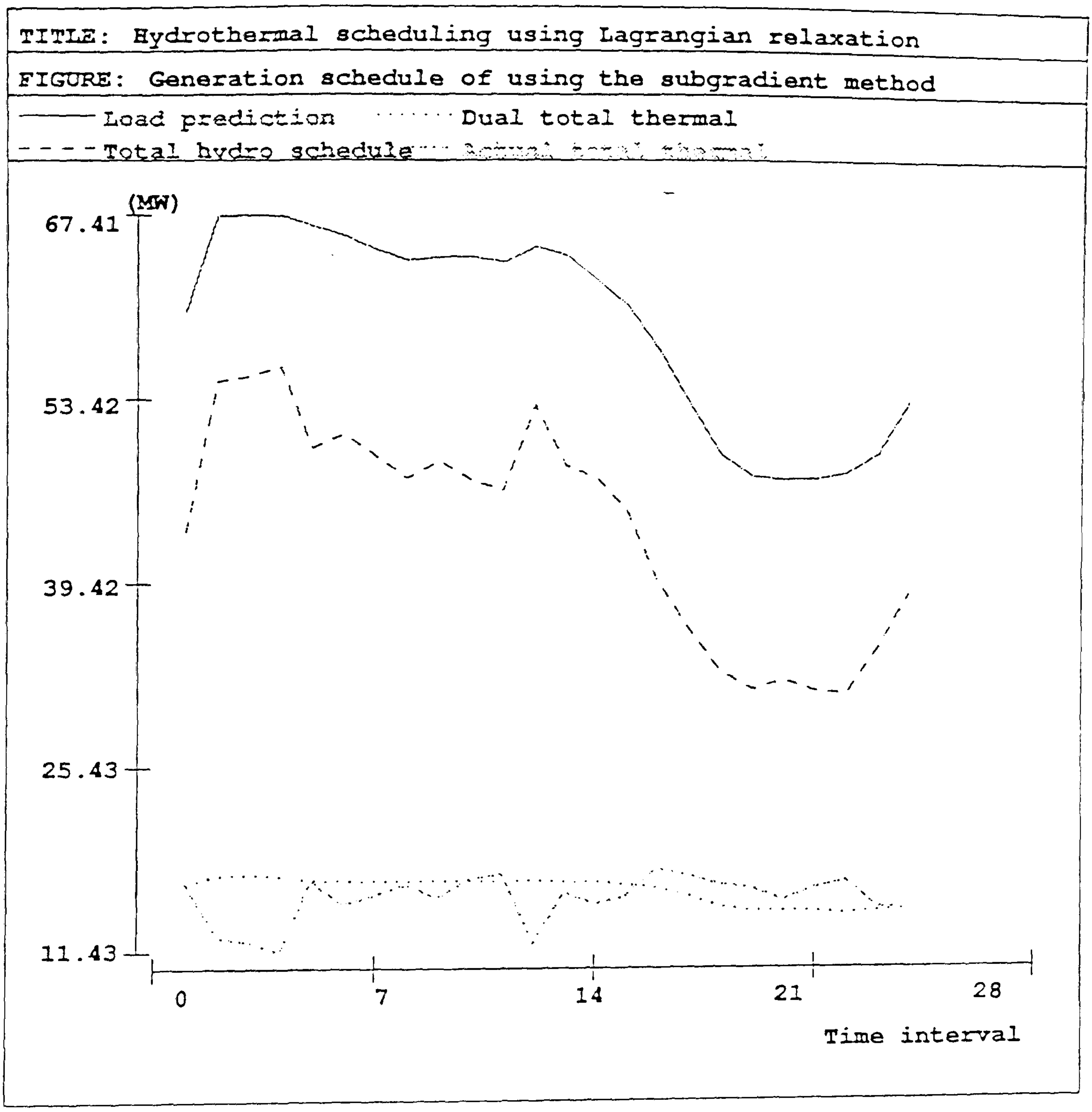

TITLE: Hydrothermal scheduling using Lagrangian relaxation
FIGURE: Generation schedule of using the subgradient method
Load prediction
Dual total thermal
Total hydro schedule
(MW)
67.41
53.42
39.42
25.43
11.43
0
7
14
21
28
Time interval

Figure 7.16

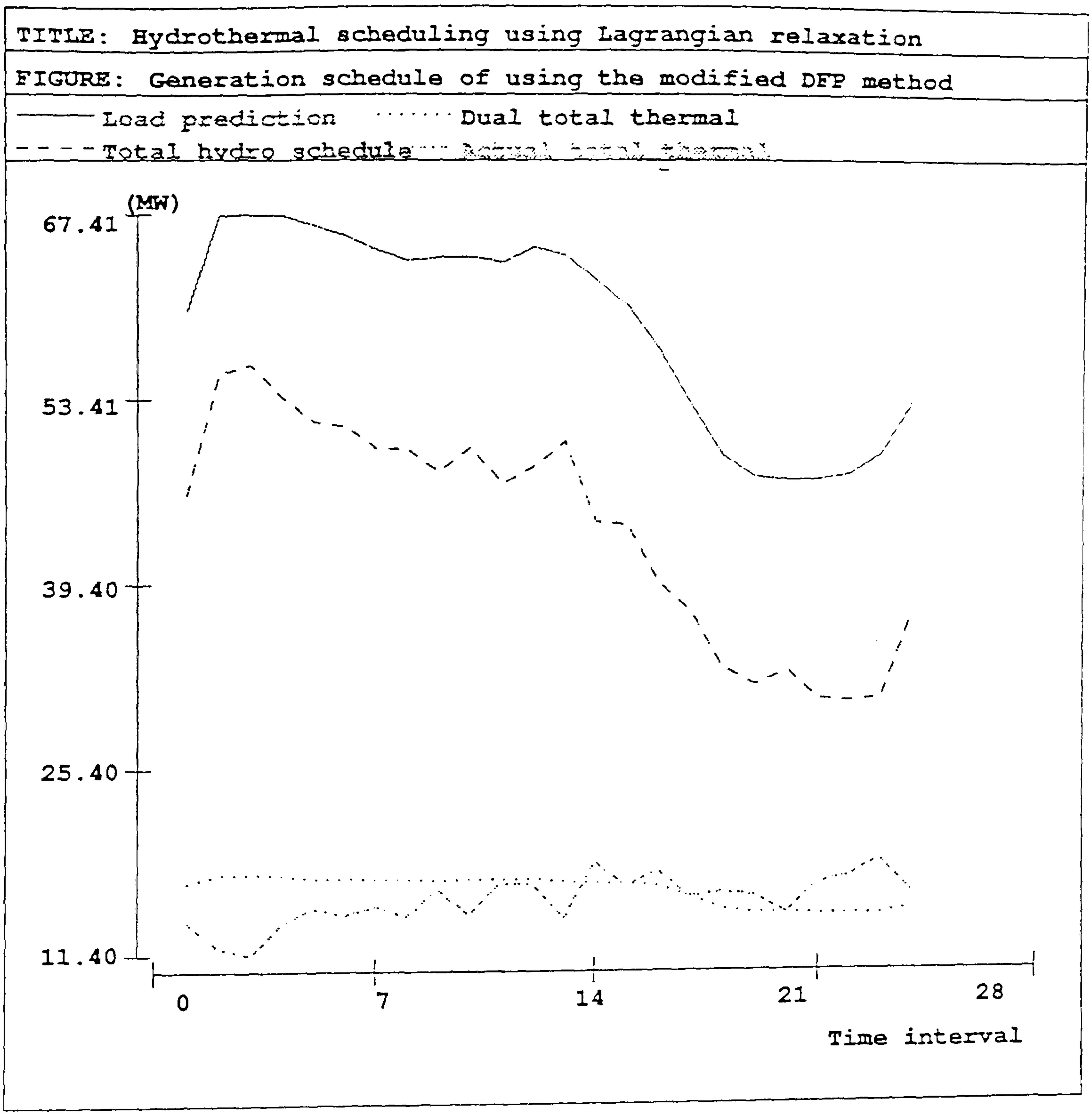

TITLE: Hydrothermal scheduling using Lagrangian relaxation
FIGURE: Generation schedule of using the modified DFP method
Load prediction
Dual total thermal
Total hydro schedule
Actual total thermal
(MW)
67.41
53.41
39.40
25.40
11.40
0
7
14
21
28
Time interval

Figure 7.17

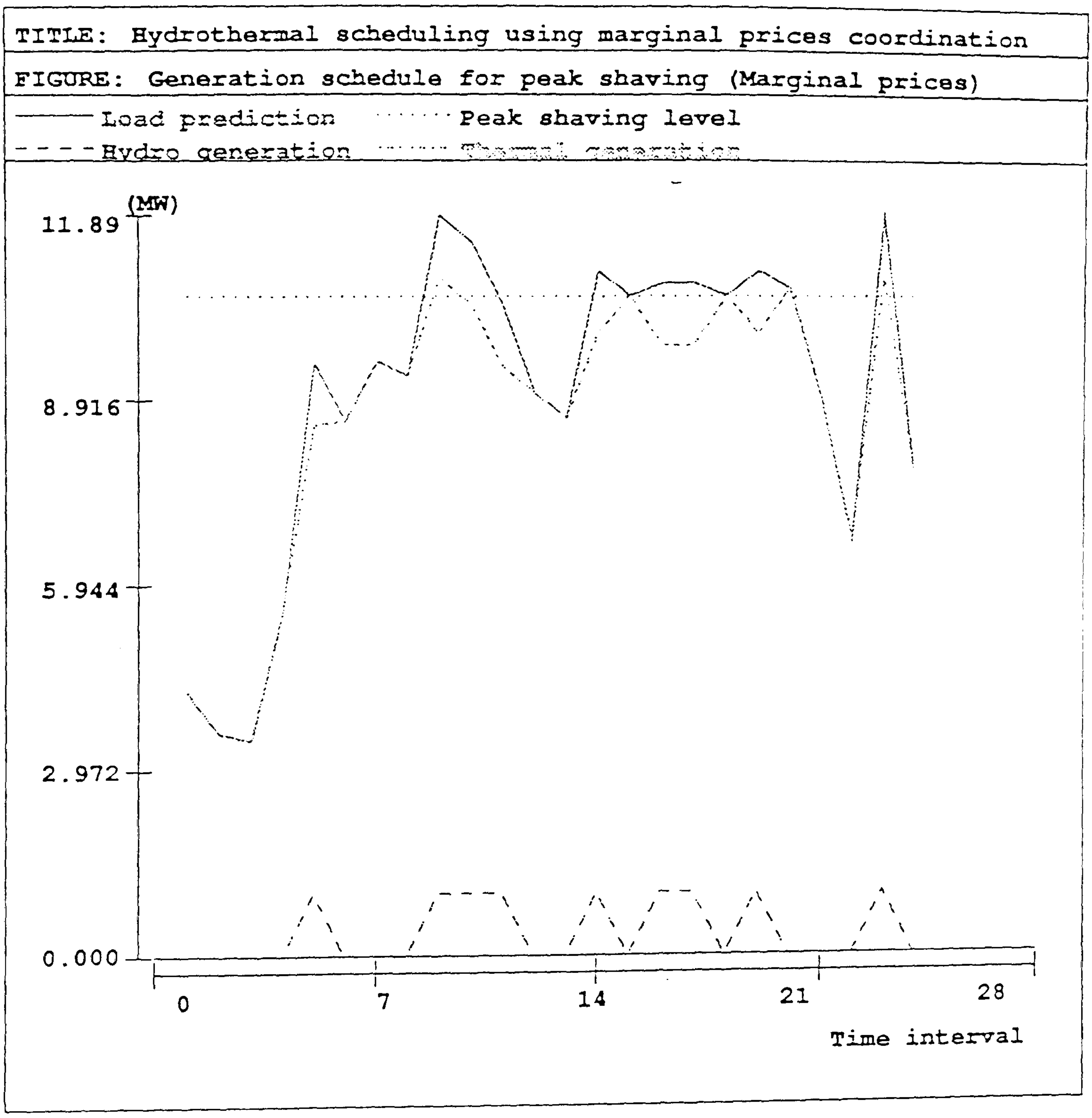

TITLE: Hydrothermal scheduling using marginal prices coordination
FIGURE: Generation schedule for peak shaving (Marginal prices)
Load prediction
Peak shaving level
Hydro generation
(MW)
11.89
8.916
5.944
2.972
0.000
0
7
14
21
28
Time interval

Figure 7.18

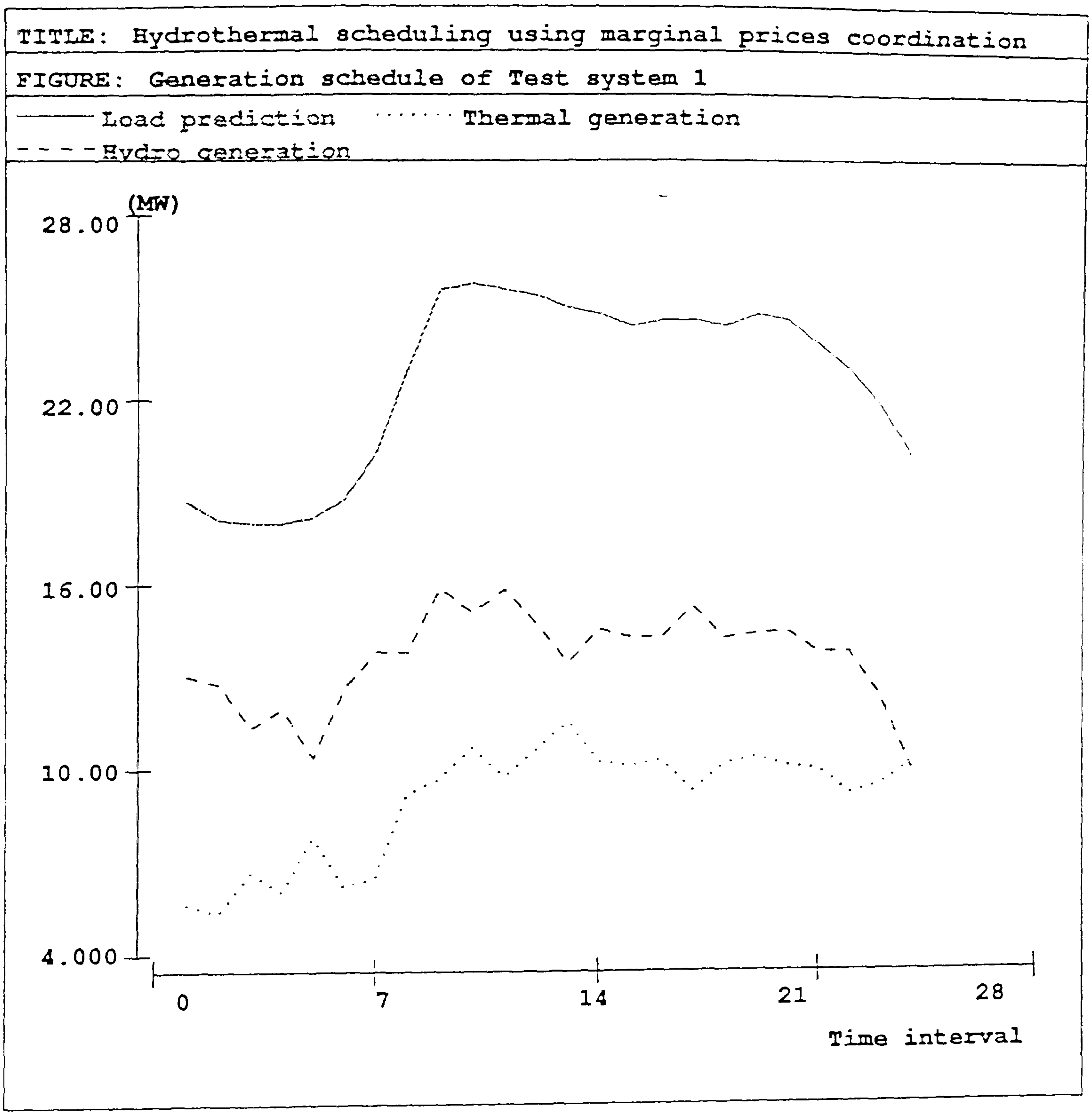
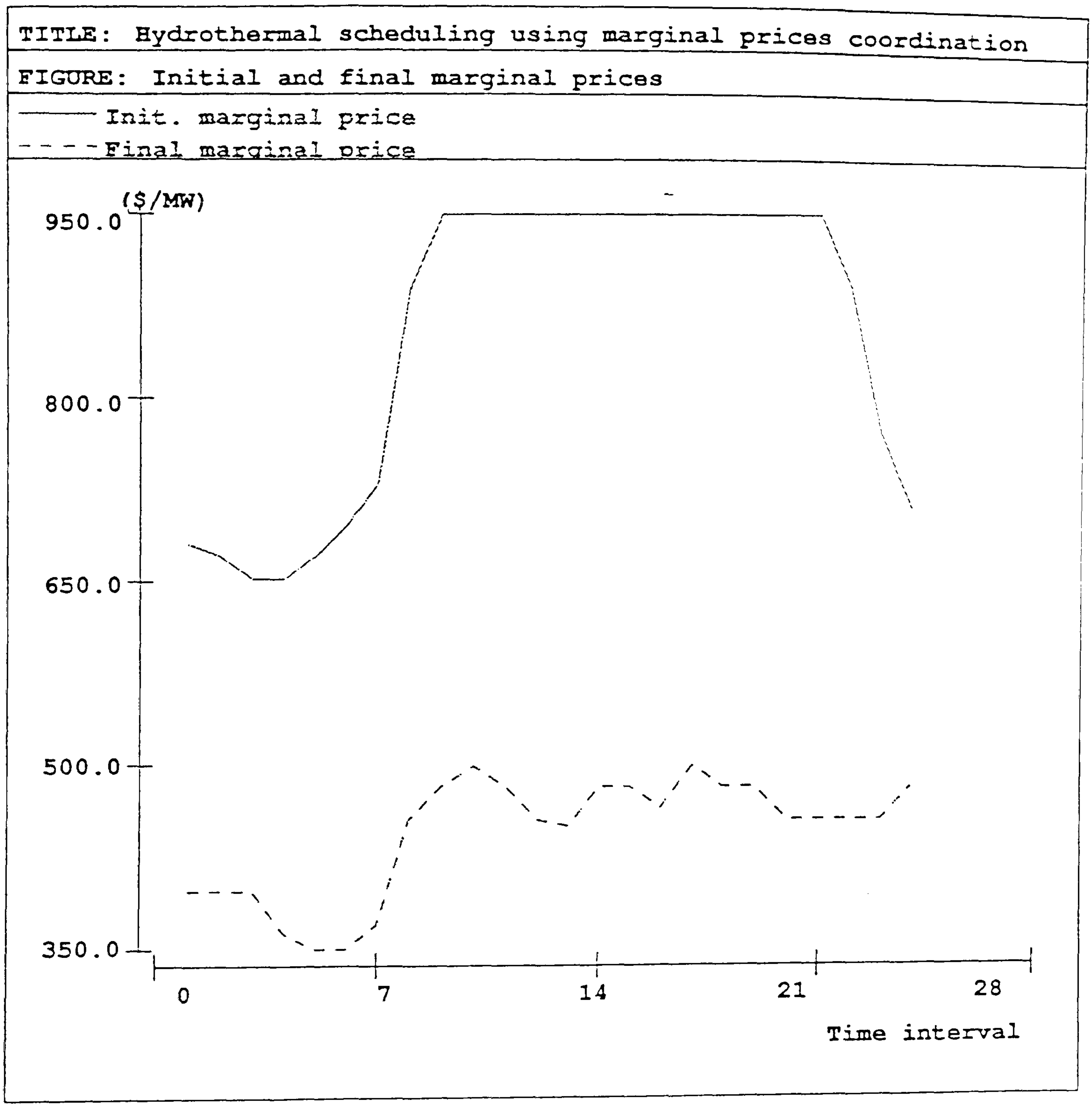

TITLE: Hydrothermal scheduling using marginal prices coordination
FIGURE: Generation schedule of Test system 1
Load prediction
Thermal generation
Hydro generation
(MW)
28.00
22.00
16.00
10.00
4.000
0
7
14
21
28
Time interval

Figure 7.19



TITLE: Hydrothermal scheduling using marginal prices coordination
FIGURE: Initial and final marginal prices
Init. marginal price
Final marginal price
($/MW)
950.0
800.0
650.0
500.0
350.0
0
7
14
21
28
Time interval

Figure 7.20

CHAPTER 8

CONCLUSIONS AND FUTURE WORK

8.1 CONCLUSIONS

This thesis has presented the results of a study of using mathematical decomposition techniques for short-term generation scheduling problems in various types of power generation systems including purely hydroelectric power systems, purely thermal generation systems and mixed hydrothermal power systems. The mathematical model developed has incorporated the principal costs and operating constraints which must be considered when scheduling the hydro and thermal generating units. Various algorithms have been developed and applied accordingly for the generation scheduling in all types of large scale, realistic sized power systems.

The mathematical decomposition techniques applied have been shown to be an efficient way of solving the large and complex models used for hydrothermal scheduling in electric utilities. The single large scheduling problem can be replaced by many smaller subproblems which can be solved independently. Near-optimal solutions can be found efficiently for the hydrothermal unit commitment and economic dispatch problems. Memory and CPU computational time have been found to be substantially less than those using the standard linear or nonlinear programs. Decomposition techniques have also been shown to offer the means for improving existing models by making them more realistic. Efficient algorithms can thus be developed separately for the solution of many subproblems. The only coordination between the subproblems is carried out by efficient procedures. Nonlinearities in the model are handled naturally and the efficient solution encourages the incorporation of more features in the scheduling subproblems.

The description of the hydrothermal scheduling problem has been quite comprehensive. The important complex operating constraints imposed on the units and the system, representing various physical, environmental, contractual and reliability conditions have been included in the modelling including min/max up/down time constraints, generating capacity limits, electricity demand constraints, cooling-time dependent startup cost, shutdown cost, hydraulic coupling constraints as well as other practical considerations. This results in a rigorous and concise mathematical formulation of the problem.

Original algorithms for short-term generation scheduling in a hydrothermal power system have been developed based on a price directive decomposition approach, the Lagrangian relaxation technique. However, since the hydrothermal generation scheduling problem is usually formulated as a large scale, mixed-integer nonlinear programming problem, due to the integer variables involved, there exists a non-convexity of the primal objective function. If the Lagrangian relaxation technique is applied, the solution of the dual problem may have a cost difference (i.e. a duality gap) with the solution value of the primal. It has been observed in the tests that some coupling constraints may not be satisfied when the dual problem converges or is near to convergence. In fact, the Lagrangian dual function can only provide a lower bound solution for the primal (original) problem. To avoid this difficulty, the simplest way is the enumeration. However, due to computational limitations, this approach is not practically acceptable. Even the branch and bound approach can only examine a few nodes due to the heavy computational burden unless a good feasible solution could be found soon with proper starting points. The importance of obtaining tight bounds and quality feasible solutions is thus crucial for the application of Lagrangian relaxation techniques.

For these reasons, an original and practical methodology is presented here for solving the large scale thermal unit commitment problems as well as short-term hydrothermal generation scheduling problems without the drawbacks of the enumeration or the branch-and-bound approach. The strength of the proposed methodology lies mainly in its ability to generate good feasible solutions from the system characteristics and the information provided by the solution of the dual.

Thus there is no need to apply the branch-and-bound approach for evaluation of integer variables. Computational experience with the proposed approach has indicated that the new technique does not increase the computational processing time since the continuous problem is an economic dispatch process which is performed after the unit commitment schedule is found in the discrete problem. Large scale thermal unit commitment problems with more than 200 units over a 24 hour time period have been easily solved in a reasonable computation time, and large scale hydrothermal scheduling problems with up to 50 hydro stations and multi-chained reservoirs in cascade can be readily solved within reasonable CPU time by applying the efficient algorithms for the hydro subproblems and dynamic programming for the thermal problems. The duality gaps between the solution of the dual and the primal have been shown to be less than 0.5% in most cases.

The proposed decomposition technique for hydrothermal scheduling problems has many special features. Decomposition makes the process of mathematical modelling of the problem efficient and modular. The structure of the model as well as the optimization techniques for the local subproblems can be easily revised independently. The technique performs much better than the dynamic programming methods in computational efficiency for medium-sized and large-sized systems. The computational time only increases linearly with respect to the number of thermal units involved and time periods and increases approximately polynomially with the number of coupled hydro units. Experience also shows that the quality of the solution can actually be improved if a large number of units are involved since the duality gap decreases relatively when the size of the problem increases.

Some non-differentiable optimization methods have been used for solving the dual problem of maximizing the dual function with respect to the Lagrangian multipliers. A sub-gradient optimization algorithm, as well as a steepest ascent method and the DFP quasi-Newton method have been applied and the performance of these algorithms has been presented. A modified DFP sub-gradient optimization method has been developed for the dual function maximization. The results have indicated that the modified DFP sub-gradient optimization algorithm is the most efficient.

To ensure near-optimal solutions, the dual function maximization should provide sufficient information to generate a near-optimal and feasible solution to the primal. This is assured by making a considerable effort at each iteration of the dual problem to find whether a feasible solution to the primal is possible at that stage or not. This ensures the feasibility of the primal solution in the continuous subproblem where the integer variables are fixed.

Furthermore, the application of network flow algorithms enables the hydro subproblem, which is one of the most difficult aspects of hydrothermal generation scheduling with multi-rivers and multi-chained reservoirs, to be solved very efficiently. This ensures the capability of the developed algorithms to deal with large scale hydrothermal scheduling problems with a high proportion of hydro generation capacity.

8.2 SUGGESTIONS FOR FUTURE WORK

Despite all the achievements mentioned above, much more profound work still needs to be carried out, which can be broadly suggested from the following aspects.

8.2.1 Modelling

Although a number of features and characteristics of real hydrothermal power systems were omitted in the presentation of the hydrothermal scheduling model here (such as the transmission network constraints), conceptually, the mathematical decomposition technique applied should have the capability of taking into account more security constraints and environmental considerations. The technique should allow a broad range of operating constraints and complexities to be incorporated in the model and handled flexibly without much theoretical and computational difficulty. It should also be easy to develop new algorithms for the solution of the corresponding local subproblems without the interference of the other constraints.

A more detailed hydrothermal generation scheduling model can be included in future research work. On the thermal unit commitment side, the model can be further extended to incorporate constraints on the maximum change in hourly power production for each generator (i.e. ramping rate constraints). This will make the dynamic programming method of thermal subproblems more complicated; however, since the thermal dynamic programming method only needs to solve one generator at a time, the computation effort should be little compared with standard dynamic programming algorithms for the solution of the thermal unit commitment problem. The crew constraints and the uncertainty of the thermal units can be included as well.

In the coordinating constraint model, the stochastic nature of electricity demand, probabilistic reserve requirement constraints and more detailed electrical transmission network representation including transmission losses as a quadratic function of generator outputs and inter-regional transmission capacity limitations, interchange contracts and other export/import constraints for large multi-area power systems can be included to generate more feasible operating schedules.

Future hydro generation scheduling models should include more detailed modelling for water time delays, the head variations and also the coupling of cascaded reservoirs and the uncertainty or the random aspects of the natural water inflows.

8.2.2 Decomposition Techniques

The mathematical decomposition techniques are flexible and modular solution methodologies. Consequently, the developments in computer science concerning parallel processing and multi-processors may also be exploited by mathematical decomposition algorithms in future work.

Research has concentrated mainly on the application of Lagrangian relaxation to the thermal unit commitment problem and the short-term hydrothermal scheduling problem. Other decomposition techniques can also be applied to compare the convergence characteristics and the performance of the algorithms.

The recent developments in mathematical decomposition and optimization techniques could also be applied to the problem considered in this project, such as the cross decomposition technique[197.],[198.], simulated annealing and the genetic optimization algorithms.[58.],[217.] It is perhaps worth noting that the cross decomposition technique can be used for the solution of large scale mixed-integer programming problems. It is a hybrid technique that unifies the primal and the dual decomposition techniques in a single framework. It is actually an improvement on both the Benders' decomposition and the Lagrangian decomposition which allows the simultaneous exploitation of the features from both the primal and the dual structures of the problem. Accordingly, the ideas of price directive decomposition and resource directive decomposition can both be applied. This can result in a drastic reduction of the computation time for some classes of large scale optimization problems.

8.2.3 Algorithms for Hydro Subproblem Scheduling

The reduced gradient network flow algorithm[18.],[40.],[116.],[167.] is known to be especially efficient for solving nonlinear network flow problems with linear constraints. This algorithm may be implemented in the hydrothermal scheduling program, as a good alternative for nonlinear network flow problems compared with the Frank-Wolfe feasible direction algorithm. This algorithm can also be combined with the Lagrangian relaxation technique, which may be very efficient for the solution of purely hydro generation scheduling problems as well.

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APPENDIX 1

GOLDEN SECTION SEARCH

The golden section search method will search on a predefined interval $[a, b]$ for the minimum point of a unimodal function $f(x)$. The process can be stated as follows:

Set

$$\alpha_1 = 0.3819660$$

$$\alpha_2 = 1 - \alpha_1$$

and

$$x_0 = a$$

$$x_1 = a + \alpha_1 * (b - a)$$

$$x_2 = a + \alpha_2 * (b - a)$$

$$x_3 = b$$

Calculate the value of function $f(x)$ at x_1 and x_2. If $f(x_1) < f(x_2)$, set the new interval as $[x_0, x_2]$; if $f(x_1) > f(x_2)$, set the new interval as $[x_1, x_3]$. The new interval length will then be $x_2 - x_0$ or $x_3 - x_1$. Using the similar calculation procedure to proceed the calculation until the interval is sufficiently small. Finally, calculate the minimum value between this sufficiently small interval and determine the value of x.

The golden section search is a simple and straightforward approach. The accuracy can of course be varied by the choice of interval value. The only drawback is that the golden section search has a linear convergence, hence may be quite slow, also the interval where the function is evaluated must be defined prior to the search.

APPENDIX 2

QUADRATIC INTERPOLATION SEARCH

The quadratic interpolation line search is one of the curve fitting approach that is used for the gradient search. The idea of the curve fitting approach is to use a few known function values, at particular points, to approximate the real function by a simple polynomial over a limited range of values. The position of the function minimum can then be approximated by the position of the polynomial function's minimum, which is much easier to evaluate.

For the quadratic interpolation approach, if the values of a function $f(x)$ at three distinct points α, β and γ are known to be f_α, f_β and f_γ respectively, then an approximation of $f(x)$ can be represented by a quadratic function as:

$$g(x) = a * x^2 + b * x + c$$

where a, b and c are determined by the following equations:

$$f_\alpha = a * \alpha^2 + b * \alpha + c$$

$$f_\beta = a * \beta^2 + b * \beta + c$$

$$f_\gamma = a * \gamma^2 + b * \gamma + c$$

Thus, the values of a, b and c can be determined as:

$$a = [(\gamma - \beta) * f_\alpha + (\alpha - \gamma) * f_\beta + (\beta - \alpha) * f_\gamma]/\Delta$$

$$b = [(\beta^2 - \gamma^2) * f_\alpha + (\gamma^2 - \alpha^2) * f_\beta + (\alpha^2 - \beta^2) * f_\gamma]/\Delta$$

$$c = [\beta * \gamma * (\gamma - \beta) * f_\alpha + \gamma * \alpha * (\alpha - \gamma) * f_\beta + \alpha * \beta * (\beta - \alpha) * f_\gamma]/\Delta$$

Where

$$\Delta = (\alpha - \beta) * (\beta - \gamma) * (\gamma - \alpha)$$

It is obvious that a minimum for $g(x)$ will be at $x = -b/(2 * a)$ when $a > 0$. Thus the position of the minimum of $f(x)$ can be approximated by x' with

$$x' = \frac{1}{2} \cdot \frac{(\beta^2 - \gamma^2) * f_\alpha + (\gamma^2 - \alpha^2) * f_\beta + (\alpha^2 - \beta^2) * f_\gamma}{(\beta - \gamma) * f_\alpha + (\gamma - \alpha) * f_\beta + (\alpha - \beta) * f_\gamma}$$

If the minimum of function $f(\mathbf{x})$ must be found at points on the line $\mathbf{x_0} + \alpha \cdot \mathbf{d}$ where $\mathbf{x_0}$ is a given point, and $\mathbf{d}$ specifies a given direction, then the values of $f(\mathbf{x} + \alpha \cdot \mathbf{d})$ on this line are functions of the one variable α as

$$g(\alpha) = f(\mathbf{x} + \alpha \cdot \mathbf{d})$$

The idea of the line search is to find α that will minimize the function value along the line direction.

The procedure of quadratic interpolation line search can be summarized as follows:

1. Calculate $f(\mathbf{x_0})$ and $f(\mathbf{x_0} + \mathbf{s})$ where s is an initial step length used.

2. If $f(\mathbf{x_0}) < f(\mathbf{x_0} + \mathbf{s})$, take the third point as $x_0 - s$ and evaluate $f(\mathbf{x_0} - \mathbf{s})$, otherwise take the third point as $x_0 + 2 * s$ and evaluate $f(\mathbf{x_0} + \mathbf{2} * \mathbf{s})$.

3. Use the three points to evaluate the quadratic approximation of minimum position x' and $f(x')$.

4. If the difference between the positions of the lowest function value and the next lowest function value is less than the required accuracy, terminate the search process with the current lowest function value and its position. Otherwise go to Step 5.

5. Discard the point with the highest function value and return to Step 3.

APPENDIX 3

NETWORK FLOW PROBLEMS AND RESULTS FROM GRAPH THEORY

A network is composed of two types of entities: arcs and nodes.

The structure of a network can be described by an $I * J$ node-arc incidence matrix **A** where an element of **A** matrix is defined as follows:

$$A_{ij} = \begin{cases} +1, & \text{if arc } j \text{ is directed away from node } i, \\ -1, & \text{if arc } j \text{ is directed towards node } i, \\ 0, & \text{otherwise.} \end{cases}$$

Here I stands for the total number of nodes and J the total number of arcs.

A characteristic of this matrix is that each column has exactly two nonzero entries, one being a $+1$ and the other -1. Any matrix having this characteristic will be called a node-arc incidence matrix.

Given that the decision variable x_j denotes the amount of flow through arc j, the unit cost for flow through arc j is denoted by c_j, the arc capacity for flow through arc j is denoted by u_j and the requirement at node i is denoted by r_i, then mathematically the linear minimal cost network flow problem may be stated as follows:

$$\min \quad \sum_{j}^{J} c_j * x_j$$

$$s.t. \quad \sum_{j}^{J} A_{ij} * x_j = r_i, \qquad i = 1, 2, ...I$$

$$0 \leq x_j \leq u_j, \qquad j = 1, 2, ...J$$

Many special cases of linear minimal cost network flow problems such as the shortest path problem, the maximal flow problem have also been studied.

The simplex on a graph algorithm is a specialization of the primal simplex method for linear programs. This specialization is very important since it completely eliminates the need for carrying and updating the basis inverse. This simplex on a graph algorithm as well as the out-of-kilter minimal cost network flow algorithm can be used to solve the linear network flow problems.

Some results from graph theory are summarized as follows:

The arcs of a path are distinct.

The arcs of a cycle are distinct.

A graph is said to be acyclic if no cycles can be formed.

A graph is said to be connected if for every pair of nodes (i, j), a path can be formed that links i to j.

A tree is a connected acyclic graph.

A tree that is a spanning subgraph of a graph is called a spanning tree for this graph.

If a graph is connected, there exists a spanning tree for this graph.

Let $\mathbf{A}$ be the node-arc incidence matrix for a proper graph that is connected and has I nodes, then the rank of $\mathbf{A}$ is $I - 1$.

Given the root node and the root arc, the corresponding graph is called a rooted graph and a rooted graph that is a tree is termed a rooted tree.

Let $\mathbf{A}$ be a node-arc incidence matrix for a proper rooted graph with the root node that is connected. The only bases from $\mathbf{A}$ will correspond to a spanning tree for the graph.

Let $\mathbf{A}$ be a node-arc incidence matrix for a proper rooted graph with the root node that is connected. Let $\mathbf{B}$ be any basis. Then $\mathbf{B}$ is triangular. The dual variables are corresponded to the node prices of the graph.